ECONOMETRICS

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ECONOMETRICS

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ECONOMETRICS

Week 12.

Time series regressions I

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Plan

Regression on stationary time series

Consequences of autocorrelated error terms

Testing autocorrelation

Handling autocorrelation

Textbook: M 6.1–6.5., 6.8.

Reminder: cross sectional regression with stochastic regressors

- Fixed regressors are less sensible in case of time series
- Model with stochastic variables: $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots \beta_k x_{ik} + u_i$
- Conditions for unbiasedness of OLS
  - $(y_i, x_{i1}, x_{i2}, \ldots, x_{ik})$ $(i = 1,..,n)$ random sample of the model
  - $E(u|x_1, x_2, \ldots, x_k) = 0$
  - No perfect collinearity
- If homoscedasticity is also assumed, the following statements are true
  - The usual formula of variance is valid, the OLS estimator is asymptotically normal
  - OLS is BLUE.
- If normality of error term is also assumed then small sample tests t-test, F-test) are also valid.

Regression with stationary variables

$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \ldots \beta_k x_{tk} + u_t$

If x_{ti} $(i = 1,\ldots,k)$ and y_t are stationary then sufficient conditions for consistency of OLS:

- $E(u_t | x_{t1}, x_{t2}, \ldots, x_{tk}) = 0$ and
- No perfect collinearity

Further assumptions are needed for asymptotic validity of the usual tests (validity of formulas of variance etc.):

- Homoscedasticity and
- No autocorrelation in the error terms:
- $E(u_t u_s | x_{t1}, \ldots, x_{tk}, x_{s1}, \ldots, x_{sk}) = 0 \ (t \neq s)$

Regression with stationary variables (cont.)

The same is true in case of trend stationarity, but trend has to be included as regressor

Some x_{ti} might be lagged y_t (but the exogeneity condition has to hold!)

E.g. stationary AR(1) model: $k = 1$, $x_{t1} = y_{t-1}$

- OLS estimation of the coefficient is consistent, asymptotically normal (but not unbiased!)

Autocorrelation of the error terms in the stationary regression

If the error terms are autocorrelated in a stationary regression then

- OLS is still consistent,
- But not BLUE,
- And the common formula of variance and the usual tests are not valid!

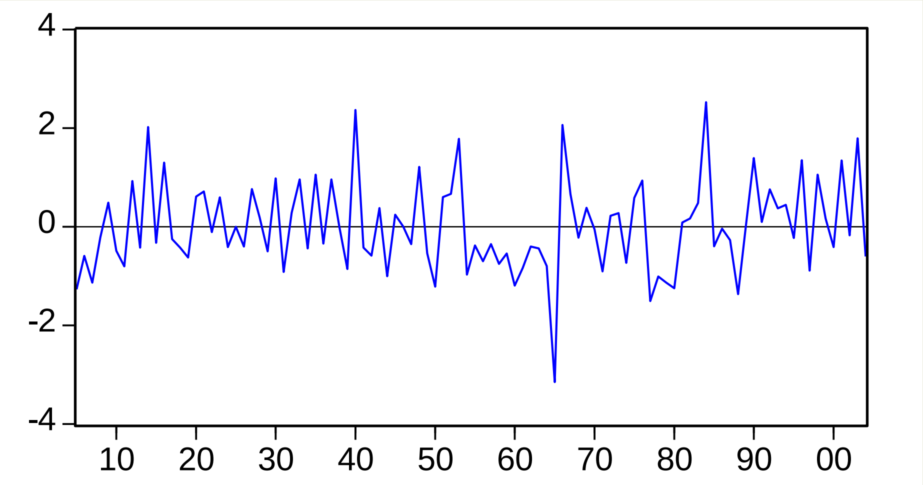
Size of bias in variance:
M page 285–286.

Testing autocorrelation of error terms

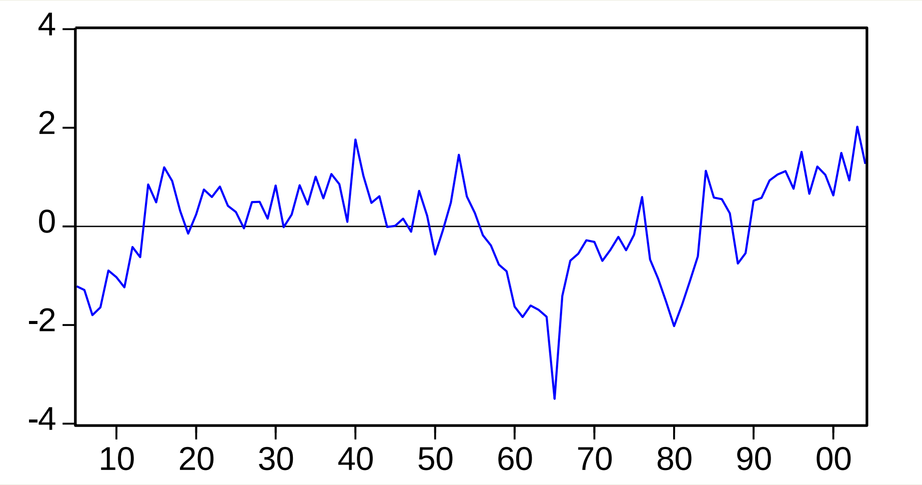
Durbin–Watson-test

Breusch–Godfrey-test

white noise error term



autocorrelated error term



Durbin–Watson-test

Analyzes the residuals of OLS regression:

$$d = \frac{\sum_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n}\hat{u}_t^2} = \frac{\sum_{t=2}^{n}\hat{u}_t^2 - 2\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1} + \sum_{t=2}^{n}\hat{u}_{t-1}}{\sum_{t=1}^{n}\hat{u}_t^2} \approx 2 - 2\frac{\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}}{\sum_{t=1}^{n}\hat{u}_t^2}$$

Estimator of first order autocorrelation:

$$\hat{\rho} = \frac{\sum_{t=2}^{n}\hat{u}_t\hat{u}_{t-1}}{\sum_{t=1}^{n}\hat{u}_t^2}$$

$$d \approx 2(1-\hat{\rho})$$

$0 \le d \le 4$ $(-1 \le \rho \le 1)$

$d = 2$ $\Leftrightarrow$ $\rho = 0$ (white noise)

$0 < d < 2$ $\Leftrightarrow$ $\rho > 0$ (positive autocorrelation)

$2 < d < 4$ $\Leftrightarrow$ $\rho < 0$ (negative autocorrelation)

DW-test, cont.

$H_0: \rho = 0$, $H_1: \rho > 0$ (one sided test!)

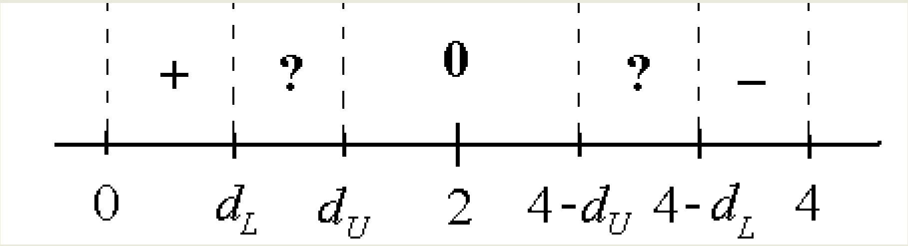
The test has two critical values because the distribution of the test statistic depends on the regressors: d_L (lower value), d_U (upper value)

Decision rule:

- Accept H_0 if $d > d_U$
- Reject H_0 if $d < d_L$
- Cannot decide if $d_L < d < d_U$ (neutral, grey zone)

Testing negative autocorrelation:

- Use $4 - d$ instead of d, otherwise everything is the same
- Accept H_0 if $4 - d > d_U$
- Reject H_0 if $4 - d < d_L$
- Cannot decide if $d_L < 4 - d < d_U$ (neutral, grey zone)



Limitations of DW-test

Can be used only for AR(1) residuals

In some cases ($d_L<d<d_U$) the test is not conclusive

Cannot be used for distributed lag models (in some cases, see later)

Breusch–Godfrey-test

AR(p) model of error terms:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \ldots + \rho_q u_{t-p} + e_t$$

$H_0: \rho_1 = \rho_2 = \ldots = \rho_p = 0$

Regress the estimated error term on the explanatory variables and p lags of the error term

Under H_0, asymptotically $nR^2 \sim \chi_p^2$

Handling autocorrelation

- Adjusting the standard error of OLS estimation: Newey-West
  - Just as the White-procedure adjusts the standard error of OLS in case of heteroscedasticity
- Generalized Least Squares (GLS) types of estimations, e.g. Cochrane-Orcutt procedure

Cochrane–Orcutt-procedure

Model

$y_t = \beta_0 + \beta_1 x_{1t} + \beta_1 x_{2t} + \beta_1 x_{kt} + u_t$

$u_t = \rho u_{t-1} + e_t$, $e_t \sim IN$

Quasi differencing

$(y_t - \rho y_{t-1}) = (1 - \rho)\beta_0 + \beta_1(x_{1t} - \rho x_{1,t-1}) + \ldots + \beta_k(x_{kt} - \rho x_{k,t-1}) + e_t$

Regress $y_t - \rho y_{t-1}$ on $x_{it} - \rho x_{i,t-1}$ variables

Procedure

- OLS estimation, then estimation of ρ based on the residuals
- OLS of the quasi-differenced time series
- Iteration can also be used (does not improve asymptotically the efficiency)

Since ρ is estimated, if ρ is close to 0 then not necessarily better than OLS

Example: estimation of static Phillips-curve (USA) with OLS
Considerable autocorrelation

Dependent Variable: INF
Method: Least Squares
Date: 02/24/09 Time: 12:23
Sample: 1948 2003
Included observations: 56

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.053565	1.547957	0.680617	0.4990
UNEM	0.502378	0.265562	1.891752	0.0639

R-squared	0.062154	Mean dependent var	3.883929
Adjusted R-squared	0.044786	S.D. dependent var	3.040381
S.E. of regression	2.971518	Akaike info criterion	5.051084
Sum squared resid	476.8157	Schwarz criterion	5.123418
Log likelihood	-139.4304	Hannan-Quinn criter.	5.079128
F-statistic	3.578726	Durbin-Watson stat	0.801482
Prob(F-statistic)	0.063892		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	31.52897	Prob. F(1,53)	0.0000
Obs*R-squared	20.88778	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 02/24/09 Time: 12:26
Sample: 1948 2003
Included observations: 56
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.555356	1.318280	1.938402	0.0579
UNEM	-0.457243	0.227341	-2.011267	0.0494
RESID(-1)	0.656330	0.116887	5.615066	0.0000

R-squared	0.372996	Mean dependent var	5.43E-16
Adjusted R-squared	0.349335	S.D. dependent var	2.944380
S.E. of regression	2.375048	Akaike info criterion	4.619996
Sum squared resid	298.9653	Schwarz criterion	4.728497
Log likelihood	-126.3599	Hannan-Quinn criter.	4.662061
F-statistic	15.76448	Durbin-Watson stat	1.808715
Prob(F-statistic)	0.000004		

Example, cont.: correcting autocorrelation

Strong autocorrelation, the regression is basically on the differences! These estimates are more reliable.

Dependent Variable: INF
Method: Least Squares
Date: 02/24/09 Time: 13:20
Sample (adjusted): 1949 2003
Included observations: 55 after adjustments
Convergence achieved after 5 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.287078	2.287763	3.185242	0.0024
UNEM	-0.663959	0.330235	-2.010564	0.0496
AR(1)	0.782009	0.094683	8.259202	0.0000

R-squared	0.503681	Mean dependent var	3.807273
Adjusted R-squared	0.484592	S.D. dependent var	3.013295
S.E. of regression	2.163302	Akaike info criterion	4.434150
Sum squared resid	243.3536	Schwarz criterion	4.543641
Log likelihood	-118.9391	Hannan-Quinn criter.	4.476491
F-statistic	26.38569	Durbin-Watson stat	1.600203
Prob(F-statistic)	0.000000		

Inverted AR Roots	.78

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.734056	Prob. F(1,51)	0.3956
Obs*R-squared	0.780397	Prob. Chi-Square(1)	0.3770

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 02/24/09 Time: 13:21
Sample: 1949 2003
Included observations: 55
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.198041	2.305251	0.085908	0.9319
UNEM	-0.017397	0.331705	-0.052446	0.9584
AR(1)	-0.055103	0.114662	-0.480565	0.6329
RESID(-1)	0.151531	0.176863	0.856771	0.3956

R-squared	0.014189	Mean dependent var	-1.33E-12
Adjusted R-squared	-0.043800	S.D. dependent var	2.122863
S.E. of regression	2.168855	Akaike info criterion	4.456223
Sum squared resid	239.9006	Schwarz criterion	4.602211
Log likelihood	-118.5461	Hannan-Quinn criter.	4.512678
F-statistic	0.244685	Durbin-Watson stat	1.715550
Prob(F-statistic)	0.864729		

Distributed lag models

Model: $y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \ldots + \beta_k x_{t-k} + u_t$

β_0: immediate effect of a unit shock in x on y

$\beta_0, \beta_1, \beta_2, \ldots$: lag distribution

$\beta_0 + \beta_1 + \ldots + \beta_k$: effect of a permanent unit shock in x on y (long run multiplier)

There are models with infinite lags, e.g. geometric lags (geometrically declining β-s)

Seminar

Time series regressions I

AR(2)-process:

- Stationarity, necessary conditions of stationarity
- Solution of Yule–Walker equation

Autocorrelations of ARMA(1,1) process

Simulation of ARMA and ARIMA processes

Comparison and simulation difference and trend stationary processes

Box–Jenkins modelling, forecasting from ARIMA model

- Example: modelling time series of (seasonally adjusted) industrial production