Exercise in tensile testing

Exercise:

A round test specimen with d_0=10 mm initial diameter was tensile tested. After the test process the next data were measured:

- offset yield point at 0,2% strain: $F_{p0,2}$=22 kN;
- ultimate force: F_m=29 kN;
- fracture force: F_u=23 kN;
- necking rate: Z=60%;
- diameter at the ultimate force: d_m=9,2 mm.

Let us determine 3 points of the true stress-strain diagram and the approximate value of the specific work of rupture!

Solution:

The 1st point is the offset yield point. 0,2 means that the force belonging to it was measured at $\varepsilon = 0{,}2\% = 0{,}002$ engineering elongation. Knowing it we can calculate a $\varphi_{p0,2}$ true elongation and then a $d_{p0,2}$ diameter a $\sigma_{p0,2}$ true stress.

On the one hand:

$$\varphi_{p0,2} = \ln(1+\varepsilon) = \ln(1+0{,}002) = 0{,}001998\,.$$

On the other hand:

$$\varphi_{p0,2} = 2\ln\frac{d_0}{d_{p0,2}} \Rightarrow d_{p0,2} = \frac{d_0}{e^{\frac{\varphi_{p0,2}}{2}}} = \frac{d_0}{e^{\frac{0{,}001998}{2}}} = 9{,}99\text{mm}\,.$$

Now we can calculate a true stress:

$$\sigma_{p0,2} = \frac{F_{p0,2}}{S_{p0,2}} = \frac{F_{p0,2}}{\frac{d_{p0,2}^2\pi}{4}} = \frac{22000\text{N}}{\frac{(9{,}99\text{mm})^2\pi}{4}} = 280{,}67\text{MPa}\,.$$

It is approximately equal to the engineering stress:

$$R_{p0,2} = \frac{F_{p0,2}}{\frac{d_0^2\pi}{4}} = \frac{22000\text{N}}{\frac{(10\text{mm})^2\pi}{4}} = 280{,}11\text{MPa}\,.$$

The 2nd point is the point of ultimate strenght, where we can use the next equations:

$$\varphi_m = 2\ln\frac{d_0}{d_m} = 2\ln\left(\frac{10\text{mm}}{9{,}2\text{mm}}\right) = 0{,}1168\,,$$

and

$$\sigma_m = \frac{F_m}{S_m} = \frac{F_m}{\frac{d_m^2\pi}{4}} = \frac{29000\text{N}}{\frac{(9{,}2\text{mm})^2\pi}{4}} = 436{,}25\text{MPa}\,.$$

The 3rd point is the point of rupture, but firstly we can calculate the cross sectional are of the specimen.

From the equation of necking:

$$Z = \frac{S_0 - S_u}{S_0} \Rightarrow S_u = S_0(1-Z) = \frac{d_0^2\pi}{4}(1-Z) = \frac{(10\text{mm})^2\pi}{4}(1-0{,}6) = 31{,}4159\text{mm}^2\,.$$

Now we can calculate a real elongation:

$$\varphi_u = \ln\left(\frac{S_0}{S_u}\right) = \ln\left(\frac{78{,}5398\text{mm}^2}{31{,}4159\text{mm}^2}\right) = 0{,}9163,$$

and the true stress before the rupture:

$$\sigma_u = \frac{F_u}{S_u} = \frac{23000\text{N}}{31{,}4159\text{mm}^2} = 732{,}11\text{MPa}.$$

Now the 3 point of the true stress-strain curve has been calculated, what can be plotted in a *σ*–*φ* diagram. In this diagram the area below the curve is equal to the specific work of rupture. The approximate value of specific work of rupture can be calculated by the next equation below:

$$W_c \approx \frac{R_m + \sigma_u}{2}\varphi_u = \frac{(280{,}11 + 732{,}11)\text{MPa}}{2} \cdot 0{,}9163 = 1855\frac{\text{J}}{\text{cm}^3}.$$

Because:

$$1\frac{\text{N}}{\text{mm}^2} = 1\frac{\text{Nmm}}{\text{mm}^3} = \frac{0{,}001\text{J}}{0{,}001\text{cm}^3} = \frac{\text{J}}{\text{cm}^3}.$$