ECONOMETRICS

NEW SZÉCHENYI PLAN

ECONOMETRICS

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ECONOMETRICS

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ECONOMETRICS

Week 6.

Multivariate regression III

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Content

F-test (cont.),

Stability tests

Adjusted R^2, model selection

Dummy variables

F-test more generally

Joint test of r constraints in a regression with k explanatory variables

Nested hypotheses: the parameter set of the model is a subset of that of the original one

Example

$$U: y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

$$H_0: \beta_2 = 0, \beta_3 = 0$$

$$R: y = \alpha + \beta_1 x_1 + v$$

F-test, test statistic

Sum of squares decompositon

$$TSS = RESS + RRSS = RESS + (RRSS - URSS) + URSS$$

Degrees of freedom

$$n - 1 = (k - r) + (n - k + r - 1) = (k - r) + r + (n - k - 1)$$

$$F = \frac{(RRSS - URSS) / r}{URSS / (n - k - 1)} \sim F_{r,(n-k-1)}$$

$\sim \chi_r^2 / r$, in large samples, approximately the Wald-test)

$$RRSS = S_{yy}(1 - R_R^2) \quad URSS = S_{yy}(1 - R_U^2)$$

$$F = \frac{(R_U^2 - R_R^2) / r}{(1 - R_U^2) / (n - k - 1)} \sim F_{r,(n-k-1)}$$

Testing a linear function of the parameters

Example: Cobb-Douglas production function

$$\log X = \alpha + \beta_1 \log L + \beta_2 \log K + u$$

$$H_0: \beta_1 + \beta_2 = 1$$

t-test: $\theta = \beta_1 + \beta_2$ $\beta_2 = \theta - \beta_1$

$$\log X = \alpha + \beta_1(\log L - \log K) + \theta \log K + u$$

$$H_0: \theta = 1$$

t-test directly on $\beta_1 + \beta_2$, using that the variance of the sum is:

$$Var(\beta_1\hat{} + \beta_2\hat{}) = Var(\beta_1\hat{}) + Var(\beta_2\hat{}) + 2cov(\beta_1\hat{}, \beta_2\hat{})$$

F-test: $\beta_2 = 1 - \beta_1$

$$R: \log X - \log K = \alpha + \beta_1 (\log L - \log K) + u$$

Stability test: two independent data sets (sometimes referred to as Chow-test)

1. $y_i = \alpha_1 + \beta_{11} x_{1i} + \beta_{21} x_{2i} + \ldots + \beta_{k1} x_{ki} + u_i,\ i = 1 \ldots n_1$
2. $y_i = \alpha_2 + \beta_{12} x_{1i} + \beta_{22} x_{2i} + \ldots + \beta_{k2} x_{ki} + v_i,\ i = 1 \ldots n_2$

$$H_0: \alpha_1 = \alpha_2,\ \beta_{11} = \beta_{12},\ \ldots,\ \beta_{k1} = \beta_{k2}$$

$$F = \frac{(RRSS - RSS_1 - RSS_2) / (k+1)}{(RSS_1 + RSS_2) / (n_1 + n_2 - 2k - 2)} \sim F_{k+1,\, n_1 + n_2 - 2k - 2}$$

RRSS: from the merged data set,

RSS_1, RSS_2: from separate regressions

Stability test – Chow-test (predictive)

1. $y_i = \alpha_1 + \beta_{11}x_{1i} + \beta_{21}x_{2i} + \ldots + \beta_{k1}x_{ki} + u_i,\ i = 1 \ldots n_1$
2. $y_i = \alpha_2 + \beta_{12}x_{1i} + \beta_{22}x_{2i} + \ldots + \beta_{k2}x_{ki} + v_i,\ i = 1 \ldots n$

$n - n_1 < k + 1$ is possible (in contrast to the previous one)

RSS_1: res. sum of squares based on the first n_1 observations

$RRSS$: res. sum of squares based on the model estimated from all ($n = n_1 + n_2$) observations

$$F = \frac{(RRSS - RSS_1)/(n - n_1)}{RSS_1/(n_1 - k - 1)} \sim F_{(n - n_1),(n_1 - k - 1)}$$

ELTE ECON

Adjusted R^2

$$\hat{\sigma}^2 = \frac{RSS}{\mathrm{df}}$$

Including new variables: RSS and the degree of freedom are both decreasing (the number of normal equations is increasing)

Adjusted R^2: $1-\bar{R}^2 = \frac{n-1}{n-k-1}(1-R^2)$

$t < 1$: omitting a variable: $\bar{R}^2$ is increasing

$F < 1$: omitting more variables: $\bar{R}^2$ is increasing

Possible: different conclusions based on t and F (e.g. multicollinearity)

Model selection

Nested hypotheses: t- and F-test

Non-nested hyp., dependent variable is the same, e.g.:

$$R\&D = \alpha + \beta \log(revenue) + u$$

$$R\&D = \alpha + \beta_1\, revenue + \beta_2\, revenue^2 + u$$

based on adjusted R^2 or information criteria (e.g. AIC)

AIC (Akaike information criterion):

$$RSS \cdot exp(2(k + 1)/n)$$

Adjusted R^2, example

Wage survey (2003): does the experience or the age explain more in the wage equation?

Equation: EQ_EXP Workfile: TARIFA2003

View | Procs | Objects | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(KER)
Method: Least Squares
Date: 11/23/08 Time: 08:59
Sample: 1 201971
Included observations: 201971

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.43052	0.005226	1995.866	0.0000
ISKVEG9	0.163335	0.000509	320.6889	0.0000
EXP01	0.020825	0.000368	56.66378	0.0000
EXP2	-0.000268	7.17E-06	-37.37194	0.0000

R-squared	0.341767	Mean dependent var	11.58047
Adjusted R-squared	0.341757	S.D. dependent var	0.592283
S.E. of regression	0.480532	Akaike info criterion	1.372175
Sum squared resid	46636.45	Schwarz criterion	1.372377
Log likelihood	-138565.8	F-statistic	34954.97
Durbin-Watson stat	0.922321	Prob(F-statistic)	0.000000

Equation: EQ_KOR Workfile: TARIFA2003

View | Procs | Objects | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(KER)
Method: Least Squares
Date: 11/23/08 Time: 09:00
Sample: 1 201971
Included observations: 201971

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.986636	0.014081	709.2486	0.000
ISKVEG9	0.154711	0.000496	312.1709	0.000
KOR	0.031483	0.000672	46.84169	0.000
KOR^2	-0.000278	7.80E-06	-35.61278	0.000

R-squared	0.342334	Mean dependent var	11.5804
Adjusted R-squared	0.342324	S.D. dependent var	0.59228
S.E. of regression	0.480325	Akaike info criterion	1.37131
Sum squared resid	46596.24	Schwarz criterion	1.37151
Log likelihood	-138478.7	F-statistic	35043.2
Durbin-Watson stat	0.923453	Prob(F-statistic)	0.00000

Logarithmic forms

Log-log (loglinear) – elasticity

$$\ln(y) = \alpha + \beta \ln(x) + u$$

$$\%\Delta\hat{y} = \left(e^{\hat{\beta}} - 1\right) \cdot \%\Delta x \approx \hat{\beta} \cdot \%\Delta x$$

Partly logarithmic forms

$$y = \alpha + \beta \ln(x) + u \qquad \Delta\hat{y} \approx \frac{\hat{\beta}}{100} \%\Delta x$$

$$\ln(y) = \alpha + \beta x + u \qquad \%\Delta\hat{y} \approx 100\hat{\beta}\, \Delta x$$

Quadratic form

Increasing or decreasing partial effect

$$y = \alpha + \beta_1 x_1 + \beta_2 x_1^{\ 2} + u$$

$$\frac{\Delta \hat{y}}{\Delta x_1} \approx \hat{\beta}_1 + 2\hat{\beta}_2 x_1$$

Example: wage survey (2003), quadratic function of experience, estimated equations:

log(Ker) = 9.83 + 0.135 ISKVEG9 + 0.0082 EXP

log(Ker) = 9.83 + 0.135 ISKVEG9 + 0.022 EXP – 0.00029 EXP^2

positive (but decreasing) partial effect for

0.022/(2*0.00029) = 39 years

Interactions

Partial effect depends on other explanatory variables as well:

$$y = \alpha + \beta_1 x_1 + \beta_2 x_1 x_2 + u$$

$$\frac{\Delta \hat{y}}{\Delta x_1} \approx \hat{\beta}_1 + \hat{\beta}_2 x_2$$

Example: wage and education premium depend on the profitability of the firm (net sales revenue – material costs)

Log(wage) = 10.304 + 0.139 Educ9 + 0.092 Log(Profit)

Log(wage) = 10.597 + 0.079 Educ9 + 0.043 Log(Profit) + 0.010 (Educ9*Log(Profit))

Dummy variables on the right hand side

So far: mainly continuous variables (quantitative information) – e.g. wage, consumption, wealth, education (?)

Binary / dummy variables

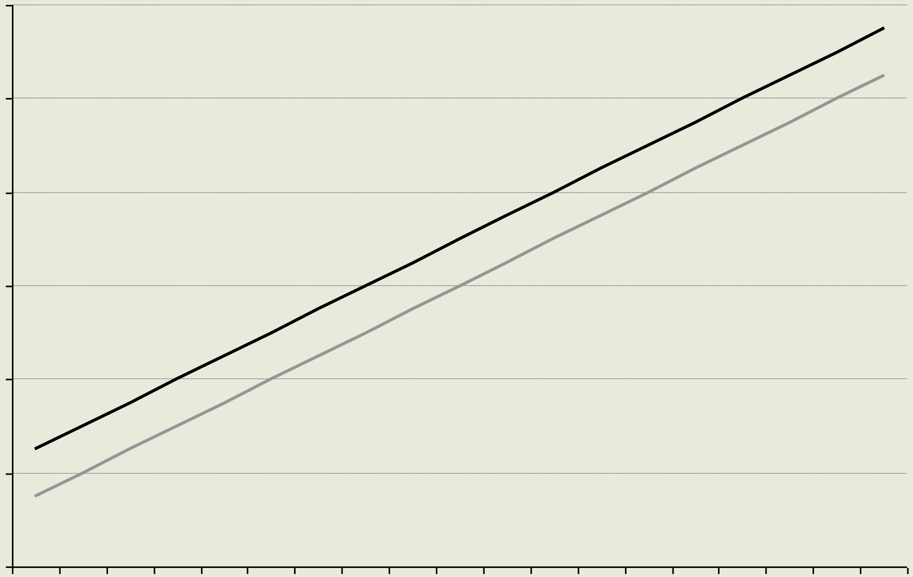
- Qualitative information
- Examples: gender, employed, country dummy…

Different intercepts – 2 groups

Example:
$$\log(wage_i) = \begin{cases} \alpha_1 + \beta Educ_i + u_i, \text{if from Budapest} \\ \alpha_2 + \beta Educ_i + u_i, \text{otherwise} \end{cases}$$

$$\Leftrightarrow \log(wage_i) = \alpha_1 + (\alpha_2 - \alpha_1) D_i + \beta Educ_i + u_i,$$

$$D_i = 0 \text{ if from Budapest}, D_i = 1 \text{ otherwise}$$



Different intercepts, example

Based on the 2003 wage survey

Dependent Variable: LOG(KER)
Method: Least Squares
Sample: 1 201971
Included observations: 199204
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.93263	0.003597	3039.442	0.0000
ISKVEG9	0.149569	0.000519	287.9107	0.0000
VIDEK	-0.164612	0.002906	-56.64754	0.0000

R-squared	0.331955	Mean dependent var	11.57456
Adjusted R-squared	0.331948	S.D. dependent var	0.590660

Log dependent variable

Estimated equation:

$$\log(Wage_i) = 10.93 - 0.16 Countryside_i + 0.15 Educ_i$$

Countryside: lower wage by approx. 16% (ceteris paribus)

Exact difference („log” is the natural logarithm in Eviews):

$$\log(Wage_1) - \log(Wage_0) = -0.16 = \log \frac{Wage_1}{Wage_0}$$

$$\Rightarrow \%\ \text{wage difference}: \left(e^{-0.16} - 1\right) \cdot 100 = -14.79$$

More than two groups

N groups (e.g. regions instead of Budapest / countryside)

$$y_i = \begin{cases} \alpha_1 + \beta x_i + u_i \text{ in Group 1} \\ \alpha_2 + \beta x_i + u_i \text{ in Group 2} \\ \vdots \\ \alpha_N + \beta x_i + u_i \text{ in Group N} \end{cases}$$

$$\Leftrightarrow y_i = \alpha_1 + (\alpha_2 - \alpha_1) D_2 + \ldots + (\alpha_N - \alpha_1) D_N + \beta x_i + u_i$$

$$D_2 = 1 \text{ in Group 2 (0 otherwise)}, \ldots,$$

$$D_N = 1 \text{ in Group N (0 otherwise)}$$

N-1 dummies in the regression (if there are N groups),

Group N: benchmark group!

Interactions between binary variables

Example: male / female wage gap is not the same in Budapest and in the countryside

Four categories:

	Budapest	countryside
female		
male		

Benchmark group: females in Budapest

Two equivalent models

$$\log(wage_i) = \alpha_0 + \alpha_1 Countryside\_Fem_i + \alpha_2 Bp\_Male_i + \alpha_3 Countryside\_Male_i + \\ + \beta Educ_i + u_i$$

$$\log(wage_i) = \alpha_0 + \alpha_1^* \cdot Male_i + \alpha_2^* \cdot Countryside_i + \alpha_3^* \cdot Countryside_i \cdot Male_i + \\ + \beta Educ_i + u_i$$

Interactions, example

Wage survey estimates (benchmark: females in Bp.):

Dependent Variable: LOG(KER)
Method: Least Squares
Included observations: 199204
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.90119	0.004225	2579.942	0.0000
VIDEK*(1-FFI)	-0.172604	0.003569	-48.36810	0.0000
(1-VIDEK)*FFI	0.053984	0.005444	9.916527	0.0000
VIDEK*FFI	-0.102617	0.003795	-27.03654	0.0000
ISKVEG9	0.150625	0.000524	287.6727	0.0000

R-squared	0.335102	Mean dependent var	11.57456
Adjusted R-squared	0.335089	S.D. dependent var	0.590660

Dependent Variable: LOG(KER)
Method: Least Squares
Included observations: 199204
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.90119	0.004225	2579.942	0.0000
VIDEK	-0.172604	0.003569	-48.36810	0.0000
FFI	0.053984	0.005444	9.916527	0.0000
VIDEK*FFI	0.016003	0.005902	2.711195	0.0067
ISKVEG9	0.150625	0.000524	287.6727	0.0000

R-squared	0.335102	Mean dependent var	11.57456
Adjusted R-squared	0.335089	S.D. dependent var	0.590660

$$-0,1026 = -0.1726 + 0.0540 + 0.0165$$

Non-constant slope parameters

In case of two groups

$$y_i = \begin{cases} \alpha + \beta_{11} x_{1i} + \beta_2 x_{2i} + u_i \text{ for Group 1} \\ \alpha + \beta_{12} x_{1i} + \beta_2 x_{2i} + u_i \text{ for Group 2} \end{cases}$$

$$\Leftrightarrow y_i = \alpha + \beta_{11} x_{1i} + (\beta_{12} - \beta_{11}) D_i x_{1i} + \beta_2 x_{2i} + u_i$$

$$D_i = 0 \text{ for Group 1, } D_i = 1 \text{ for Group 2}$$

→ Interaction of a dummy variable and an explanatory variable

Example

Effect of education gender-dependent but the effect of age is not

$$\log(Wage_i) = \alpha_0 + \beta_1 Educ_i + \beta_2 Educ_i \cdot Male_i + \beta_3 Age_i + u_i$$

Dependent Variable: LOG(KER)
Method: Least Squares
Included observations: 201971
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.45207	0.004941	2115.328	0.0000
ISKVEG9	0.149156	0.000504	295.9727	0.0000
ISKVEG9*FFI	0.014087	0.000440	31.99000	0.0000
KOR	0.007764	9.57E-05	81.14545	0.0000

R-squared	0.342540	Mean dependent var	11.58047
Adjusted R-squared	0.342530	S.D. dependent var	0.592283

Examining the stability of the coefficients

Example: is the wage model the same for males and females?

Cross sectional analysis (time series: stability of the coefficient in time)

$$\log(Wage_i) = \alpha_1 + \alpha_2 Male_i + \beta_1 Educ_i + \beta_2 Educ_i \cdot Male_i + \beta_3 Age_i + \beta_4 Age_i \cdot Male_i + u_i$$

$$\mathrm{H}_0 : \alpha_2 = 0, \beta_2 = 0, \beta_3 = 0$$

F-test (also possible for a subset of the restrictions)

Problem if $N_2 < k \rightarrow$ Chow-test (predictive) can be used (see week 5)

Examining the stability of the coefficients, cont.

Estimation results and test statistic

Dependent Variable: LOG(KER)
Method: Least Squares
Included observations: 201971
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.39458	0.006437	1614.696	0.0000
FFI	0.104694	0.009702	10.79114	0.0000
ISKVEG9	0.150973	0.000580	260.0792	0.0000
ISKVEG9*FFI	0.011260	0.001075	10.47171	0.0000
KOR	0.008858	0.000124	71.71457	0.0000
KOR*FFI	-0.002065	0.000191	-10.81957	0.0000

R-squared	0.342951	Mean dependent var	11.58047
Adjusted R-squared	0.342935	S.D. dependent var	0.592283

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	438.0596	(3, 201965)	0.0000
Chi-square	1314.179	3	0.0000

Dummy dependent variable

Examples:

- Labour market: employed or not
- Consumption: real estate owner or not
- Finance: bankruptcy of the borrower or not

Binary dependent variable ↔ linear model?

Linear probability model I.

y binary variable

$$E(y \mid x) = P(y = 1 \mid x) = \alpha + \beta x$$

$$\text{Estimated model}: y_i = \alpha + \beta x_i + u_i$$

$$\hat{y}: y = 1 \text{ estimated prob.}$$

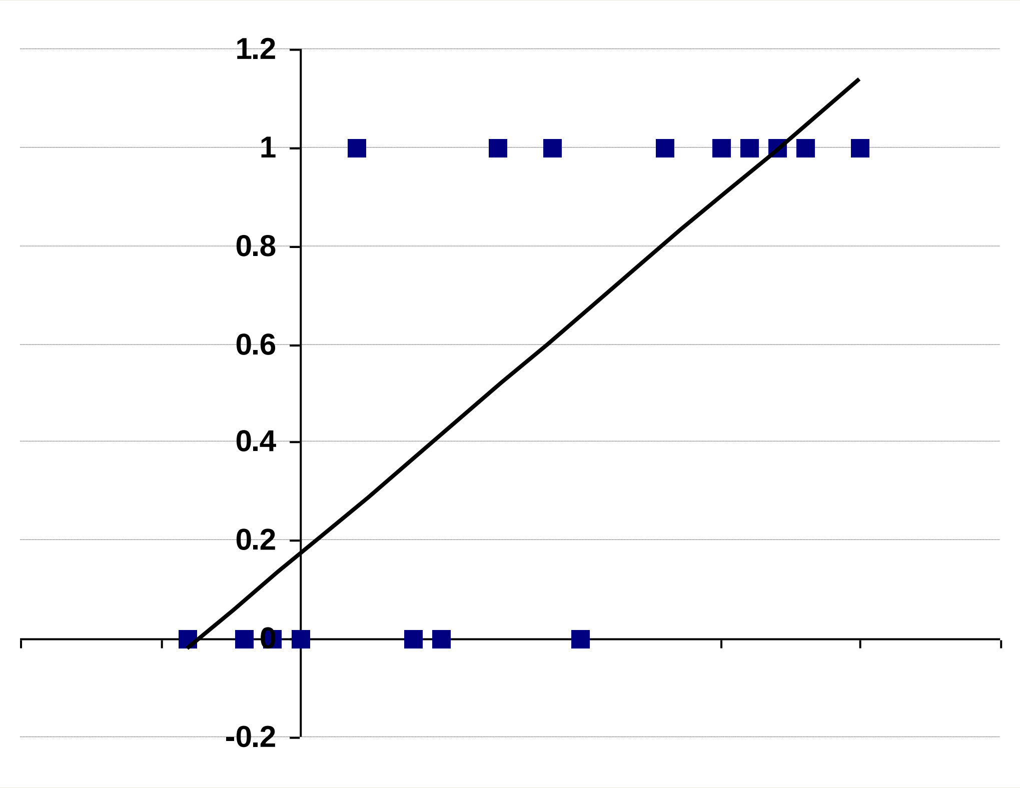
Nonlinear model: $P(y = 1 \mid x) = F(\beta x)$

If F is the Gaussian distribution function then the probit,

if $F(z) = e^z/(1 + e^z)$ then the logit model is obtained.

Linear probability model II.

Problem 1: estimated probability may lie outside the [0,1] interval



Linear probability model III.

Problem 2: heteroscedasticity

$$y_i = \beta x_i + u_i, \; E(u_i) = 0$$

$$u_i = \begin{cases} -\beta x_i & (1-\beta x_i) \text{ prob.} \\ 1-\beta x_i & \beta x_i \text{ prob.} \end{cases}$$

$$\Rightarrow \mathrm{Var}(u_i) = (1-\beta x_i)(-\beta x_i)^2 + \beta x_i (1-\beta x_i)^2 = \beta x_i (1-\beta x_i) = \hat{y}_i (1-\hat{y}_i)$$

Solution:

Using robust SE

Weighted LS: $w_i = \hat{y}_i (1-\hat{y}_i)$

Linear probability model, example

Dependent variable: whether the person has a private health insurance or not (SHARE database)

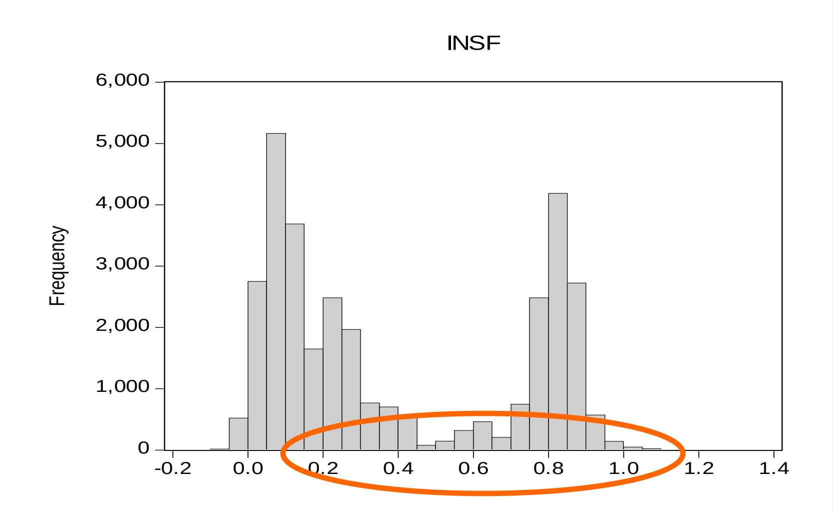
Explanatory variables: wealth, income, age, education, country dummies

Dependent Variable: INS
Method: Least Squares
Included observations: 32405
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.438462	0.015836	27.68759	0.0000
HNETW/10^6	0.008275	0.002079	3.980982	0.0001
HGTINC/10^6	0.445824	0.041066	10.85636	0.0000
AGE	-0.003281	0.000192	-17.05036	0.0000
EDU	0.000697	0.000280	2.487641	0.0129
COUNTRY=GERM	-0.026880	0.011713	-2.294858	0.0217
COUNTRY=SWE	-0.156186	0.010539	-14.81915	0.0000
COUNTRY=NETH	0.568132	0.011364	49.99388	0.0000
COUNTRY=SPAIN	-0.135671	0.010888	-12.46093	0.0000
COUNTRY=ITALY	-0.190969	0.010249	-18.63311	0.0000
COUNTRY=FRA	0.596495	0.011187	53.32010	0.0000
COUNTRY=DENM	0.115167	0.014577	7.900561	0.0000
COUNTRY=GRE	-0.186449	0.010237	-18.21386	0.0000
COUNTRY=SWITZ	0.345743	0.017138	20.17374	0.0000
COUNTRY=BELG	0.546137	0.011148	48.98888	0.0000

R-squared	0.464462	Mean dependent var	0.395957
Adjusted R-squared	0.464230	S.D. dependent var	0.489063

Histogram of predicted prob.



Seminar

Multivariate regression III.

Exercise: estimation of a wage equation based on a small sample from the wage survey

Variables

- Educ (years of education)
- Exp (experience)
- Wage
- Typ (type of settlement – qualitative variable)
- Bp (Budapest dummy)
- Male (male dummy)

Estimation of the wage equation I.

Model 1: modelling log(wage) in the private sector with the educ, exp, exp^2, bp, male variables and with the interaction of educ, exp with male

- Does the equation for males differ significantly from the equation for females?
  - Joint test of Male, Educ*Male and Exp*Male
- Experience-profile for Budapest males with 12 years of experience
  - Where is the maximum?
  - Graphical presentation of the experience-profile with confidence interval

Estimation of the wage equation II.

Model 2: previous model + dummies for “chief town of the county” and for “other town”

- Testing the equality of the two new coefficients with three methods
  - Directly
  - By a t-test after transformation
  - By comparing the R^2 of the restricted and unrestricted model
- Testing heteroscedasticity with the White- and Breusch–Pagan-tests
- Calculating robust standard errors and comparing them with the non-robust ones

Exercise: estimation of labour market participation with a linear probability model

Dependent variable: economically active or not

Explanatory variables: education, experience, age, kid below / over 6 years

OLS estimation with usual and with robust standard errors, WLS estimation

Forecasting probabilities