

ECONOMIC STATISTICS





NEW

SZÉCHENYI PLAN

ECONOMIC STATISTICS

Sponsored by a Grant TÁMOP-4.1.2-08/2/A/KMR-2009-0041

Course Material Developed by Department of Economics,

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The project is supported
by the European Union.

National Development Agency
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The projects have been supported
by the European Union.

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ECONOMIC STATISTICS

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June 2010

ECONOMIC STATISTICS

Week 11

AR models

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AR(p) model

- Up to now: AR(1) model
 - Slope – stationarity
- AR(p) model: autoregression of order p

$$Y_t = \alpha + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + e_t$$

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$

$$\rho = \phi_1 + \dots + \phi_p - 1$$

$\rho=0$ – unit root $-2 < \rho < 0$ - stationary

AR(p) model – modified form

$$Y_t = \alpha + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + e_t$$

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$

$$\gamma_{p-1} = -\phi_p$$

$$\gamma_{p-2} = -\phi_p - \phi_{p-1}$$

⋮

$$\gamma_1 = -\phi_p - \phi_{p-1} - \dots - \phi_2$$

$$\rho = \phi_1 + \dots + \phi_p - 1$$

Unit root

- Y has a unit root – cannot be included in the regression!

Exemption: cointegration

- Differenced value (ΔY) has to be used!
- ΔY stationary – Y difference stationary
- Y : has stochastic trend

Deterministic trend

- Example:

$$Y_t = \alpha + \phi Y_{t-1} + \delta t + e_t, \quad |\phi| < 1$$

- Y stationary – trend stationary
- Graph: similar to stochastic trend – not enough to make a decision on unit root

Example – AR(4) model

- AR(4) model with deterministic trend:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \gamma_3 \Delta Y_{t-3} + \delta t + e_t$$

- Generate differenced variables
- Differenced variables: 3 lags
- Trend: @trend
- Coefficient of $Y_{-1} = 0$?

Seasonality

- Pattern recurring at regular intervals
- Example: consumption, agricultural production, export
- Treatment: variables indicating seasonality
 - Quarterly: 3 dummies!
 - Monthly: 11 dummies!
- Or: seasonal adjustment
- KSH: seasonally adjusted time series

Specification choice

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + \delta t + e_t$$

- Maximal lag length (p_{\max})
- Estimate AR(p_{\max}) model with or without deterministic trend (according to the dependent variable, based on assumption!)
- Test $\Gamma_{p_{\max}-1} = 0$ (t-test) – if satisfied: decrease lag length by one

Unit root test

Testing $\rho=0$: usual t-test cannot be used!

Dickey–Fuller-test: use t-statistic, but critical values are corrected

Problem: "weak" test – can find unit root even if it is not present

- Example: trend stationary time series, structural break

Dickey–Fuller-test

- Question: include trend?
- Null hypothesis: unit root
- Large p-value: has unit root, not stationary

Unit root test – example

Monthly export data

- Seasonally adjusted
- Trend

Null Hypothesis: EXPORT_SA has a unit root

Exogenous: Constant, Linear Trend

Lag Length: 3 (Automatic based on SIC, MAXLAG=13)

| | t-Statistic | Prob. |
|------------------------------------|-------------|--------|
| Augmented Dickey–Fuller-test stat. | -2,1186 | 0,5310 |

| | |
|--------------------------------|---------|
| Test critical values: 1% level | -4,0180 |
| 5% level | -3,4389 |
| 10% level | -3,1438 |

Summary

- AR(p) model, modified form
- Unit root in AR(p) models
- Trend stationarity
- Seasonality
- Dickey–Fuller-test

AR models

Seminar 11

AR(p) model

- AR(p) model: autoregression of order p

$$Y_t = \alpha + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + e_t$$

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$

$$\rho = \phi_1 + \dots + \phi_p - 1$$

$\rho=0$ – unit root $-2 < \rho < 0$ - stationary

Unit root

- Y has a unit root – cannot be included in the regression!
Exemption: cointegration
- Differenced value (ΔY) has to be used!
- ΔY stationary – Y difference stationary
- Y : has stochastic trend

Example – monthly export

MNB data (m EUR)

- Estimation of AR(4) model with deterministic trend:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \gamma_3 \Delta Y_{t-3} + \delta t + e_t$$

- Generate differenced variables
- Differenced variables: 3 lags
- Trend: @trend
- Coefficient of $Y_{-1} = 0$?

Seasonality

- Pattern recurring at regular intervals
- Treatment: variables indicating seasonality
 - Quarterly: 3 dummies
 - Monthly: 11 dummies

Seasonality – example

- Monthly export data – 11 seasonal dummies
- @seas(1) @seas(2) ...
- 12 seasonal dummies: multicollinearity – EViews error message
- EViews: Procs/Seasonal adjustment

Specification choice

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \dots + \gamma_{p-1} \Delta Y_{t-p+1} + \delta t + e_t$$

- Maximal lag length (p_{\max})
- Estimate AR(p_{\max}) model with or without deterministic trend
- Test $\Gamma_{p_{\max}-1}=0$ (t-test) – if satisfied: decrease lag length by one
- Test the significance of trend after lag length selection
- Example: AR(p) model for first differenced log export time series (use seasonally adjusted data!)

Dickey–Fuller-test

- Test unit root
- View/Unit root test
- Option: automatic lag length selection
- Question: include trend?

- Null hypothesis: unit root
- Large p-value: has unit root, not stationary

Unit root test

- Monthly export data (MNB)
 - Seasonally adjusted
 - Trend?
 - Interpret output
 - Is the differenced variable stationary?
- Quarterly public debt data (MNB)
 - Trend?