Introduction to Logic and Computer Science
Foundation of Computer Science
Logic in Computer Science

Lecture #2: syntax

Tamás Mihálydeák, László Aszalós

This work was supported by the construction EFOP-3.4.3-16-2016-00021.
The project was supported by the European Union, co-financed by the European Social Fund.

The essence of logic

- The main role of logic
  - laws that define valid arguments (inferences, consequence relations)
- Valid arguments (inferences):
  - an argument (an inference):
    - a relation between premise(s) and conclusion
  - a consequence relation
    - input: premise(s)
    - output: conclusion
- Valid arguments (inferences, consequence relations):
  - if all premises are true, then the conclusion is true.
- Logically valid arguments:
  - when the former holds necessarily.

Language of propositional logic

Definition (Classical zero-order language)

Classical zero-order language is an ordered triple

$$L^{(0)} = \langle LC, Con, Form \rangle$$

where

1. $LC = \{\neg, \supset, \wedge, \vee, \equiv, (,)\}$
   - the set of logical constants
2. $Con \neq \emptyset$ the countable set of non-logical constants
   - propositional parameters
3. $LC \cap Con = \emptyset$

Language of propositional logic - 2

Definition (cont.)

4. The set of formulae—the set *Form*—is given by the following inductive definition:
   - $Con \in Form$
   - If $A \in Form$, then $\neg A \in Form$.
   - If $A, B \in Form$, then
     - $(A \supset B) \in Form$,
     - $(A \wedge B) \in Form$,
     - $(A \vee B) \in Form$,
     - $(A \equiv B) \in Form$.

Remark

The members of the set *Con* are the atomic formulae (prime formulae).

Subformulae

Definition (direct subformula)

- If A is an atomic formula, then it has no direct subformula;
- $\neg A$ has exactly one direct subformula: A;
- Direct subformulae of formulae $(A \supset B)$, $(A \wedge B)$, $(A \vee B)$, $(A \equiv B)$ are formulae A and B, respectively.

Definition (set of subformulae)

The set of subformulae of formula A—denoting: $SF(A)$—is given by the following inductive definition:

1. $A \in SF(A)$ (i.e. the formula A is a subformula of itself);
2. if $A' \in SF(A)$ and B is a direct subformula of A', then $B \in SF(A)$
   - i.e., if A' is a subformula of A, then all direct subformulae of A' are subformulae of A.

Construction tree

Definition (Construction tree)

The construction tree of a formula A is a finite ordered tree whose nodes are formulae,

- the root of the tree is the formula A,
- the node with formula $\neg B$ has one child: the node with the formula B,
- the node with formulae $(B \supset C)$, $(B \wedge C)$, $(B \vee C)$ or $(B \equiv C)$ has two children: the nodes with B, and C
- the leaves of the tree are atomic formulae.

Construction tree - cont.

Example