# Practice Sheets for Calculus

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# Practice Sheets for Calculus

Calculus 60A101 and 60A102 (lecture and seminar) is a first-year, first-semester course taught in English intended for foreign students studying business or economics at the University of Szeged. The following topics are presented: functions, graphs, limits, differentiation, techniques of differentiation, optimization, business applications, exponential and logarithmic functions, techniques and applications of integration.

It is assumed that the students have a basic knowledge of algebra (factoring polynomials, simplifying fractions, basic laws for exponents and roots, solving linear and quadratic equations), coordinate geometry, functions and their graphs, moreover some basic notions of economics including demand, price, cost and profit. These are reviewed at the appropriate points of the course.

The course uses the following textbook:

Business Calculus Demystified, by Rhonda Huettenmueller, Mc-Graw-Hill.

Each class covers one chapter of the textbook. There is a list of problems for each class that the students can find on Coospace. These are intended for independent practice at home.

## Learning Outcomes

### (a) Knowledge

In order to pass the course the student should be able to perform the following tasks.

- Find the equation of linear functions from various data. (Chapter 1, List 1)
- Find limits of functions analytically, numerically and graphically. (Chapter 2, List 2)
- Find the derivative of a function using the limit definition. (Chapter 3, List 3)
- Find the derivative of a function by identifying and applying the appropriate derivative formula. Find the equation of the tangent line. (Chapter 4, List 4)
- Understand the interpretation of the derivative as a rate of change. (Chapter 5, List 5)
- Find the derivative of a function using the Chain Rule. (Chapter 6, List 6)
- Use the first derivative to determine intervals on which the graph of a function is increasing or decreasing. Find and classify relative extrema of a function. Find the absolute extrema of a continuous function on a closed interval. (Chapter 8, List 7)
- Use the second derivative to determine intervals on which the graph of a function is concave upwards or concave downwards and to determine points of inflection. Find the relative extrema of a function using the Second Derivative Test. (Chapter 9, List 8)



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• Understand the business terminology of demand, cost, price, revenue, and profit, and solve applied optimization problems. (Chapter 10, List 9)

• Find the derivative of exponential and logarithmic functions. Analyze exponential and logarithmic functions. Find the derivative using logarithmic differentiation. (Chapter 11, List 10)

• Find antiderivatives and indefinite integrals using integration formulas, the method of substitution and integration by parts. (Chapter 12, List 11)

• Use the Fundamental Theorem of Calculus to evaluate definite integrals. Use definite integrals to calculate the area of the region under a curve and the area of the region between two curves. (Chapter 13, List 12)

**(b) Skills**

By the time the students finish the course, they should be able to

– use the basic vocabulary of economics and calculus correctly in writing and in oral communication;

– uncover facts and basic connections, arrange and analyse data systematically, draw conclusions and make critical observations along with preparatory suggestions;

– analyze and solve problems in economics using the tools of calculus keeping in mind the conditions under which the methods work and the limitations of the models used.

**(c) Attitude**

By the time the students finish the course, they should be able to

– work together with others in teams and projects in a constructive, cooperative and active way;

– conduct the tasks independently and responsibly as a member of projects or teams.

**(d) Autonomy/Responsibility**

By the time the students finish the course, they should be able to

– take responsibility for their analysis, conlusions and decisions;

– know and keep the rules and ethical norms of cooperation and leadership as part of a project, a team and a work organisation.

## 1. Linear functions

Let $L$ be a straight line in the plane which is not vertical. Choose two different points $(x_1, y_1)$ and $(x_2, y_2)$ on $L$. The **slope** $m$ of $L$ is defined as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Vertical lines have no slope.

**The Point-Slope Equation.** The equation of the line with slope $m$ through the point $(x_0, y_0)$ is

$$y - y_0 = m(x - x_0).$$

**The Slope-Intercept Equation.** The equation of the line with slope $m$ and $y$-intercept $b$ is

$$y = mx + b.$$

(We say that the $y$-intercept is $b$ if the line intersects the $y$-axis at distance $b$ from the origin, that is, the line intersects the $y$-axis at the point $(0, b)$.)

### Problems

1. Find the slope of the straight line that passes through the points $(1, 2)$ and $(5, 7)$.
2. Find the slope of the straight line that passes through the points $(1, 5)$ and $(3, 1)$.
3. Find the slope of the straight line that passes through the points $(1, 5)$ and $(8, 5)$.
4. Write an equation for the straight line that passes through the point $(-2, -3)$ and has slope $1/2$.
5. Write an equation for the straight line through the points $(1, -1)$ and $(3, 5)$.
6. Write an equation for the straight line that passes through $(1, 5)$ and is parallel to the line with equation $2x + y = 8$.
7. Plot the three given points, and determine whether or not they lie on a single line.

(a) $(-1, -2), (2, 1), (4, 3)$
(b) $(-2, 5), (2, 3), (8, 0)$
(c) $(-1, 6), (1, 2), (4, -2)$

8. The Fahrenheit temperature F and the Celsius temperature C satisfy a linear equation. Given that $F = 32$ when $C = 0$ and $F = 212$ when $C = 100$, express $F$ in terms of $C$. What is the value of $F$ when $C = 36.5$? What is the value of $C$ when $F = 0$?



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**Results**

1. $5/4$
2. $-2$
3. $0$
4. $y = \frac{1}{2}x - 2$
5. $y = 3x - 4$
6. $2x + y = 7$
7. yes, no, no
8. $F = \frac{9}{5}C + 32$, 97.7, $-17.78$



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## 2. Limits

1. Find the limit to 3 decimals using a calculator.

(a) $\lim_{x\to 0^+} x^x$ (b) $\lim_{x\to 0}(1+x)^{1/x}$ (c) $\lim_{x\to 0} \frac{2^x-1}{x}$

2. Find the limit using the limit rules.

(a) $\lim_{x\to 3} \frac{x^2+2x+5}{x^2-4}$ (b) $\lim_{x\to 4} \frac{x^2-16}{x-4}$ (c) $\lim_{x\to 5} \frac{x^2-25}{x+7}$

(d) $\lim_{x\to -7} \frac{x^2-25}{x+7}$ (e) $\lim_{x\to 0} \frac{(3+x)^2-9}{x}$ (f) $\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{5+x}-\frac{1}{5}\right)$

3. Find the limit.

(a) $\lim_{x\to 3}(x^2+xy-5x+3y)$ (b) $\lim_{h\to 0} \frac{4xh^2-2h}{h}$ (c) $\lim_{h\to 0} \frac{5xh-3x^2h+h^2}{h}$



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**Results:**

1. (a) 1
(b) 2.718
(c) 0.693

2. (a) 4
(b) 8
(c) 0
(d) does not exist
(e) 6
(f) $-1/25$

3. (a) $6y - 6$
(b) $-2$
(c) $5x - 3x^2$

## 3. The derivative

Find the derivative of the given functions using the definition.
(Form the quotient $\frac{f(x+h)-f(x)}{h}$; simplify; take the limit.)

(a) $f(x) = 5 - 7x$

(b) $g(x) = 10$

(c) $h(x) = 3x^2 + 8$

(d) $j(x) = \dfrac{3}{x+5}$

(e) $k(x) = \dfrac{x}{2x+1}$



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**Results.**

(a) $f'(x) = -7$

(b) $g'(x) = 0$

(c) $h'(x) = 6x$

(d) $j'(x) = \dfrac{-3}{(x+5)^2}$

(e) $k'(x) = \dfrac{1}{(2x+1)^2}$



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## 4. Rules of differentiation

1. Find the derivative.

(a) $f(x) = \dfrac{1}{x^4}$ (b) $g(x) = \sqrt[5]{x^3}$ (c) $h(x) = \dfrac{1}{\sqrt[7]{x}}$

2. Find the derivative.

(a) $f(x) = (x^2 + 3x)(x^3 - 2x)$

(b) $g(x) = (x^3 - 2x^2 + 5)(x^5 + 1)$

(c) $h(x) = (\sqrt{x} + 1)(\sqrt[5]{x} + 1)$

3. Find the derivative.

(a) $f(x) = \dfrac{x^2 + 1}{x^3 - 2}$

(b) $g(x) = \dfrac{x^3 - 3x + 7}{x^5 + 1}$

(c) $h(x) = \dfrac{\sqrt{x} + 1}{\sqrt[3]{x} + 1}$

4. Find the equation of the tangent line to the curve at the given point.

(a) $f(x) = x^2 - 3x + 5,\ P = (1, 3)$

(b) $g(x) = 2x^3 - 5x - 9,\ P = (2, -3)$

(c) $h(x) = 3\sqrt[3]{x} - 4\sqrt{x} - 1,\ P = (1, -2)$



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1. (a) $f'(x) = -4x^{-5} = -\frac{4}{x^5}$

(b) $g'(x) = \frac{3}{5}x^{-2/5}$

(c) $h'(x) = -\frac{1}{7}x^{-8/7}$

2. (a) $f'(x) = (2x+3)(x^3-2x) + (x^2+3x)(3x^2-2)$

(b) $g'(x) = (3x^2-4x)(x^5+1) + (x^3-2x^2+5)5x^4$

(c) $h'(x) = \frac{1}{2}x^{-1/2}(\sqrt[5]{x}+1) + (\sqrt{x}+1)\frac{1}{5}x^{-4/5}$

3. (a) $f'(x) = \frac{2x(x^3-2) - (x^2+1)3x^2}{(x^3-2)^2}$

(b) $g'(x) = \frac{(3x^2-3)(x^5+1) - (x^3-3x+7)5x^4}{(x^5+1)^2}$

(c) $h'(x) = \frac{\frac{1}{2}x^{-1/2}(\sqrt[3]{x}+1) - (\sqrt{x}+1)\frac{1}{3}x^{-2/3}}{(\sqrt[3]{x}+1)^2}$

4. (a) $y = -x + 4$

(b) $y = 19x - 41$

(c) $y = -x - 1$



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## 5. Rates of change

1. We know that a falling object falls $y = 16t^2$ feet after $t$ seconds.

(a) What is the object's average velocity between 4 seconds and 8 seconds?

(b) What is the object's instantaneous velocity at 6 seconds?

2. The cost for producing $x$ units of a product is given by $C(x) = x^2 + 5x + 102$. Find the marginal cost for producing 20 units. Interpret this number.

3. The revenue function for a product is $R(t) = 0,93t^4 - 39t^3 + 562t^2 - 3332t + 8233$, where $t$ is the number of years after the introduction of the product. How fast was the revenue changing at the end of the second year?

4. The revenue for a product is $R(x) = -0,005x^2 + 11x - 5400$ and the cost is $C(x) = 0,015x + 15$, for $x$ units produced and sold. Find the marginal profit for 800 units. Interpret this number.

### Results

1. (a) 192 ft/s

(b) 192 ft/s

2. $C'(20) = 45$. As the production increases from 20 to 21 units, the cost increases by approximately 45.

3. $R'(2) = -2646,24$. The revenue was decreasing at the rate of 2646 per year.

4. $P'(800) = 2,985$. As the production and sales increase from 800 to 801 units, the profit increases by approximately $2,985$.

## 6. The Chain Rule

5. Find the derivative of the following functions:

(a) $f(x) = (3x^2 - 4)^7$

(b) $g(x) = (6x^3 - 2x^2 + 5)^5$

(c) $h(x) = \sqrt{x^2 - 3x + 8}$

(d) $j(x) = \dfrac{1}{\sqrt[3]{2x+3}}$

6. Calculate $\dfrac{dy}{dx}$.

(a) $y = u^2 - 6$ and $u = 5x^2 + 1$

(b) $y = u^3 + u^2 - 5$ and $u = 4x + 6$

(c) $y = \dfrac{4}{u}$ and $u = 14x + 9$



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## Results

1. (a) $f'(x) = 42x(3x^2 - 4)^6$

(b) $g'(x) = 5(6x^3 - 2x^2 + 5)^4(18x^2 - 4x)$

(c) $h'(x) = \frac{1}{2}(x^2 - 3x + 8)^{-1/2}(2x - 3)$

(d) $j'(x) = -\frac{2}{3}(2x + 3)^{-\frac{4}{3}}$

2. (a) $\frac{dy}{dx} = 20(5x^2 + 1)x$

(b) $\frac{dy}{dx} = 12(4x + 6)^2 + 8(4x + 6)$

(c) $\frac{dy}{dx} = -\frac{56}{(14x + 9)^2}$



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## 7. The First Derivative Test

1. Where are the functions increasing, decreasing?

(a) $f(x) = -4x^3 + 3x$ (b) $f(x) = 5x^3 + 6x$ (c) $f(x) = \dfrac{x}{x+1}$

2. Where is the function $f(x) = \sqrt[3]{(x^2-1)^2}$ increasing, decreasing?

3. What are the relative extrema for the function on the given interval?

(a) $f(x) = 3x + 2$, $[-2, 3]$ (b) $f(x) = 4 - x^2$, $[1, 3]$

(c) $f(x) = x^2 + 4x + 7$, $[-3, 0]$ (d) $f(x) = x^3 - 3x$, $[-2, 4]$

(e) $f(x) = x + \dfrac{4}{x}$, $[1, 4]$ (f) $f(x) = 3x^5 - 5x^3$, $[-2, 2]$



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**Results**

1. (a) increasing on $(-1/2, 1/2)$
decreasing on $(-\infty, -1/2)$ and $(1/2, \infty)$

(b) increasing on $(-\infty, \infty)$

(c) increasing on $(-\infty, -1)$ and $(-1, \infty)$

2. increasing on $(-1, 0)$ and $(1, \infty)$
decreasing on $(-\infty, -1)$ and $(0, 1)$

3. (a) maximum is 11, at $x = 3$
minimum is $-4$, at $x = -2$

(b) maximum is 3, at $x = 1$
minimum is $-5$, at $x = 3$

(c) maximum is 7, at $x = 0$
minimum is 3, at $x = -2$

(d) maximum is 52, at $x = 4$
local maximum at $x = -1$, $f(-1) = 2$
minimum is $-2$, at $x = -2$ and $x = 1$

(e) maximum is 5, at $x = 1$ and $x = 4$
minimum is 4, at $x = 2$

(f) maximum is 56, at $x = 2$
local maximum at $x = -1$, $f(-1) = 2$
minimum is $-56$, at $x = -2$
local minimum at $x = 1$, $f(1) = -2$



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**Solutions of selected problems**

1. (a) $f'(x) = -12x^2 + 3 = 0$
the critical points: $x = 1/2$ and $x = -1/2$
$f'(-1) = -9 < 0$; $f'(0) = 3 > 0$; $f'(1) = -9 < 0$
This implies that $f'$ is negative on $(-\infty, -1/2)$ and $(1/2, \infty)$ and positive on $(-1/2, 1/2)$.
Therefore $f$ is increasing on $(-1/2, 1/2)$ and
decreasing on $(-\infty, -1/2)$ and $(1/2, \infty)$.

(b) $f'(x) = 15x^2 + 6 = 0$
There are no critical points. $f'(x)$ is always positive.
So $f$ is increasing on $(-\infty, \infty)$.

(c) $f$ is not defined for $x = -1$.
$f'(x) = \dfrac{1}{(x+1)^2} = 0$ There are no solutions.
The only critical point is $x = -1$.
$f'(x)$ is always positive, so $f$ is
increasing on $(-\infty, -1)$ and $(-1, \infty)$.

2. $f(x) = (x^2 - 1)^{2/3}$;
$f'(x) = \dfrac{2}{3} \cdot (x^2 - 1)^{-1/3} \cdot 2x = \dfrac{4}{3} \cdot \dfrac{x}{\sqrt[3]{x^2 - 1}} = 0$
The only solution is $x = 0$ but $f'(x)$ does not exist for $x = -1$ and $x = 1$ (the denominator is 0). So the critical points are $-1, 0, 1$.
$f'(-2) < 0$, $f'(-1/2) > 0$, $f'(1/2) < 0$, $f'(2) > 0$
So $f$ is increasing on $(-1, 0)$ and $(1, \infty)$
and decreasing on $(-\infty, -1)$ and $(0, 1)$.

3. (e) $f'(x) = 1 - \dfrac{4}{x^2} = 0$
$x = 2$ and $x = -2$
$f'(1) = -3 < 0$; $f'(3) > 0$ so
$f$ is decreasing on $[1, 2)$ and increasing on $(2, 4]$.
So the minimum is at $x = 2$; $f(2) = 4$.
Since $f(1) = f(4) = 5$, the maximum is 5.



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## 8. Concavity and the Second Derivative Test

1. Determine where the function is concave up and where it is concave down. Find the inflection points.

(a) $f(x) = 5x^2 - 4$

(b) $f(x) = -x^4 + 6x^2 + 7x + 5$

(c) $f(x) = x^4 - 9x^3 + 12x^2 + x - 2$

(d) $f(x) = \dfrac{2x^3}{x^2 - 9}$

2. Find the relative extrema of the function using the Second Derivative Test.

(a) $f(x) = 3x^4 + 14x^3 - 12x^2 + 10$

(b) $f(x) = x^2(x-1)^2$

(c) $f(x) = \dfrac{x^2}{x^2 + 1}$



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**Results**

1.

(a) concave up: $(-\infty, \infty)$, no inflection points

(b) concave up: $(-1, 1)$, concave down: $(-\infty, -1)$ and $(1, \infty)$,
inflection points: $(-1, 3)$ and $(1, 17)$

(c) concave up: $(-\infty, 1/2)$ and $(4, \infty)$, concave down: $(1/2, 4)$,
inflection points: $(1/2, 7/16)$ and $(4, -126)$

(d) concave up: $(-3, 0)$ and $(3, \infty)$, concave down: $(-\infty, -3)$ and $(0, 3)$,
inflection point: $(0, 0)$

2.

(a) $f(0) = 10$: local maximum
$f(1/2) = 143/16$: local minimum
$f(-4) = -310$: local minimum

(b) $f(0) = 0$: local minimum
$f(1) = 0$: local minimum
$f(1/2) = 1/16$: local maximum

(c) $f(0) = 0$: local minimum



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**Solutions**

1. (a) $f''(x) = 10$ so $f''(x) > 0$ for all $x$;
concave up: $(-\infty, \infty)$, no inflection points

(b) $f''(x) = -12x^2 + 12 = 0$
$x = -1$ and $x = 1$
$f''(-2) < 0$; $f''(0) > 0$; $f''(2) < 0$
concave up: $(-1, 1)$, concave down: $(-\infty, -1)$ and $(1, \infty)$,
inflection points: $(-1, 3)$ and $(1, 17)$

(c) $f''(x) = 12x^2 - 54x + 24 = 0$
$x = 1/2$ and $x = 4$
$f''(0) = 24 > 0$; $f''(1) < 0$; $f''(5) = 54 > 0$
concave up: $(-\infty, 1/2)$ and $(4, \infty)$, concave down: $(1/2, 4)$,
inflection points: $(1/2, 7/16)$ and $(4, -126)$

2.
(a) $f'(x) = 12x^3 + 42x^2 - 24x = 6x(2x^2 + 7x - 4) = 0$
$x = 0$, $x = -4$, $x = 1/2$
$f''(x) = 36x^2 + 84x - 24$
$f''(0) < 0$ so $f(0) = 10$: local maximum
$f''(1/2) = 27 > 0$ so $f(1/2) = 143/16$: local minimum
$f''(-4) = 216 > 0$ so $f(-4) = -310$: local minimum

(b) $f(x) = x^2(x^2 - 2x + 1) = x^4 - 2x^3 + x^2$
$f'(x) = 4x^3 - 6x^2 + 2x = 2x(2x^2 - 3x + 1) = 0$
$x = 0$, $x = 1$, $x = 1/2$
$f''(x) = 12x^2 - 12x + 2$
$f''(0) = 2 > 0$ so $f(0) = 0$: local minimum
$f''(1) = 2 > 0$ so $f(1) = 0$: local minimum
$f''(1/2) = -1 < 0$ so $f(1/2) = 1/16$: local maximum

(c) $f'(x) = \dfrac{2x}{(x^2 + 1)^2} = 0$
$x = 0$
$f''(x) = \dfrac{2 - 4x^2 - 6x^4}{(x^2 + 1)^4}$
$f''(0) = 2 > 0$ so $f(0) = 0$: local minimum

## 9. Business applications

1. The profit for selling $x$ units of a product can be approximated by
$P(x) = -x^3 + 45x^2 + 1200x + 80000$, $(0 \leq x \leq 50)$. What level of sales maximizes the profit?

2. The revenue function for a product is $R(p) = -0{,}04p^2 + 0{,}06p + 9{,}9775$, where $p$ is the price of the product. What price maximizes the revenue?

3. The revenue for a product is $R(x) = \dfrac{8x}{0{,}2x+1}$, and the cost is $C(x) = 0{,}5x$, for $x$ units produced and sold. How many units should be sold to maximize profit?

4. The cost for producing $x$ units of a product is given by $C(x) = 0{,}004x^2 - 9{,}6x + 7840$. What level of production minimizes the average cost?

### Results

1. $x = 40$; the maximum profit is 136000.

2. $p = 0{,}75$; the maximum revenue is 10.

3. $x = 15$; the maximum profit is 22,5.

4. $A(1400) = 1{,}6$

## 10. Exponential and logarithmic functions

1. Find the derivative of the following functions.

(a) $y = e^{3x^2-4}$

(b) $y = 5^{6x^3-2x^2+5}$

(c) $y = \ln(\sqrt{x^2-3x+8})$

(d) $y = \log_3 \frac{5x-4}{2x+3}$

(e) $y = 3^{3^x}$

2. Where is the function $f(x) = \frac{\ln x}{x}$, $x > 0$, increasing, decreasing? Find the inflection points.

### Results

1. (a) $y' = 6xe^{3x^2-4}$

(b) $y' = \ln 5 \cdot (18x^2 - 4x) \cdot 5^{6x^3-2x^2+5}$

(c) $y' = \frac{1}{2} \cdot \frac{2x-3}{x^2-3x+8}$

(d) $y' = \frac{1}{\ln 3} \cdot \left( \frac{5}{5x-4} - \frac{2}{2x+3} \right)$

(e) $y' = (\ln 3)^2 \cdot 3^{3^x+x}$

2. increasing on $(0, e)$
decreasing on $(e, \infty)$
inflection point at $x = e^{3/2}$



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## 11. The Indefinite Integral

1. Find the indefinite integrals.

(1) $\int (5x^3 + 7x^2 + 11)\,dx$

(2) $\int (\sqrt{x} + \frac{1}{\sqrt{x}})\,dx$

(3) $\int (\sqrt[3]{x^5} + \frac{1}{\sqrt{x^7}})\,dx$

(4) $\int 2xe^{x^2}\,dx$

(5) $\int x^4 e^{-x^5}\,dx$

(6) $\int x^6(1 + x^7)^3\,dx$

(7) $\int \frac{\ln x}{x}\,dx$

(8) $\int x\sqrt{x^2 - 10}\,dx$

(9) $\int \frac{x}{1+x}\,dx$

2. Find the indefinite integrals using integration by parts.

(1) $\int 5xe^{3x+7}\,dx$

(2) $\int 8x(4x + 1)^{77}\,dx$

(3) $\int x^2 e^{-6x}\,dx$



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SZÉCHENYI 2020
MAGYARORSZÁG KORMÁNYA
Európai Unió
Európai Szociális Alap
BEFEKTETÉS A JÖVŐBE

**Results**

1.

(1) $\frac{5}{4}x^4 + \frac{7}{3}x^3 + 11x + C$

(2) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$

(3) $\frac{3}{8}x^{8/3} - \frac{2}{5}x^{-5/2} + C$

(4) $e^{x^2} + C$

(5) $-\frac{1}{5}e^{-x^5} + C$

(6) $\frac{1}{28}(1 + x^7)^4 + C$

(7) $\frac{(\ln x)^2}{2} + C$

(8) $\frac{1}{3}(x^2 - 10)^{3/2} + C$

(9) $x - \ln|1 + x| + C$

2.

(1) $\frac{5}{3}xe^{3x+7} - \frac{5}{9}e^{3x+7} + C$

(2) $\frac{1}{39}x(4x + 1)^{78} - \frac{1}{12324}(4x + 1)^{79} + C$

(3) $-\frac{1}{6}x^2e^{-6x} - \frac{1}{18}xe^{-6x} - \frac{1}{108}e^{-6x} + C$



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## 12. The Definite Integral and Areas

1. Evaluate the integrals.

(1) $\int_{-1}^{3}(4x^3-6x^2+1)\,dx$ (2) $\int_{2}^{4}\frac{3x^2+1}{x^3+x-2}\,dx$ (3) $\int_{0}^{3}2e^{4x}\,dx$

2. Calculate the area of the regions enclosed by the given curves.

(1) $y=-x^2+4x-3,\ x=5,\ y=0$

(2) $y=\ln x,\ x=3,\ x=4,\ y=0$

(3) $y=3\sqrt{x},\ y=x$

(4) $y=\frac{1}{x},\ y=-x+\frac{5}{2}$

### Results

1.
(1) 28
(2) $\ln\frac{33}{4}\approx 2{,}11$
(3) 81376,90

2.
(1) $-6\frac{2}{3}$
(2) 1,25
(3) 13,5
(4) 0,49



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