ECONOMETRICS

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Course Material Developed by Department of Economics,

Faculty of Social Sciences, Eötvös Loránd University Budapest (ELTE)

Department of Economics, Eötvös Loránd University Budapest

Institute of Economics, Hungarian Academy of Sciences

Balassi Kiadó, Budapest

Authors: Péter Elek, Anikó Bíró

Supervised by Péter Elek

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Week 11

Nonstationary time series

Content

Testing nonstationarity: unit root tests

Trends and seasonal components

Material: M 613–617., 301–306., 597–602.

Example: estimating the parameter of a random walk the limit distribution of the t-statistic is not t-distribution!

$$\Delta X_t = c + 0 \cdot X_{t-1} + \varepsilon_t$$

Dependent Variable: D(RW)
Method: Least Squares
Date: 02/22/09 Time: 18:23
Sample: 2 1000
Included observations: 999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.016216	0.032184	0.503864	0.6145
RW(-1)	-0.006548	0.004169	-1.570679	0.1166

R-squared	0.002468	Mean dependent var	0.016010
Adjusted R-squared	0.001468	S.D. dependent var	1.017977
S.E. of regression	1.017230	Akaike info criterion	2.874043
Sum squared resid	1031.652	Schwarz criterion	2.883866
Log likelihood	-1433.584	F-statistic	2.467031
Durbin-Watson stat	1.990226	Prob(F-statistic)	0.116575

Dependent Variable: D(RW)
Method: Least Squares
Date: 02/22/09 Time: 18:24
Sample: 2 1000
Included observations: 999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.072756	0.059453	-1.223754	0.2213
RW(-1)	-0.003267	0.002384	-1.370530	0.1708

R-squared	0.001880	Mean dependent var	-0.004060
Adjusted R-squared	0.000879	S.D. dependent var	1.011009
S.E. of regression	1.010564	Akaike info criterion	2.860894
Sum squared resid	1018.176	Schwarz criterion	2.870718
Log likelihood	-1427.017	F-statistic	1.878353
Durbin-Watson stat	2.052062	Prob(F-statistic)	0.170830

Testing nonstationary: Dickey-Fuller test

$Y_t = \alpha Y_{t-1} + \varepsilon_t$

Equivalent: $\Delta Y_t = (\alpha - 1)Y_{t-1} + \varepsilon_t$

H_0: $\alpha = 1$, H_1: $\alpha < 1$

Test: the usual t-statistic,

Under H_0: the so-called Dickey–Fuller-distribution

Asymptotic critical values:
5%: –1,95 (t-critical value: –1,65)
1%: –2,58 (t-critical value: –2,33)

Versions of Dickey–Fuller-test

AR(1) + constant

$Y_t = c + \alpha Y_{t-1} + \varepsilon_t$

Asympt. critical value: –2,86 (5%), –3,43 (1%)

AR(1) + constant + trend

$Y_t = c + \delta_t + \alpha Y_{t-1} + \varepsilon_t$

Asympt. critical value: –3,41 (5%), –3,96 (1%)

Augmented DF test:

$Y_t = c + \delta_t + \alpha Y_{t-1} + \theta_1 \cdot \Delta Y_{t-1} + \theta_2 \cdot \Delta Y_{t-2} + \ldots + \theta_k \cdot \Delta Y_{t-k} + \varepsilon_t$

There are other stationarity tests as well
(eg. KPSS)

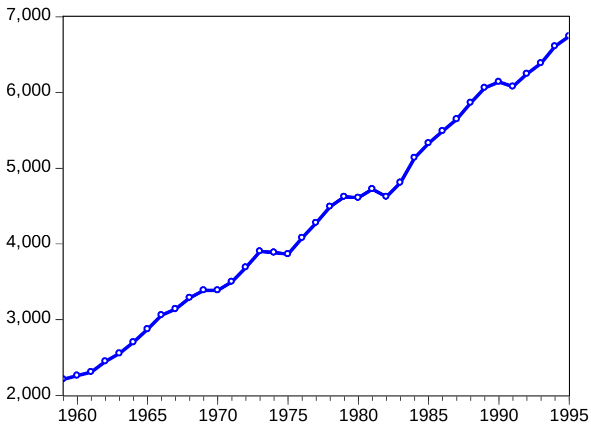
Example: is USA GDP difference or trend stationary?

Are the effects of shocks persistent or temporary?

Supply side: random walk (technological shocks)

Demand side: trend stationary

Which shocks dominate?



Unit root test for the GDP series

Null Hypothesis: LOGGDP has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Fixed)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.420733	0.3632
Test critical values:	1% level	-4.243644	
	5% level	-3.544284	
	10% level	-3.204699	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOGGDP)
Method: Least Squares
Date: 02/23/09 Time: 17:21
Sample (adjusted): 1961 1995
Included observations: 35 after adjustments

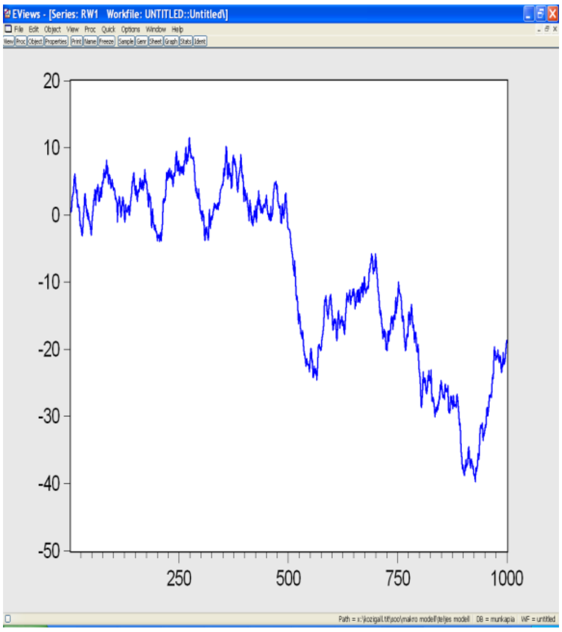
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOGGDP(-1)	-0.209621	0.086594	-2.420733	0.0215
D(LOGGDP(-1))	0.263751	0.164739	1.601017	0.1195
C	1.656791	0.669068	2.476268	0.0189
@TREND(1959)	0.005870	0.002696	2.177186	0.0372

R-squared	0.268005	Mean dependent var	0.031211
Adjusted R-squared	0.197167	S.D. dependent var	0.022448
S.E. of regression	0.020114	Akaike info criterion	-4.867591
Sum squared resid	0.012542	Schwarz criterion	-4.689837
Log likelihood	89.18285	Hannan-Quinn criter.	-4.806230
F-statistic	3.783335	Durbin-Watson stat	1.980899
Prob(F-statistic)	0.020148		

The hypothesis of unit root cannot be rejected.

The conclusions are similar on larger samples, but final decision in the debate cannot be made.

Why should we bother about stationarity? Spurious trend in time series

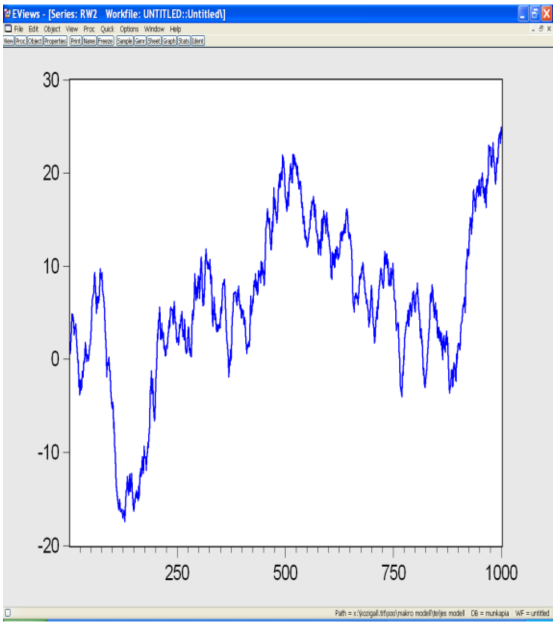


Dependent Variable: RW1
Method: Least Squares
Date: 11/03/09 Time: 17:59
Sample: 2 1000
Included observations: 999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	11.11926	0.417487	26.63378	0.0000
@TREND	-0.039853	0.000723	-55.09992	0.0000

R-squared	0.752790	Mean dependent var	-8.807344
Adjusted R-squared	0.752542	S.D. dependent var	13.25314
S.E. of regression	6.592797	Akaike info criterion	6.611833
Sum squared resid	43334.58	Schwarz criterion	6.621656
Log likelihood	-3300.610	Hannan-Quinn criter.	6.615566
F-statistic	3036.001	Durbin-Watson stat	0.022017
Prob(F-statistic)	0.000000		

Spurious trend in time series II



Dependent Variable: RW2
Method: Least Squares
Date: 11/03/09 Time: 18:03
Sample: 2 1000
Included observations: 999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.463948	0.481105	-3.042886	0.0024
@TREND	0.015076	0.000834	18.08697	0.0000

R-squared	0.247058	Mean dependent var	6.073859
Adjusted R-squared	0.246302	S.D. dependent var	8.751208
S.E. of regression	7.597428	Akaike info criterion	6.895496
Sum squared resid	57547.74	Schwarz criterion	6.905320
Log likelihood	-3442.301	Hannan-Quinn criter.	6.899230
F-statistic	327.1385	Durbin-Watson stat	0.017946
Prob(F-statistic)	0.000000		

Spurious regression in time series

Two independent random walks:

$X_t = X_{t-1} + \varepsilon_{1t}$

$Y_t = Y_{t-1} + \varepsilon_{2t}$

Regression: $Y_t = c + \beta X_t + u_t$

$\beta = 0$ because of independence, but the t-test is significant!

The t-statistic does not have a limit distribution

Reason: u_t is nonstationary!

Dependent Variable: RW2
Method: Least Squares
Date: 02/22/09 Time: 18:03
Sample: 2 5000
Included observations: 4999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-24.18274	0.901345	-26.82963	0.0000
RW1	0.784565	0.024531	31.98294	0.0000

R-squared	0.169921	Mean dependent var	-47.98849
Adjusted R-squared	0.169755	S.D. dependent var	39.44434
S.E. of regression	35.94081	Akaike info criterion	10.00202
Sum squared resid	6454835.	Schwarz criterion	10.00463
Log likelihood	-24998.06	F-statistic	1022.909
Durbin-Watson stat	0.001251	Prob(F-statistic)	0.000000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	15.65639	0.434744	36.01291	0.0000
RW1	-0.362172	0.007234	-50.06680	0.0000

R-squared	0.334060	Mean dependent var	-2.177867
Adjusted R-squared	0.333927	S.D. dependent var	21.59167
S.E. of regression	17.62167	Akaike info criterion	8.576536
Sum squared resid	1551685.	Schwarz criterion	8.579143
Log likelihood	-21435.05	F-statistic	2506.684
Durbin-Watson stat	0.003573	Prob(F-statistic)	0.000000

Fitting trends in the trend stationary and the difference stationary case

Simplest trend stationary case
$y_t = \beta_0 + \beta_{1t} + u_t,\ u_t \sim IN$

Trend fitting by OLS is consistent and in this case efficient (becaues of the independence of the error term)

Differentiation also yields consistency, but the independence of the error term does not hold any more: $\Delta y_t = \beta_1 + u_t - u_{t-1}$

Difference stationary case
$y_t = y_{t-1} + \beta_1 + u_t,\ u_t \sim IN$

Trend fitting by OLS is inconsistent!

Differentiation yields a consistent (and in this case efficient) estimate:
$\Delta y_t = \beta_1 + u_t$

Hodrick–Prescott filter

y_t: original time series
s_t: filtered time series

$$\min_{s_1, s_2, \ldots, s_T} \left\{ \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} ((s_{t+1} - s_t) - (s_t - s_{t-1}))^2 \right\}$$

If $\lambda = 0$ then $y_t = s_t$ for all t
If $\lambda = \infty$ then the linear trend is obtained.
A possible choice for λ: $\lambda = 1600^*(i/4)^2$, where i is the frequency

Annual data:	$\lambda =$	100
Quarterly data:	$\lambda =$	1600
Monthly data:	$\lambda =$	14400

Seasonality

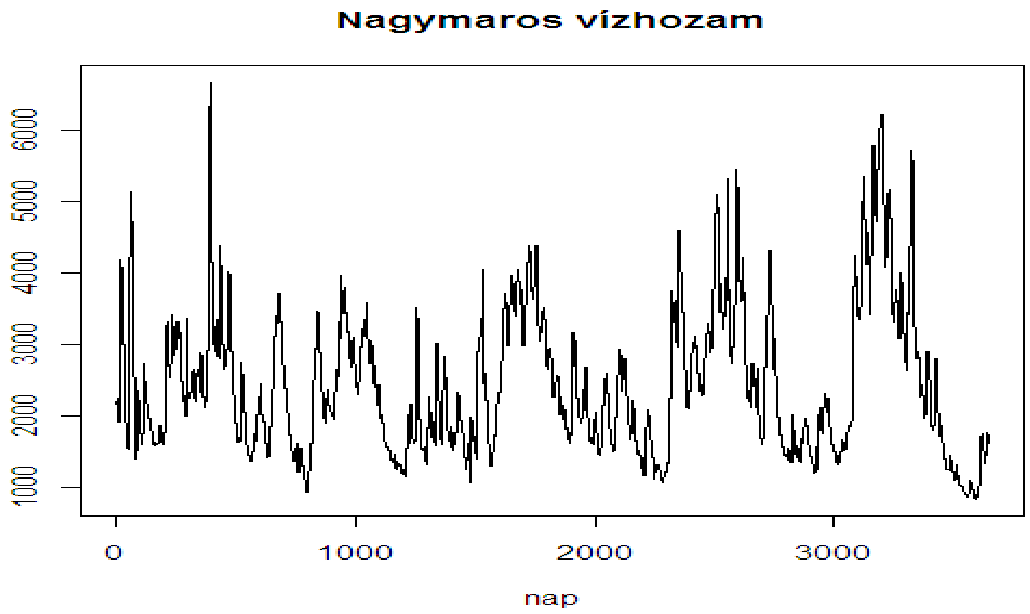
Two types of seasonality

deterministic (can be filtered out by using dummy variables)

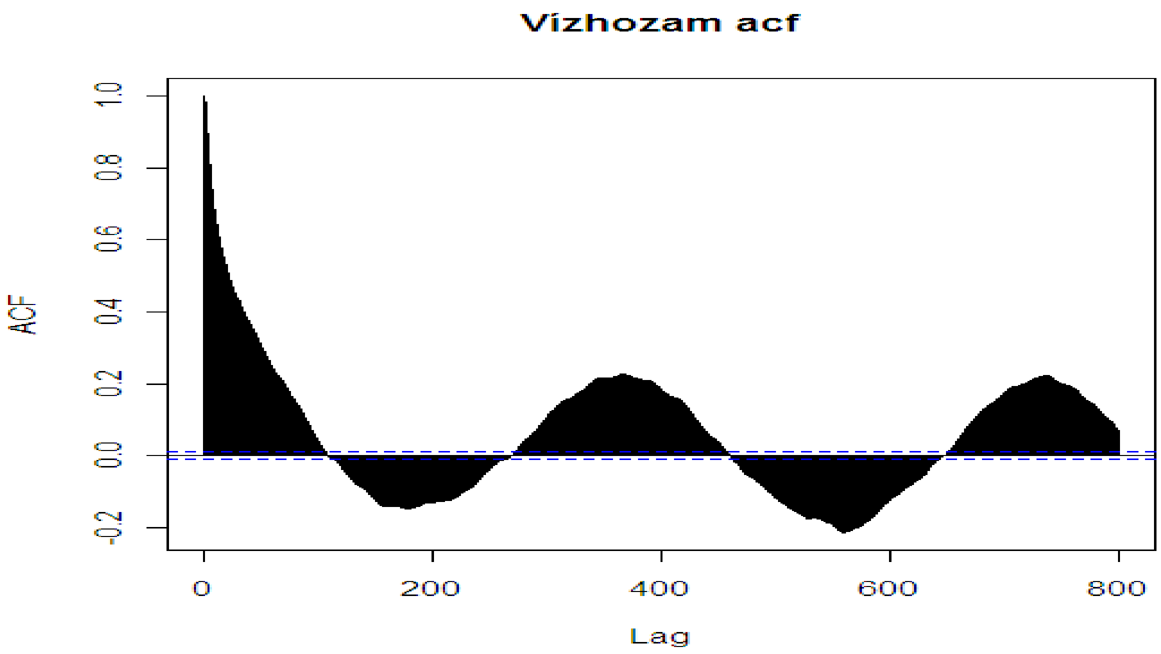
stochastic (can be filtered out by taking seasonal differences)

In practice: more difficult filtering methods
(e.g. TRAMO-SEATS)

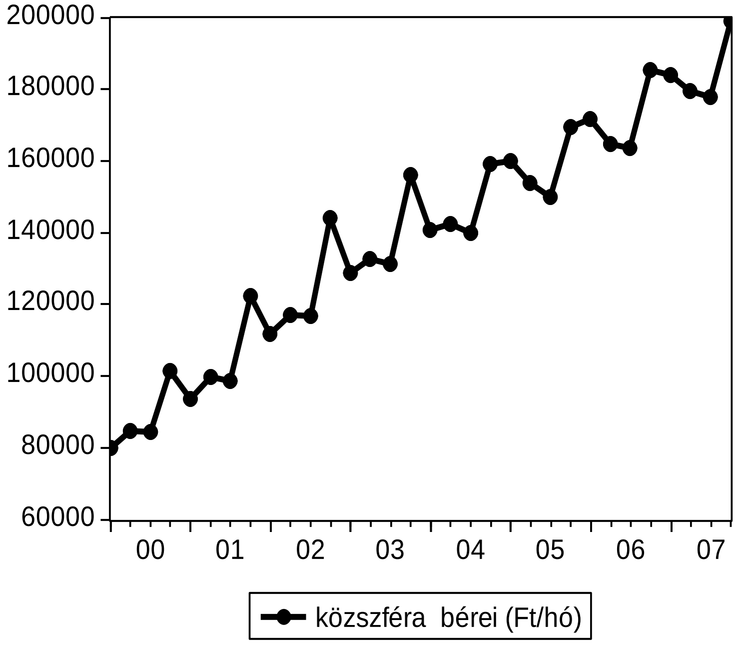
Example: daily water discharge



ACF



Example: wages



Example: seasonality in the quarterly growth rate of private sector wages

Dependent Variable: DLOG(GWAGE_PR_NSA)
Method: Least Squares
Date: 02/24/09 Time: 14:48
Sample (adjusted): 1998Q2 2008Q3
Included observations: 42 after adjustments

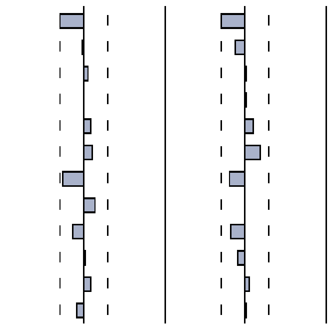
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.048859	0.007687	-6.356093	0.0000
@TREND	-0.000546	0.000183	-2.992150	0.0049
@SEAS(2)	0.127645	0.006265	20.37477	0.0000
@SEAS(3)	0.061046	0.006262	9.748293	0.0000
@SEAS(4)	0.188217	0.006412	29.35326	0.0000

R-squared	0.964217	Mean dependent var	0.027069
Adjusted R-squared	0.960348	S.D. dependent var	0.071975
S.E. of regression	0.014332	Akaike info criterion	-5.541277
Sum squared resid	0.007600	Schwarz criterion	-5.334412
Log likelihood	121.3668	Hannan-Quinn criter.	-5.465453
F-statistic	249.2503	Durbin-Watson stat	2.573560
Prob(F-statistic)	0.000000		

ACF of residuals

Date: 02/24/09 Time: 14:49
Sample: 1998Q2 2008Q3
Included observations: 42

Autocorrelation / Partial Correlation



	AC	PAC	Q-Stat	Prob
1	-0.305	-0.305	4.2044	0.040
2	-0.012	-0.116	4.2110	0.122
3	0.053	0.016	4.3462	0.226
4	-0.007	0.016	4.3485	0.361
5	0.092	0.113	4.7742	0.444
6	0.107	0.193	5.3574	0.499
7	-0.270	-0.192	9.2163	0.237
8	0.143	-0.005	10.328	0.243
9	-0.152	-0.184	11.630	0.235
10	0.015	-0.092	11.642	0.310
11	0.093	0.057	12.160	0.352
12	-0.087	0.020	12.625	0.397

Seminar
Nonstationary time series

Examples I.: analysis of the import time series of barium-clorid

Choice between trend stationarity and difference stationarity by the inspection of ACF and by a formal test

Fitting a linear trend and filtering by HP-filter

Fitting an AR(1) + trend to the original, and an ARMA(1,1) to the differentiated time series
Testing the uncorrelatedness of the residuals

Forecasting from the model

Examples II

Analysis of quarterly macro time series

Fitting a deterministic seasonal component
+ AR(1) term, and seasonal component
+ ARMA(1,1) term

Forecasting