Mathematics for Engineers

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Functions

This work was supported by the construction EFOP-3.4.3-16-2016-00021. The project was supported by the European Union, co-financed by the European Social Fund.

Functions

The velocity of a body falling freely to Earth increases with time, i.e. the velocity of fall depends on the time. The pressure of a gas maintained at a constant temperature depends on its volume. The periodic time of a simple pendulum depends on its length. Such dependencies between observed quantities are frequently encountered in physics and engineering and they lead to the formulation of natural laws.

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Two quantities are measured with the help of suitable instruments such as clocks, rulers, balances, ammeters, voltmeters etc.; one quantity is varied and the change in the second quantity observed. The former is called the **independent quantity**, or argument, and the latter the **dependent quantity**, all other conditions being carefully kept constant.

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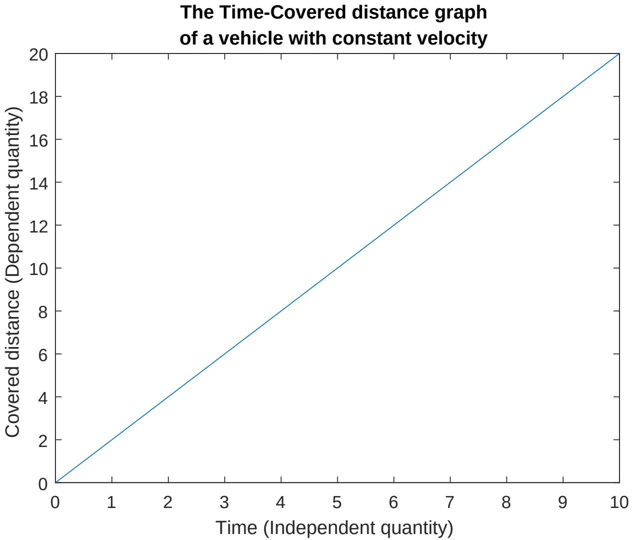
Relationships obtained experimentally may be tabulated or a graph drawn showing the variation at a glance. Such representations are useful but in practice we prefer to express the relationships mathematically.

Example


The Time-Covered distance graph
of a vehicle with constant velocity
Covered distance (Dependent quantity)
Time (Independent quantity)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8
9
10

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Example

Consider a spring fixed at one end and stretched at the other end. This results in a force which opposes the stretching or displacement. Two quantities can be measured: the displacement x in metres (m); the force F in newtons (N). Measurements are carried out for several values of x. Thus we obtain a series of paired values for x and F associated with each other.

Example (continued)

The paired values are tabulated below. The direction of the force is opposite to the direction of the displacement.
The range of x is called the **domain** of definition. The corresponding range of the functional values is called the **range** of values (sometimes referred to as the **co-domain**).
We plot each paired value on a graph and draw a curve through the points. This enables us to obtain, approximately, intermediate values.

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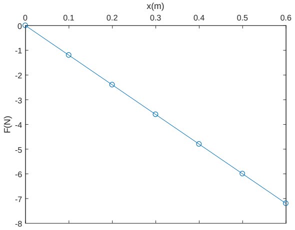
Displacement (m)	Force (N)
0.0	0.0
0.1	-1.2
0.2	-2.4
0.3	-3.6
0.4	-4.8
0.5	-6.0
0.6	-7.2



The relationship between x and F can be expressed by a formula which must be valid with the domain of definition. In this case the formula is

$$F = -\alpha x, \quad \text{where} \quad \alpha = 12N/m.$$

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If y depends on x then y is said to be a function of x; the relationship is expressed as

$$y = f(x)$$

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In order to define the function completely we must state the set of values of x for which it is valid, i.e. the domain of definition. The quantity x is called the argument or independent variable and the quantity $f(x) = y$ the dependent variable.

Functions

Formal definition of a real function

Given two sets of real numbers, a domain (often referred to as the x-values) and a co-domain (often referred to as the $f(x) = y$-values), a real function assigns to each x-value a unique $f(x) = y$-value.

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Examples

- $y = f(x) = 3x^2$.
- $y = f(t) = 2t$, the independent variable is t instead of x.
- piecewise defined function

$$y = f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x & \text{otherwise.} \end{cases}$$

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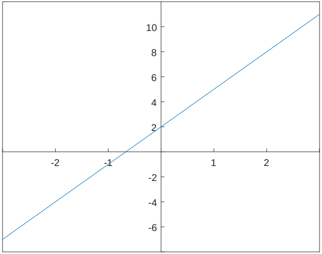
Exercise: Determine the domain and the co-domain of the previous functions!

Affine functions

$f:\mathbb{R}\to\mathbb{R},\ \ f(x)=ax+b$, where $a,b\in\mathbb{R}$ are given.

Figure: Affine function

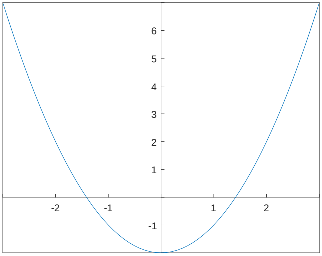


Functions

Parabola

$f : \mathbb{R} \to \mathbb{R}, \; f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{R}, \; a \neq 0$ are given.

Figure: Parabola



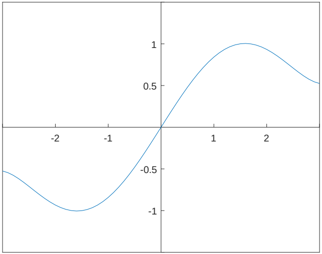
Polynomials

Let $a_0, a_1, \ldots, a_n \in \mathbb{R}$ are given, where $a_n \neq 0$, then the function $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = a_0 + a_1 x + \cdots + a_n x^n$$

is called a *real polynomial with degree* n.

Figure: $x - \frac{x^3}{6} + \frac{x^5}{120}$



Functions

Figure: The sine and the cosine function

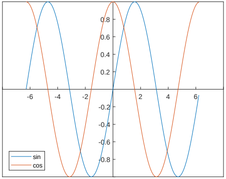


Figure: The arcsine and the arccosine function

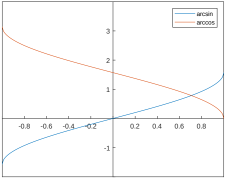


Figure: The tangent function

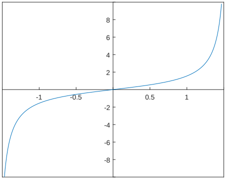
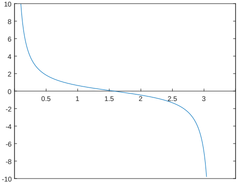
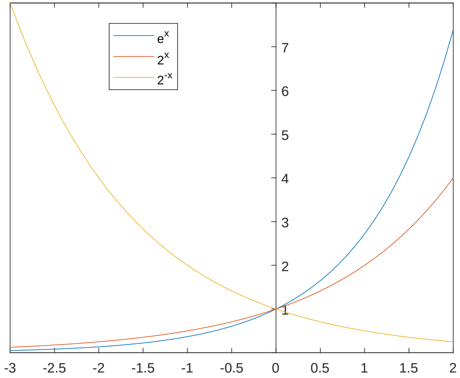


Figure: The cotangent function



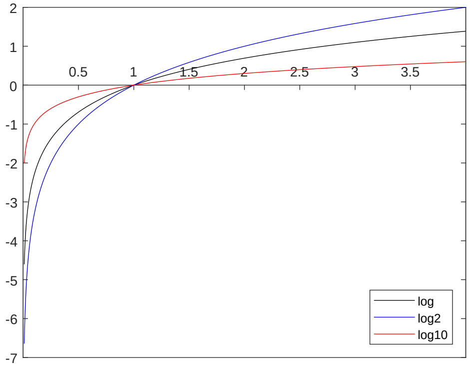
Exponential functions

Figure: The e^x, the 2^x and the $\left(\frac{1}{2}\right)^x$ function



Logarithmic functions

Figure: The $\log_2(x)$ the $\log(x)$ and the $\log_{10}(x)$ function



Functions

Let $f : \mathbb{R} \to \mathbb{R}$ be a function and $K \in \mathbb{R} \setminus \{0\}$ is a given constant, then

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- the graph of $f(Kx)$ is the graph of $f(x)$ stretching it along the x axis by the factor $\frac{1}{K}$.

Functions, Examples

Figure: Translation

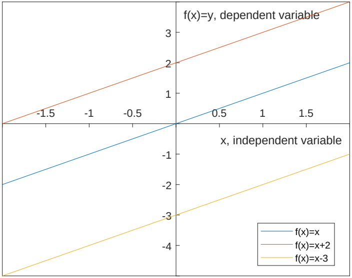
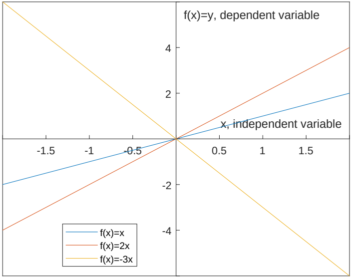


Figure: Stretching



Functions, Examples

Figure: Translation along the y axis

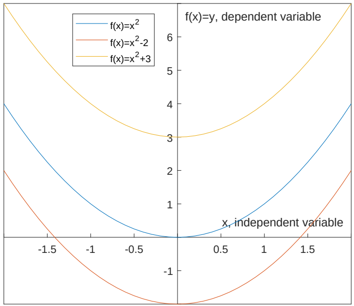
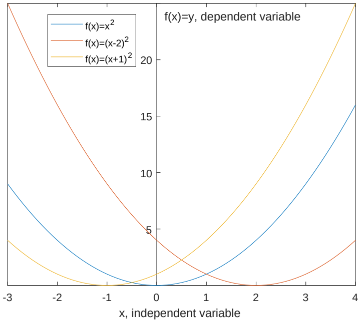


Figure: Translation along the x axis



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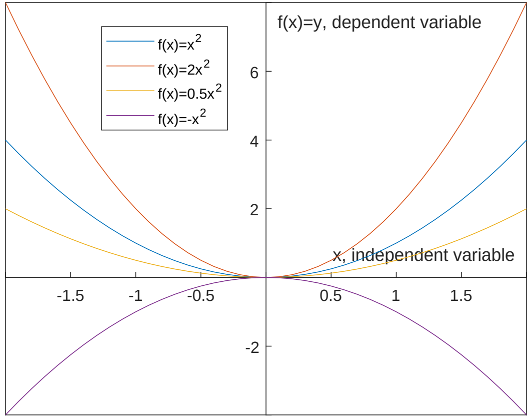

Sketch the graph of the following functions!

- $f(x) = \sin x, \; g(x) = sin(x - \frac{\pi}{2}), \; h(x) = \sin(2x), \; i(x) = 2\sin(x).$
- $f(x) = 2(x-1)^2 + 2.$
- $f(x) = \frac{1}{x+1}.$
- $f(x) = x^2 - 2x + 3.$