ECONOMICS I.

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week 9

Strategic behavior

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Game theory

Basic notions of game theory

Game theory (theory of games) deals with the general analysis of strategic interactions.

Representation of games

- Who are the players? (set of players): $\{1, \ldots, n\}$
- What are the alternatives? (set of moves (or strategies) available to all players)

$$S_i = \left\{s_i^1, \ldots, s_i^m\right\} \ (i = 1, \ldots, n)$$

- What is the payoff? (specification of payoffs for each combination of strategies) (definition of profit and utility curves)

$$f_i : S_1 \times S_2 \times \ldots \times S_n \to \mathbb{R} \, (i = 1, \ldots, n)$$

- How does the game proceeds? (definition of the scenario)

Two more assumption:

- Players maximize their payoff-functions (rationality assumption)
- Everything given is common knowledge

E.g. A 'hides' a coin in his right or left hand and B tries to guess the place of the coin. If he guesses right, then A pays B 100 HUF, if he guesses wrong then B pays A 50 HUF.

- Players: A, B
- Strategies:
  - Strategies of A:
    - s_{A1}: hides in the left hand (hr)
    - s_{A2}: hides in the right hand (hr)
  - Strategies of B:
    - s_B1: guesses left (gl)
    - s_{B2}: guesses rightl (gr)

$$S = \{(hl, gl), (hl, gr), (hr, gl), (hr, gr)\}$$

- Payoffs:

$$f_A(hl, gl) = -100, f_A(hr, gr) = -100$$
$$f_A(hl, gr) = +50, f_A(hr, gl) = +50$$
$$f_B(hl, gl) = +100, f_B(hr, gr) = +100$$
$$f_B(hl, gr) = -50, f_B(hr, gl) = -50$$

Types of games:

- Cooperative
- Non-cooperative
- Perfect information
- Total information
- Zero-sum
- Non-zero-sum

Representation of the game (payoff matrix and trees):

- Normal form
- Extensive form

	left	righ
up	a,a	c,b
down	b,c	d,d

E.g. Prisoners' dilemma:

- Players: {1st prisoner; 2nd prisoner}={1;2}
- Strategies (strategy sets): $S_1 = \{confess,\ don't\ confess\}\,; S_2 = \{confess,\ don't\ confess\}$
- Payoffs (the first argument is the strategy of the 1st prisoner, negative payoff=loss):
  - f_1(confess, confess) $= -5$; f_2(confess, confess) $= -5$
  - f_1(confess, don't confess) $= 0$; f_2(confess, don't confess) $= -10$
  - f_1(don't confess, confess) $= -10$; f_2(don't confess, confess) $= 0$
  - f_1(don't confess, don't confess) $= -2$; f_2(don't confess, don't confess) $= -2$
- Rules of the game: prisoners are questioned isolated from one another, etc.
- Payoff matrix:

	confess	don't confess
confess	(–5;–5)	(0;–10)
don't confess	(–10;0)	(–2;–2)

1. Definition. *Equilibrium based on dominant strategies: Decisions of players are best answers to any decision of the other player.*

$$\Pi(s_i^*, s_j^*) \geq \Pi(s_i, s_j)\,(i = 1, \ldots, n)$$

Interactive elimination of dominated strategies:

(2;0)	(1;1)	(4;2)
(1;4)	(5;2)	(2;3)
(0;3)	(3;2)	(3;4)

(2;0)	(4;2)
(1;4)	(2;3)
(0;3)	(3;4)

(2;0)	(4;2)

Example: Fight of genders

	opera	football match
opera	(2;1)	(0;0)
football match	(0;0)	(1;2)

1. Note. *Equilibrium based on dominant strategies does not always exist.*

2. Definition. *Nash equilibrium based on pure strategies: Decisions of players are mutually best answers, i.e. decisions of each player are best answers to decisions of other players.*

$$\Pi(s_i^*, s_j^*) \geq \Pi(s_i, s_j^*)\,(i = 1, \ldots, n)$$

1. Consequence. *In case of Nash equilibrium neither party would benefit from a unilateral change of move.*

Example continued: Fight of genders (after 30 years of marriage)

	opera	football match
opera	(2;0)	(0;2)
football match	(0;1)	(1;0)

2. Note. *Nash equilibrium based on dominant strategies does not always exist.*

Zero-sum game: land or sea?

		Defender's choice of strategy	
		land	sea
Attacker's choice of strategy	land	–10,+10	+25,–25
	sea	+25,–25	–10,+10

Mutuality of interests: the coordination game

		Choice of B	
		righ	left
Choice of A	right	+15,+15	–100,–100
	left	–100,–100	+10,+10

The prisoners' dilemma: two versions

		Months of imprisonment	
		Don't confess	Confess
Panel (a)	Don't confess	–1,–1	–36,0
	Confess	0,–36	–24,–24
		Rank-ordered payoffs	
		Small output	Large output
Panel (b)	Small output	3,3	1,4
	Large output	4,1	2,2

Farm drainage as a public good: a prisoners' dilemma

	Pump	Don't pump
Pump	2,2	–3,5
Don't pump	5,–3	0,0

Farm drainage as a multiperson prisoners' dilemma

		Number of other farmers pumping				
		0	1	2	3	4
Farmer A's	Pump	–3	2	7	12	17
choices	Don't pump	0	5	10	15	20

3. Definition. *Mixed expansion of the game: players choose a probability distribution instead of a specific strategy.*

	opera (q)	football match ($1-q$)
opera (p)	(2;0)	(0;2)
football match ($1-p$)	(0;1)	(1;0)

Methods of determining it:

- Solution of linear programming task
- Mini-Max principle
- Calculation of multiple-variable extreme values

$$2pq + 0p(1-q) + 0(1-p)q + 1(1-p)(1-q) \to \max_p$$

$$2pq + 0p(1-q) + 0(1-p)q + 1(1-p)(1-q) \to \max_q$$

4. Definition. *A game is finite if the number of participants and the strategy sets are finite.*

1. Statement. *Nash-theorem Every finite game has a Nash equilibrium regarding its mixed expansion.*

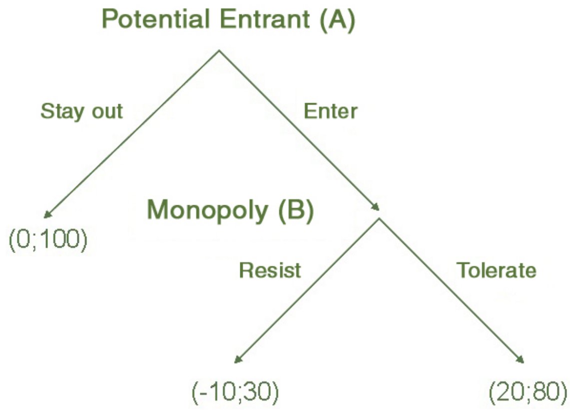
2. Consequence. *In the simultaneous-play protocol, a dominant strategy - one that is better in the strong or weak sense no matter what the opponent does - should be chosen if available. A dominant equilibrium exists if even only player has such a strategy available (since then the other player can predict what his opponent will do). In the absence of a dominant equilibrium, the Nash equilibrium concept applies. At a Nash equilibrium, no player has an incentive to alter his or her decision, given the other's choice. There may be one, several, or no Nash equilibria in pure strategies. If mixed strategies - probabilistic mixtures of pure strategies aimed at keeping the opponent guessing - are also permitted, a Nash equilibrium always exists.*

Sequential and repeated games

5. Definition. *Sequential game: later players have some knowledge about earlier actions. This type of games should be represented in extensive form.*

The entry-deterrence game

		Monopolist	
		resist	tolerate
Potential	enter	–10,30	20,80
entrant	stay out	0,100	0,100



Sub-games: The total game, choice of the monopoly

6. Definition. *Subgame-perfect equilibrium: Equilibrium in all sub-games of the sequential game.*

Solution method: backward induction

3. Consequence. *In the sequential-play protocol, the perfect equilibrium concept has each player make a rational (payoff-maximizing) choice on the assumption that the opponent will do the same when it comes to his or her turn. A perfect equilibrium always exists, though it may not be unique. In the simultaneous-play protocol, a dominant strategy - one that is better in the strong or weak sense no matter what the opponent does - should be chosen if available.*

7. Definition. *Repeated game: the game is played many times consecutively so previous outcomes are known before the next game.*

8. Definition. *Tit for tat strategy: cooperate in the first play, after that play always the same as the other player played in the previous play.*

2. Statement. *Selten's theorem: If a game with a unique equilibrium is played finitely many times its solution is that equilibrium played each and every time. Finitely repeated play of a unique Nash equilibrium is the equilibrium of the repeated game.*

4. Consequence. *Equilibrium qualities of games repeated finitely and (potentially) infinitely are substantially different.*