Introduction to Logic and Computer Science
Foundation of Computer Science
Logic in Computer Science

Lecture #12: Finite automaton and regular language

Tamás Mihálydeák, László Aszalós

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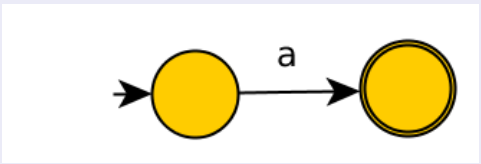
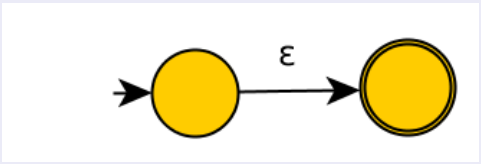
1. Automaton based on regular expression
2. Regular expression based on automaton
3. Construction

Automaton based on regular expression

By following the inductive definition we construct a non-deterministic automaton. Inductive hypothesis: for R_1 and for R_2 we have accepting automata A_1 and A_2

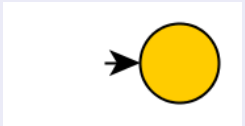
$R = a$, where $a \in \Sigma \cup \{\varepsilon\}$





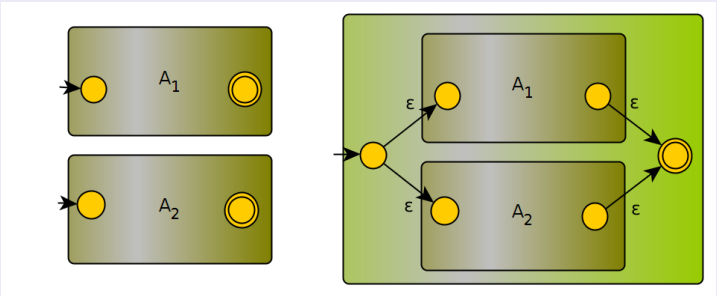
$R = \emptyset$

There is no final state, it does not accept anything!

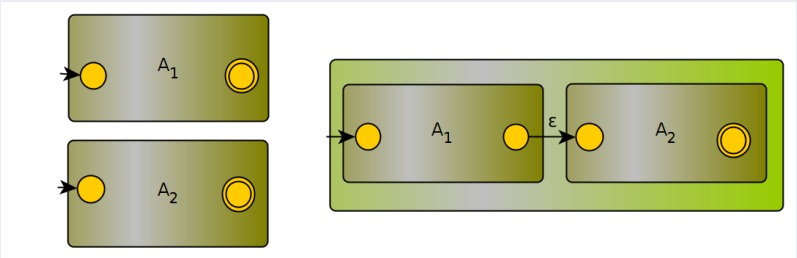


Automaton based on regular expression

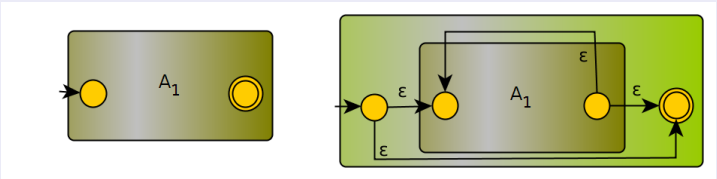
$R = R_1 \cup R_2$



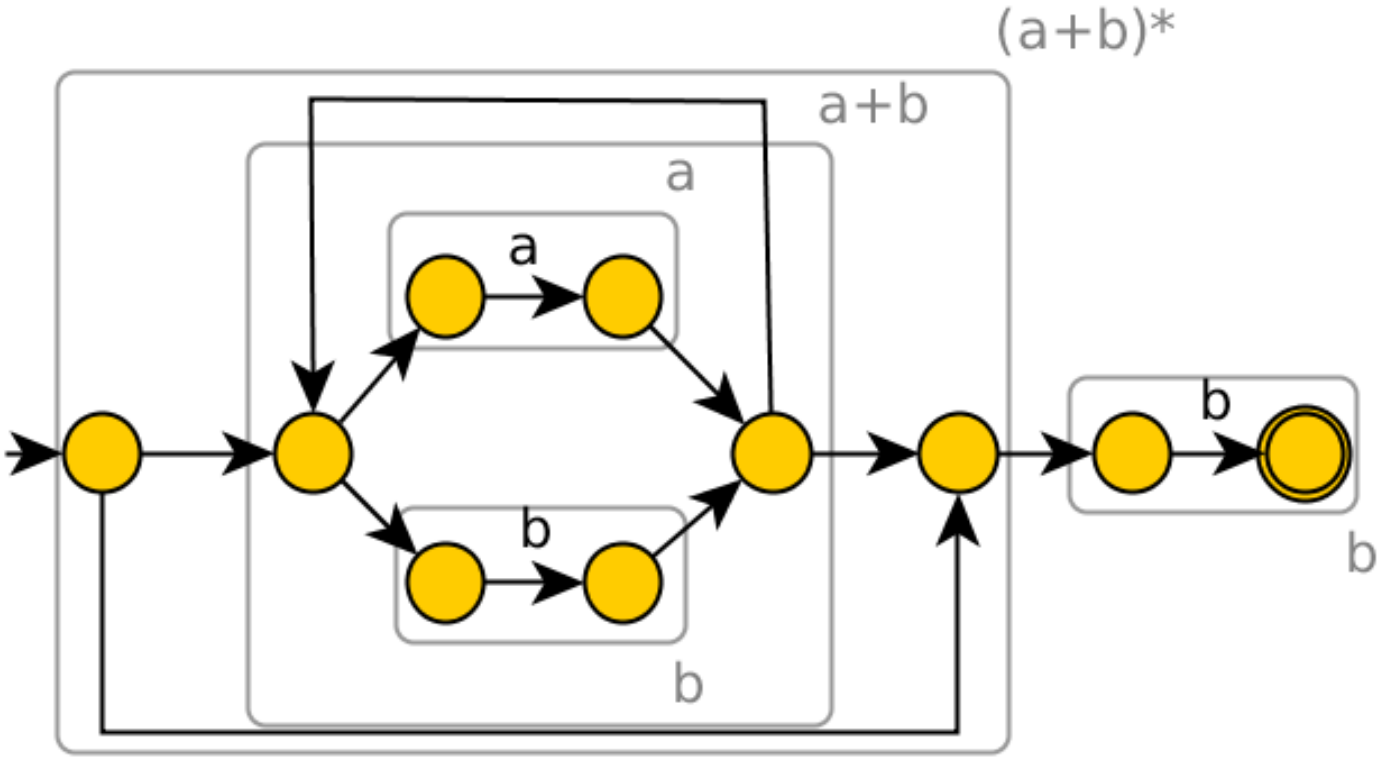
$R = R_1 \circ R_2$



$R = R_1^\star$



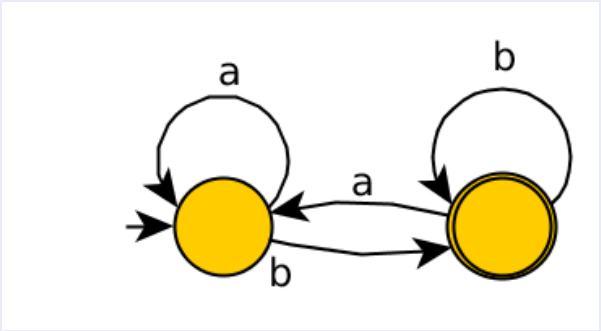
Example: $(a + b)^\star b$



Example: $(a + b)^\star b$

Remark

Following this step, Thompson's algorithm produced a deterministic version of the non-deterministic finite automaton and then minimized it. Applying this algorithm, we get the following automaton:



The implementation of this algorithm can be find in the lexers *lex* and *flex*.

Regular expression based on automaton

Theorem

For any deterministic automaton A there exist a regular expression R, such that $L_A = L_R$.

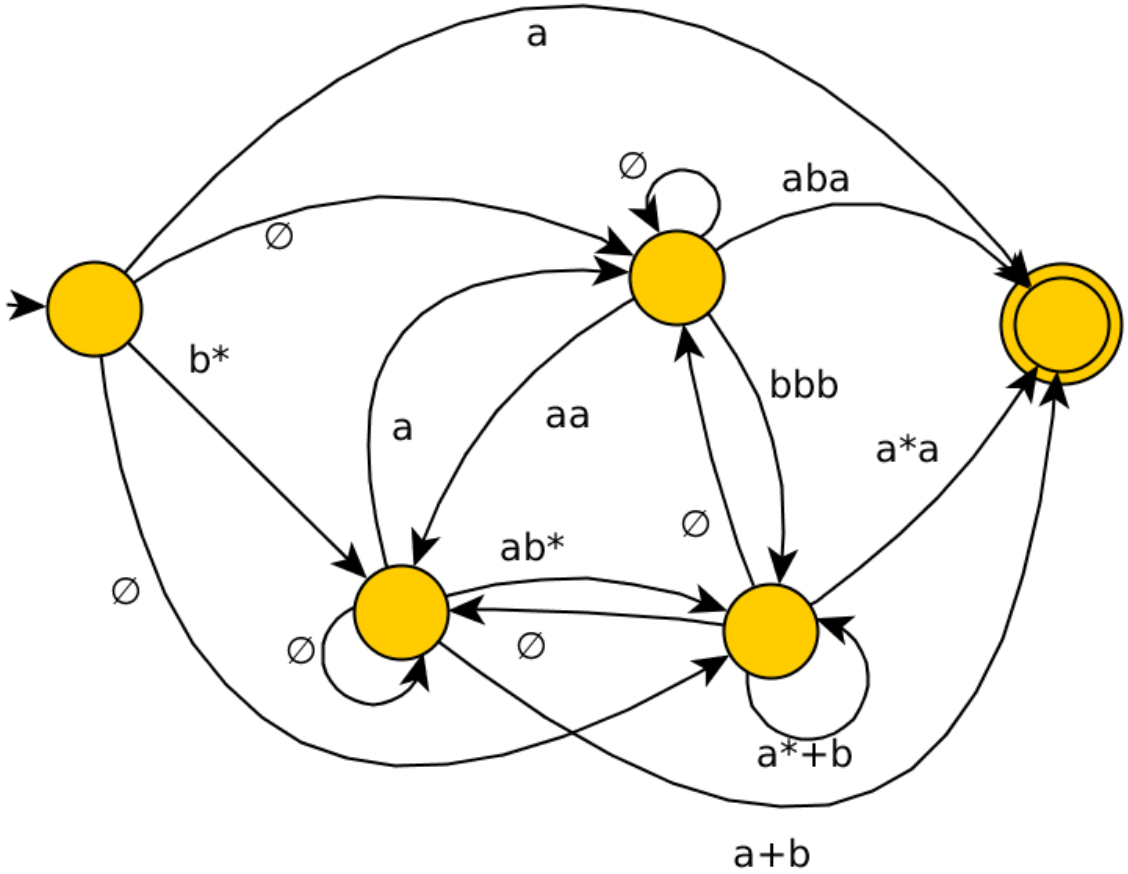
Remark

We need a more general automaton to do this.

Construction

- The transition function of the more general non-deterministic finite automaton can even contain a regular expression (instead of one letter).
- The starting state has no incoming edges, and has outgoing edges into every other state.
- It has one final state only. This state has no outgoing edge, and has ingoing edges from every other state. (The starting state and the final state are different.)
- All the states (except the starting and final) have outgoing edges to every states, even to itself.

Example: more general non-deterministic final automaton



More general NFA based on finite automaton

Construction

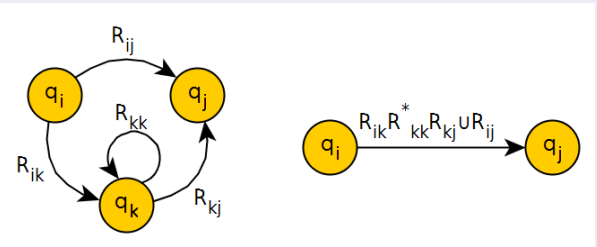
1. Create a new starting state and connect it with the old starting state with (outgoing) edge labelled ε
2. Create a new final state and connect it with the all the old final states with (incoming) edge labelled ε
3. If there are parallel edges between two states replace with one labelled by the union of the previous labels.
4. If there is no edge between two edges, add a new one with label $\emptyset$ (so it will be an unusable edge)

RE based on more general NFA

Construction

Let's assume that the automaton has k states.

- If $k = 2$, the only label contains the regular expression.
- If $k > 2$, then we construct a more general NFA with $k - 1$ states:
  - Let denote R_{ij} the label of the edge between q_i and q_j.
  - Let choose to delete the state q_k.
  - For any states i and j (different from k let replace R_{ij} with $R_{ik} \circ R_{kk}^{\star} \circ R_{kj} \cup R_{ij}$



Corollary

Theoram

The languages accepted by finite automata are regular.