ECONOMETRICS

Sponsored by a Grant TÁMOP-4.1.2-08/2/A/KMR-2009-0041

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The project is supported by the European Union.

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June 2010

Week 13

Time series regressions II.

Plan

Stationer variables: distributed lag models, ADL models

Spurious regression

Regression with non-stationary time series
- Filtering trend and seasonality components
- Cointergration and error correction
- VAR models

Distributed lag models

Assumption: Y and X stationary
E.g. 4-period distributed lag model

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + e_t$$

Coefficients: effect of temporary change in X
Sum of coefficients: long run (or total) effect

Example: patents

1960-1993 USA annual data (Ramanathan)

Y: number of patents (thousand)

X: R&D expenditures (bn USD)

Are lagged regressors needed?

How many lags?

Estimation result

Dependent Variable: PATENT
Method: Least Squares
Sample(adjusted): 1964 1993
Included observations: 30 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	26.327	4.148	6.347	0.000
RD	–0.597	0.459	–1.298	0.207
RD(–1)	0.867	0.971	0.893	0.381
RD(–2)	0.013	1.098	0.012	0.991
RD(–3)	–0.640	0.995	–0.649	0.526
RD(–4)	1.347	0.494	2.727	0.012

R-squared 0.964

ADL(p,q) model

Autoregressive distributed lag model
– ADL(p,q):

$$Y_t = \alpha + \delta t + \phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p} +$$
$$+ \beta_0 X_t + \beta_1 X_{t-1} + \ldots + \beta_q X_{t-q} + e_t$$

X, Y: stationary

Asymptotic properties

Assumptions

$$E(e_t \mid Y_{t-1},\ldots,Y_{t-p}, X_t, X_{t-1},\ldots, X_{t-q}) = 0$$

Stationary variables

No perfect collinearity

→ OLS is consistent

But: unbiasedness does not hold! E.g. $E(e_{t-1} \mid Y_t) \neq 0$

Asymptotic properties, cont.

Generally NOT true: OLS is inconsistent if the error terms are serially correlated

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t$$
$$E(u_t \mid y_{t-1}) = 0$$
$$\Rightarrow Cov(u_t, u_{t-1}) = Cov(u_t, y_{t-1} - \beta_0 - \beta_1 y_{t-2}) = ?0$$

OLS is inconsistent if the error term is stable AR(1) process

$$u_t = \rho u_{t-1} + e_t$$
$$\Rightarrow Cov(y_{t-1}, u_t) = \rho Cov(y_{t-1}, u_{t-1}) \neq 0$$

Assumptions: homoscedasticity, no autocorrelation

$$Var(e_t \mid Y_{t-1},\ldots Y_{t-p}, X_t, X_{t-1},\ldots, X_{t-q}) = \sigma^2$$
$$E(e_t e_s \mid Y_{t-1},\ldots Y_{t-p}, X_t, X_{t-1},\ldots, X_{t-q}, Y_{s-1},\ldots Y_{s-p}, X_s, X_{s-1},\ldots, X_{s-q}) = 0$$

→ Asymptotic normality

→ Usual tests are valid

Autocorrelation of error terms is often the consequence of misspecified dynamics!

Why is non-stationarity important? Spurious regression of time series

Two indep. Random walks

$X_t = X_{t-1} + \varepsilon_{1t}$

$Y_t = Y_{t-1} + \varepsilon_{2t}$

Regression: $Y_t = c + \beta X_t + u_t$

$\beta = 0$ since independent, but the t-test is significant!

The t-test has no marginal distribution!

Reason: u_t not stationary

Dependent Variable: RW2
Method: Least Squares
Date: 02/22/09 Time: 18:03
Sample: 2 5000
Included observations: 4999

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-24.18274	0.901345	-26.82963	0.0000
RW1	0.784565	0.024531	31.98294	0.0000

R-squared	0.169921	Mean dependent var	-47.98849
Adjusted R-squared	0.169755	S.D. dependent var	39.44434
S.E. of regression	35.94081	Akaike info criterion	10.00202
Sum squared resid	6454835.	Schwarz criterion	10.00463
Log likelihood	-24998.06	F-statistic	1022.909
Durbin-Watson stat	0.001251	Prob(F-statistic)	0.000000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	15.65639	0.434744	36.01291	0.0000
RW1	-0.362172	0.007234	-50.06680	0.0000

R-squared	0.334060	Mean dependent var	-2.177867
Adjusted R-squared	0.333927	S.D. dependent var	21.59167
S.E. of regression	17.62167	Akaike info criterion	8.576536
Sum squared resid	1551685.	Schwarz criterion	8.579143
Log likelihood	-21435.05	F-statistic	2506.684
Durbin-Watson stat	0.003573	Prob(F-statistic)	0.000000

Regression with non-stationary time series

Be careful in non-stationary case

- The coefficient estimates are generally not consistent
- Very common mistake (see: spurious regression)

"Safe" procedure: for I(1) time series write up the regression on differenced variables

- If higher order of integration: do differencing until the variables become stationary

This way we do not make any mistakes, but: we can lose information on long run behavior (see later: cointegration)

Seasonality

Two types of seasonality

Deterministic (can be filtered with dummy variables)

Stochastic (can be filtered with differencing)

Similarly to the trend, the two types of seasonality can be present at the same time

In practice: more complex filtering methods (e.g. TRAMO-SEATS)

Cointegration

y_t and x_t I(1) time series

If there exists a β such that $y_t - \beta x_t$ is stationary, then the two time series are cointegrated.

In this case the estimation of β is consistent.

Test: estimate β, then DF-test on the estimated error terms

Critical values have to be adjusted due to the estimated β

Example: 3 and 6 month interest rates, cointegration due to arbitrage

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.908	0.908	104.65	0.000
		2	0.835	0.064	193.96	0.000
		3	0.801	0.193	276.91	0.000
		4	0.737	-0.154	347.55	0.000
		5	0.682	0.039	408.54	0.000
		6	0.624	-0.100	460.17	0.000
		7	0.546	-0.125	499.91	0.000
		8	0.514	0.190	535.52	0.000
		9	0.485	-0.003	567.45	0.000
		10	0.435	-0.011	593.37	0.000
		11	0.403	-0.001	615.80	0.000
		12	0.380	0.036	635.90	0.000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.909	0.909	104.97	0.000
		2	0.846	0.114	196.66	0.000
		3	0.817	0.184	282.79	0.000
		4	0.760	-0.117	357.93	0.000
		5	0.699	-0.059	422.01	0.000
		6	0.647	-0.034	477.47	0.000
		7	0.577	-0.132	521.97	0.000
		8	0.546	0.180	562.09	0.000
		9	0.508	-0.035	597.19	0.000
		10	0.467	0.046	627.02	0.000
		11	0.438	0.005	653.52	0.000
		12	0.409	-0.022	676.85	0.000

Correlograms of r_6 és r_3 (top)

Correlogram of $r_6 - r_3$ (bottom)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.329	0.329	13.737	0.000
		2	0.301	0.216	25.331	0.000
		3	0.120	-0.034	27.183	0.000
		4	0.170	0.096	30.935	0.000
		5	-0.022	-0.125	30.998	0.000
		6	0.087	0.080	32.011	0.000
		7	0.295	0.344	43.630	0.000
		8	0.222	0.026	50.271	0.000
		9	0.243	0.073	58.269	0.000
		10	0.150	-0.025	61.338	0.000
		11	0.177	0.011	65.694	0.000
		12	0.045	0.028	65.977	0.000

Error correction

y_t and x_t I(1) processes

Generally we estimate the regression on differences, e.g.

$\Delta y_t = \alpha_0 + \gamma_1 \Delta x_t + u_t$

In case of cointegration we can include also the deviation from the long run equilibrium:

$\Delta y_t = \alpha_0 + \delta(y_{t-1} - \beta x_{t-1}) + \gamma_1 \Delta x_t + u_t$

where $\delta<0$.
This is the error correction model (ECM).

$\Delta y_t = \alpha_0 + \delta(y_{t-1} - \beta x_{t-1}) + \gamma_1 \Delta x_t + u_t$
$\delta<0$

“Engle-Granger two step procedure”
Step 1: estimate β, test cointegration

If cointegrated:
Step 2: estimate error correction model

Engle-Granger: t-test is valid for the estimated coefficients (two step estimation can be neglected)

Error correction – example

Agricultural and fuel price indices (MNB) relative to the same period of previous year
Cointegrated time series (test!)

Dependent Variable: AGR
Method: Least Squares

Variable	Coeff	Std. Error	t-Statistic	Prob.
C	9.502	0.867	10.961	0.000
FUEL	0.284	0.056	5.103	0.000

Dependent Variable: D(AGR)
Method: Least Squares

Variable	Coeff	Std. Error	t-Statistic	Prob.
C	–0.155	0.128	–1.208	0.228
D(FUEL)	0.039	0.036	1.085	0.279
RESID(-1)	–0.046	0.0145	–3.183	0.002

VAR model

Genearlization of AR model to more variables

Matrix notation:

$$\mathbf{Y}_t = \mathbf{A}_1\mathbf{Y}_{t-1} + \ldots + \mathbf{A}_p\mathbf{Y}_{t-p} + \mathbf{e}_t$$

Uncretain driection of causality, e.g.

Interest rate – exchange rate, inflation – exchange rate

Price of substitutes

“Atheoretical”

Good forecasting properties

Seminar
Time series regressions II

Excercises: M 14/9, 14/10a

Discussion:

Filtering trend and seasonality from time series, forecasting based on the models

Unit root test on Hungarian price level and inflation data

Model of retail turnover and household consumption, analysis of the relationship between the two

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