1. Introduction

Stability problems of thin walled structures are important in engineering practice. As regards circular plates loaded in their own plane we remark that a number of papers have dealt with the problem of how to determine the critical load since the first paper was published (Bryan, 1890). Without striving for completeness we cite paper (Nádai, 1915), which investigated some fundamental stability issues, together with some further papers (Meissner, 1933; Kawamoto, 1936; Iwato, 1939; Yamaki, 1959; Mansfield, 1960; Majumdar, 1971; Ramaiah and Vijayakumar, 1975). The effect of elastic restraints has been investigated by Thevendran (Thevendran and Wang, 1996).

The stability of a structure can be increased in various ways. We can apply, for example, an appropriate corrugation. Or, in the case of a circular plate, we can attach a stiffening along the diameter of the plate. It is Turvey who has dealt with a plate stiffened in this manner (Turvey and Salehi, 2008). However the stability issues are left out of consideration in his paper.

Another way to improve the resistance of a circular plate against buckling is the application of a stiffening ring. Paper (Turvey and Der Avanessien, 1989) investigates a plate stiffened such way. This paper is concerned among others with experimental results however stability issues are again left out of consideration.

The influence of a stiffening ring attached between the two boundaries to the middle surface of a circular plate on the stability is investigated in papers (Frostig and Simitses, 1987, 1988). The ring analysis is based on the engineering theory of curved beams. It turns out from the references that the authors do not know about the corresponding results of Szilassy (Szilassy, 1971, 1976).

In his PhD thesis (Szilassy, 1971) and in a further article (Szilassy, 1976) Szilassy investigated some stability problems of circular plates stiffened by a cylindrical shell on the outer boundary. Szilassy dealt with the stability problems of a solid circular plate and those of some annular plates. It was assumed that (i) the load is an in-plane axisymmetric dead one and (ii) the deformations of the annular plate and the cylindrical shell are also axisymmetric. For the circular plates he used the solution of a differential equation set up for the rotation field. As regards the cylindrical shell the solution was based on the theory of thin shells. After studying the work of Szilassy we think that the following questions are unanswered: (a) can we expand the range of solvable boundary value problems by using a differential equation set up for the displacement perpendicular to the middle plane of the plate, (b) under the condition of axisymmetric loading which deformation state does belong to the lowest critical load, i.e. the axisymmetric one or a non-axisymmetric deformation state (c) what happens if the load is not axisymmetric.

The present paper is concerned with the buckling of shell-stiffened circular plates under the assumption of non-axisymmetric deformations. Section 2 is the problem formulation. Sections 3 and 4 present the field equations for the cylindrical shell and the annular plates. Section 5 clarifies the boundary conditions and the continuity (discontinuity) conditions between the shell and plate. Computational results can be found in Section 6. The last Section is a conclusion. We remark that the paper lays an emphasis on the line of thought and the details are in general omitted.

For determining the non-axisymmetric deformations of the shell we shall utilize some results of Vlasov (Vlasov, 1962) and Jezső (Jezső, 1980).
2. Problem formulation

Fig. 1 shows the cross section of the structure we are concerned with. The structure shown consists of an annular plate (or a solid circular plate) which is stiffened by a cylindrical shell on its external boundary. The inner and external radii of the plate are denoted by \( R_i \) and \( R_e \), respectively. By assumption the latter coincides with the radius of the shell middle surface. The thicknesses of the plate and the shell are denoted by \( b_p \) and \( b_s \), respectively. The shell is symmetric with respect to the middle plane of the plate. Its height is \( h \). The load of the structure is a radially distributed constant dead force \( f_o \) which is exerted in the middle plane of the plate.

![Figure 1. The structure and its load](image)

We shall assume that the plate and the shell are thin, consequently we can use the Kirchhoff theory of plates and shells. We shall also assume that the problem is linear in respect of the kinematic equations and the material law. Heat effects are not taken into account. The plate and the shell are made of the same homogenous isotropic material for which \( E \) and \( \nu \) are the Young modulus and the Poisson ratio.

![Figure 2. Possible supports](image)

Under the assumption of small, non-axisymmetric and linearly elastic deformations we shall determine (a) the critical load of the structure and (b) the effect of the stiffening shell on the critical load.

Fig. 3 shows the coordinate systems and the displacement components both for the shell and for the plate.

![Figure 3. Coordinate systems](image)

3. Governing equations for the cylindrical shell

3.1. Equations for axisymmetric deformations

Since the load of the structure is axisymmetric, the in-plane load of the shell is axisymmetric as well. Consequently the deformations in the shell due to the in-plane load are also axisymmetric. For the axisymmetric part of the shell deformation we use the following differential equation (Thimosenko and Woinowski-Krieger, 1987, Chapter 15, p. 468):

\[
\frac{d^4 u_\zeta}{dx^4} + 4\beta^4 u_\zeta = \frac{1}{I_1 s E_1 s} \left( -p_2 - \nu \frac{N_{xz}}{R_e} \right)
\]

where \( p_2 \) is the constant radial load exerted on the middle surface of the shell, \( N_{xz} \) is the inner force in direction \( x \) (both values are zero), furthermore \( \beta = \nu_0 \sqrt{\frac{R_e}{b_s R_i}} \), \( \nu_0 = \sqrt{3(1-\nu_s^2)} \), \( I_1 s = b_s^2/12 \), \( E_1 s = E_s / (1-\nu_s^2) \).

Equation (1) is associated with the following boundary conditions – see Fig. 4 for the axisymmetric forces acting on the shell –

\[
Q_{xz} \big|_{x=0} = -\frac{f_o - f}{2},
\]

\[
\frac{du_\zeta}{dx} \big|_{x=0} = 0,
\]

\[
Q_{xz} \big|_{x=h} = 0,
\]

\[
M_{xx} \big|_{x=h} = 0,
\]

where \( Q_{xz} \) and \( M_{xx} \) denote the shear force and bending moment. Since \( u_\zeta(x) = u_\zeta(-x) \) due to the load, the rotation about the axis \( \varphi \) is zero.
– cf. equation (2b). The other boundary conditions are obvious.

\[ \xi = \frac{r}{R_e} \]

\[ Q_{zz} \]

\[ f \]

\[ f_0 \]

\[ \zeta \]

**Figure 4.** Axisymmetric forces on the shell

The solution for this partial load includes the distributed force \( f \) as unknown parameter. Theoretically this quantity can be calculated from the continuity condition

\[ u|_{R=R_e} = u|_{x=0} \]

which we prescribe on the intersection line of the middle surfaces of the plate and the shell. After some hand made calculations we obtain

\[ u|_{x=0} = u|_{R=R_e} = -\alpha (f_0 - f) \]

where

\[ \alpha = \frac{\nu_o}{2E} \left( R_e \right)^{\frac{3}{2}} \frac{\cos 2h\hat{\beta} + \cosh 2h\hat{\beta} + 2}{\sin 2h\hat{\beta} + \sinh 2h\hat{\beta}} \]

The actual displacement \( u_\zeta \) under axisymmetric deformations is a superposition of two solutions. The first one belongs to the above described partial load, the other to the partial load shown in Fig. 5.

\[ \kappa = \frac{\nu_o}{E} \sqrt{\frac{R_e \cos 2h\hat{\beta} + \cosh 2h\hat{\beta} + 2}{\sin 2h\hat{\beta} - \sinh 2h\hat{\beta}}} \]

**Eq. (6)** is a condition to be satisfied on the outer boundary of the plate under axisymmetric deformations.

### 3.2. Equations if the deformations are non-axisymmetric

The equations for non-axisymmetric deformations are presented in a bit more detail. Let \( u_\xi \), \( u_\varphi \) and \( u_\zeta \) be the three displacement components on the middle surface of the shell in the coordinate system shown in Fig. 3. Deformations on the middle surface are characterized by the axial strains \( \varepsilon_{xx} \) \( \varepsilon_{\varphi\varphi} \) \( \varepsilon_{x\varphi} \), the shear strain \( \varepsilon_{x\varphi} \), the rotation \( \vartheta_x \), and the elements \( \kappa_{xx} \) \( \kappa_{x\varphi} \) \( \kappa_{\varphi\varphi} \) of the curvature tensor (Jeszsõ, 1980):

\[ \varepsilon_{xx} = \frac{1}{R_e} \frac{\partial u_\xi}{\partial \xi} \]

\[ \varepsilon_{\varphi\varphi} = \frac{1}{R_e} \left( \frac{\partial u_\varphi}{\partial \varphi} + u_\zeta \right) \]

\[ \varepsilon_{x\varphi} = \frac{1}{2R_e} \left( \frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_\varphi}{\partial \xi} \right) \]

\[ \vartheta_x = \frac{1}{R_e} \frac{\partial u_\zeta}{\partial \xi} \]

\[ \kappa_{xx} = -\frac{1}{R_e^2} \frac{\partial^2 u_\zeta}{\partial \xi^2} \]
\[ \kappa_{\varphi\varphi} = -\frac{1}{R_s^2} \frac{\partial^2 u_\zeta}{\partial \varphi^2}, \quad (7f) \]
\[ \kappa_{x\varphi} = -\frac{1}{R_s^2} \frac{\partial^2 u_\zeta}{\partial x \partial \varphi}, \quad (7g) \]

The corresponding inner forces and bending moments are obtained from the Hooke law:

\[ N_{xx} = \dot{E}_s b_s (e_{xx} + \nu e_{\varphi\varphi}) \quad (8a) \]
\[ N_{\varphi\varphi} = \dot{E}_s b_s (e_{\varphi\varphi} + \nu e_{xx}) \quad (8b) \]
\[ N_{x\varphi} = \dot{E}_s b_s (1 - \nu) e_{x\varphi} \quad (8c) \]
\[ M_{xx} = \dot{E}_s \frac{b_s^3}{12} (\kappa_{xx} + \nu \kappa_{\varphi\varphi}) \quad (8d) \]
\[ M_{\varphi\varphi} = \dot{E}_s \frac{b_s^3}{12} (\kappa_{\varphi\varphi} + \nu \kappa_{xx}) \quad (8e) \]
\[ M_{x\varphi} = \dot{E}_s \frac{b_s^3}{12} (1 - \nu) \kappa_{x\varphi} \quad (8f) \]

The above equations are associated with the equilibrium equations:

\[ \frac{\partial N_{xx}}{\partial \xi} + \frac{\partial N_{x\varphi}}{\partial \varphi} + R_e p_x = 0 \quad (9a) \]
\[ \frac{\partial N_{x\varphi}}{\partial \xi} + \frac{\partial N_{\varphi\varphi}}{\partial \varphi} + \frac{1}{R_e} \left( \frac{\partial M_{x\varphi}}{\partial \xi} + \frac{\partial M_{\varphi\varphi}}{\partial \varphi} \right) + R_e p_\varphi = 0 \quad (9b) \]
\[ \frac{\partial^2 M_{xx}}{\partial \xi^2} + 2 \frac{\partial^2 M_{x\varphi}}{\partial \xi \partial \varphi} + \frac{\partial^2 M_{\varphi\varphi}}{\partial \varphi^2} - \frac{R_e N_{\varphi\varphi}}{R_e p_x} = 0 \quad (9c) \]

Observe that we have as many equations as there are unknowns (fifteen equations in fifteen unknowns).

For \( p_x = p_\varphi = 0 \) the fundamental equations (obtained after we have eliminated the intermediate variables) set up for the displacement coordinates \( u_\zeta, u_\varphi \) and \( u_\xi \) will be fulfilled identically if we calculate the displacement coordinates in terms of the Galerkin function \( \phi \) using the relations

\[ u_\zeta = \frac{\partial^2 \phi}{\partial \xi \partial \varphi^2} - \nu \frac{\partial^3 \phi}{\partial \xi^3}, \quad (10a) \]
\[ u_\varphi = -\frac{\partial^3 \phi}{\partial \varphi^3} - (2 + \nu) \frac{\partial^3 \phi}{\partial \xi^3 \partial \varphi}, \quad (10b) \]
\[ u_\xi = \nabla^2 \nabla^2 \phi, \quad (10c) \]
in which \( \phi \) should satisfy the differential equation

\[ \nabla^2 \nabla^2 \nabla^2 \phi + 4 \beta^2 \frac{\partial^4 \phi}{\partial \xi^4} = \frac{4 \beta^4 R_s^2}{Eb_s} p_z \quad (11a) \]
since the distributed load \( p_z \) exerted on the shell is zero in the present problem. In the above equation

\[ \nabla^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \varphi^2}, \quad (11b) \]
\[ \beta^4 = 3 \left( 1 - \nu^2 \right) \frac{R_s^2}{b_s^2} \quad (11c) \]

Let us expand \( \phi \) into a Fourier series:

\[ \phi (\xi, \varphi) = \mathcal{F}_0 (\xi) + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \mathcal{F}_m (\xi) \cos \left( n \varphi - m \frac{\pi}{2} \right). \quad (12) \]

It follows from (11) that the Fourier coefficients should satisfy the equation

\[ \frac{d^6 \mathcal{F}_m}{d \xi^6} - 4n^2 \frac{d^6 \mathcal{F}_m}{d \xi^6} + 6n^4 \frac{d^4 \mathcal{F}_m}{d \xi^4} - 4n^6 \frac{d^4 \mathcal{F}_m}{d \xi^4} - 4 \beta^4 \frac{d^4 \mathcal{F}_m}{d \xi^6} = 0. \quad (13) \]

The characteristic polynomial of this equation takes the form

\[ (\lambda_n^2 - n^2)^4 = -4 \beta^4 \lambda_n^2. \quad (14) \]

The roots of this polynomial are as follows:

\[ \lambda_{n1} = -\beta_{n1} + i \alpha_{n1}, \quad (15a) \]
\[ \lambda_{n2} = -\beta_{n2} + i \alpha_{n2}, \quad (15b) \]
\[ \lambda_{n3} = \lambda_{n1}, \quad (15c) \]
\[ \lambda_{n4} = \lambda_{n2}, \quad (15d) \]
\[ \lambda_{n+l} = -\lambda_n, \quad l = 1, \ldots, 4 \quad (15e) \]

where

\[ \beta_{n1} = b_n + \frac{\beta}{2}, \quad \beta_{n2} = b_n - \frac{\beta}{2}, \quad (15f) \]
\[ \alpha_{n1} = \frac{\beta}{2} + a_n, \quad \alpha_{n2} = \frac{3}{2} - a_n, \quad (15g) \]
\[ b_n = \frac{\beta}{2} \sqrt{1 + 4 \left( \frac{n}{\beta} \right)^4 + 2 \left( \frac{n}{\beta} \right)^2}, \quad (15h) \]
It can be shown that the real solution of equation (13) is
\[ m \mathcal{F}_n = \sum_{k=1}^{m} \left[ K_{nk} \sinh (\beta_{nk} \xi) \sin (\alpha_{nk} \xi) + \right. \\
+ M_{nk} \sinh (\beta_{nk}) \cos (\alpha_{nk} \xi) + \\
+ \left. P_{nk} \cosh (\beta_{nk} \xi) \sin (\alpha_{nk} \xi) + \\
+ S_{nk} \cosh (\beta_{nk} \xi) \cos (\alpha_{nk} \xi) \right] \] 
\quad (16)
where the quantities $K_{nk} \ldots S_{nk}$ are altogether eight integration constants.

In what follows the Fourier series of a physical quantity – denoted say by $Q$ – will be written in the same form as series (12).

\[ Q = Q_0 + \sum_{m=1}^{\infty} \sum_{n=1}^{m} Q_n (\xi) \cos \left( n \varphi - m \frac{\pi}{2} \right). \] 
\quad (17)

Omitting the hand made calculations, we shall present the amplitude functions of those physical quantities here which are involved in the boundary- and continuity conditions:
\[ u_{\xi_n} = -n^2 \mathcal{F}_n (1) - m \mathcal{F}_n (3), \quad (18a) \]
\[ u_{\varphi_n} = -n^3 \mathcal{F}_n (2) + (2 + \nu) \mathcal{F}_n (2), \quad (18b) \]
\[ u_{\xi_n} = n^4 \mathcal{F}_n - 2n^2 \mathcal{F}_n (2) + \mathcal{F}_n (4), \quad (18c) \]
\[ \dot{\varphi}_x = \frac{1}{Re} \left( n^4 \mathcal{F}_n (1) - 2n^2 \mathcal{F}_n (3) + \mathcal{F}_n (5) \right), \quad (18d) \]
\[ \frac{m}{N_{xx}} = \frac{2b_{z}E}{Re} \mathcal{F}_n (2), \quad (18e) \]
\[ \frac{m}{N_{x\varphi}} = \frac{2b_{z}E}{Re} \mathcal{F}_n (3), \quad (18f) \]
\[ \frac{m}{M_{xx}} = b_{z}E \left[ \frac{m}{F_n (6)} - n^2 (2 + \nu) \mathcal{F}_n (4) + \\
+ (1 + \nu) n^4 \mathcal{F}_n (2) - n^6 \mathcal{F}_n \right], \quad (18g) \]
\[ \frac{m}{Q_{xx}} = b_{z}E \left[ -n^6 \mathcal{F}_n (1) + 3n^4 \mathcal{F}_n (3) - \\
- 3n^2 \mathcal{F}_n (5) + \mathcal{F}_n (7) \right]. \quad (18h) \]

4. Governing equations for the circular plate

4.1. Equations for the in-plane load

We shall assume that the plane stress problem in the plate due to the load $f$ is axisymmetric i.e., the inner forces $N_R$, $N_\varphi$ and $N_{R\varphi}$ satisfy the relations
\[ N_R = -A + \frac{B}{\rho^2} \quad (19a) \]
\[ N_\varphi = -A + \frac{B}{\rho^2} \quad (19b) \]
\[ N_{R\varphi} = 0 \quad (19c) \]
where we use the dimensionless coordinate $\rho = \frac{R_i}{R_e} = \frac{R_i}{R_e}$. The constants $A$ and $B$ depend on the boundary conditions. For a solid circular plate
\[ A = f \quad \text{and} \quad B = 0. \quad (20) \]
The radial displacement on the external can be calculated by using the relation
\[ u_{\rho} = - (1 - \nu) \frac{Re}{b_{\rho}} \frac{f}{E}. \quad (21) \]

4.2. Equations for the displacement field after stability loss

If the plate is solid the displacement $w$ vertical to the middle surface should fulfill the differential equation
\[ \ddot{w} + \ddot{w} = 0 \quad (22) \]
where
\[ \ddot{w} = \frac{R_i^2 f}{I_{1p}E_{1p}}, \quad \ddot{w} = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \quad (23a) \]
and
\[ I_{1p} = \frac{b_{\rho}^3}{12}, \quad E_{1p} = \frac{E_{\rho}}{1 - \nu^2}. \quad (23b) \]

Similarly to equation (12), let us expand the solution for $w$ into a Fourier series:
\[ w = w_0 + \sum_{m=0}^{\infty} \sum_{n=1}^{m} \frac{m}{m} w_n (\rho) \cos \left( n \varphi - m \frac{\pi}{2} \right). \quad (24) \]
After substituting solution (24) into (22) we obtain that the amplitudes $w_0(\rho)$ and $m$ $w_n(\rho)$
should fulfill the following differential equations:

\[ \hat{\Delta}_n \hat{\Delta}_n w + \hat{\delta}_n \hat{\delta}_n w = 0 \]  
\( m = 0, 1 \quad n = 0, 1, 2, \ldots \)  

where

\[ \hat{\Delta}_n = \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} - \frac{n^2}{\rho^2} . \]  

(25b)

These equations have closed form solutions:

\[ w_o = c_1 + c_2 \ln \rho + c_3 J_o(\sqrt{\delta_\rho}) + c_4 Y_o(\sqrt{\delta_\rho}) \]  
\( m = 0, 1 \quad n = 1, 2 \)  

(26a)

\[ w_m = c_1 \rho^m + c_2 \rho^{-n} + 
+ c_3 J_m(\sqrt{\delta_\rho}) + c_4 Y_m(\sqrt{\delta_\rho}) \]  
\( m = 0, 1 \quad n = 1, 2 \)  

(26b)

Every physical quantity can be written in a form similar to that of equation (17) – we should write \( \rho \) instead of \( \xi \) there. One can show that the Fourier coefficients – the amplitude functions – for the rotation \( \psi_x \) the bending moment \( M_{Rn} \) and the shear force \( Q_R \) can all be given in terms of the amplitudes of \( w \) as follows:

\[ \psi_{\varphi n} = -\frac{1}{\rho} \frac{d}{d\rho} \frac{d^m w_n}{d\rho} \]  

(27a)

\[ M_{Rn} = -\frac{1}{R_e} \frac{E_1}{\rho} \left[ \frac{d^m w_n}{d\rho} + \frac{\nu m^2}{\rho^2} \frac{d^m w_n}{d\rho} \right] \]  

(27b)

\[ Q_{Rn} = \frac{f}{R_e} \frac{d^m w_n}{d\rho} + \]  

\[ + \frac{1}{R_e} \frac{d}{d\rho} \left( \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} - \frac{n^2}{\rho^2} \right) \frac{d^m w_n}{d\rho} \]  

(27c)

5. Boundary and continuity conditions

A solution for the amplitude of the displacement field on the middle surface of the plate contains 4, while a solution for \( \mathcal{F}_n \) involves 8 integration constants. The stiffening shell is mentally divided in two separate shells at the intersection line of the middle surfaces of the plate and the shell. Thenthere we need two solutions for each of the two parts of the shell, consequently we have to determine altogether 20 integration constants.

In what follows we shall present those boundary- and continuity conditions, which provide the integration constants.

The boundary conditions on the inner boundary of the plate depend on the supports applied. It is clear that two boundary conditions can be prescribed on the inner boundary. If the plate has no hole in it then the displacement \( w_n \) and the rotation \( \psi_n \) has to be finite:

\[ \frac{m}{w_n} = \text{finite} \]  
\[ \frac{m}{\psi_n} = \text{finite} \]  

(28a)

(28b)

The shell and plate deform together on the intersection line of the middle surfaces of the shell and the plate which results in the following kinematic continuity conditions

\[ u_{\xi n}(\xi = +0) = -w_n(\rho = 1) \]  
\[ u_{\xi n}(\xi = -0) = -w_n(\rho = 1) \]  
\[ \vartheta_{x n}(\xi = +0) = \psi_n(\rho = 1) \]  
\[ \vartheta_{x n}(\xi = -0) = \psi_n(\rho = 1) \]  
\[ u_{\zeta n}(\xi = +0) = u_n(\rho = 1) = 0 \]  
\[ u_{\zeta n}(\xi = -0) = u_n(\rho = 1) = 0 \]  
\[ u_{\varphi n}(\xi = +0) = v_n(\rho = 1) = 0 \]  
\[ u_{\varphi n}(\xi = -0) = v_n(\rho = 1) = 0 \]  

(29a)

(29b)

(29c)

(29d)

(29e)

(29f)

(29g)

(29h)

Here the two sides of the plate at \( \xi = 0 \) are designated by \( \xi = +0 \) and \( \xi = -0 \) – see Fig. 4. Observe that conditions (29e,f,g,h) reflect the fact that the plane stress problem is axisymmetric.

For the shear force \( Q_{x x n} \) we can not prescribe any condition, since \( u_{z n}(\xi = 0) = 0 \) As regards the axisymmetric part, equations (4) and (21) should also be fulfilled. Since \( \frac{m}{v_n}(\rho = 1) = 0 \) we can not prescribe continuity conditions for the inner forces \( \hat{N}_{R_x \varphi n} \) and \( \hat{N}_{x \varphi \varphi n} \). However the axisymmetric parts of these quantities are also equal to zero.

It follows from the global equilibrium of the structure that the axisymmetric part of the shear force should meet the condition \( Q_{Rn} = 0 \). Otherwise the continuity condition

\[ \frac{m}{Q_{Rn}}(\rho = 1) - \hat{N}_{x x n}(\xi = +0) + \]  

\[ + \hat{N}_{x x n}(\xi = -0) = 0 \]  

(30)
should be fulfilled.

As regards the bending moments equation
\[ M_R n (\rho = 1) - M_{x n} (\xi = +0) + M_{x n} (\xi = -0) = 0 \] (31)
is the continuity condition.

Since the boundaries of the shell with coordinates \( \xi = h/R_e \) and \( \xi = -h/R_e \) are free, the following boundary conditions should be satisfied:
\[ N_{x n} (\xi = h/R_e) = 0 , \] (32a)
\[ M_{x n} (\xi = h/R_e) = 0 , \] (32b)
\[ N_{x \varphi n} (\xi = h/R_e) + \frac{m}{R_e} M_{x \varphi n} (\xi = h/R_e) = 0 , \] (32c)
\[ Q_{x n} (\xi = h/R_e) - \frac{n}{R_e} M_{x n} (\xi = h/R_e) = 0 \] (32d)
\[ N_{x n} (\xi = -h/R_e) = 0 , \] (32e)
\[ M_{x n} (\xi = -h/R_e) = 0 , \] (32f)
\[ N_{x \varphi n} (\xi = -h/R_e) + \frac{m}{R_e} M_{x \varphi n} (\xi = -h/R_e) = 0 , \] (32g)
\[ Q_{x n} (\xi = -h/R_e) - \frac{n}{R_e} M_{x n} (\xi = -h/R_e) = 0 . \] (32h)

The boundary- and continuity conditions (28), (29), (30), (31) and (32) provide twenty homogenous equations for the twenty integration constants. These equations involve \( f \) as a parameter. Therefore the critical value of \( f \) can be determined from the condition that the system determinant should vanish.

6. Numerical results

We have made numerical computations for a solid plate. The graphs in Fig. 6 provide the critical load of the plate in terms of the height \( h \) of the stiffening shell. It is clear from Fig. 6 that the stiffening significantly increases the critical load as the height \( h \) is increased till it reaches a certain limit. The curves show the critical load for the axisymmetric deformation and the first 4 members of the Fourier-series. One can see that the lowest value of the critical load belongs to the case of the axisymmetric deformation. We have used the following data for the computations: \( E = 2 \cdot 10^5 \) MPa, \( \nu = 0.3, b_0 / R_e = 1, b_T / R_e = 0.01 \).

7. Concluding remarks

The present paper has established the equations that can be used to determine the critical load of a circular plate (solid or with a hole) stiffened by a cylindrical shell under the assumption of non-axisymmetric deformations. We have clarified what the continuity conditions are between the two separate elements of the structure. We have also presented the solutions for the critical load of the solid circular plate assuming axisymmetric and non-axisymmetric deformations. It is obvious from the results that the stiffening significantly increases the critical load.

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