Movement of a charged particle in a magnetic field

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Definitions of variables and formulas

The magnetic field

A magnet both produces its own magnetic field and responds to magnetic fields. The B magnetic field at a given point in space is a vector quantity and it is specified by two properties: (1) Its direction, in which the north pole of a compass needle points at that location. (2) Its magnitude (also called strength), which is proportional to how strongly the compass needle orients along that direction. The magnitude of the magnetic field is defined as:

$$B = \frac{F}{qv \sin\theta} \tag{1}$$

where v is the velocity of a moving test charge q in the magnetic field B and θ is the angle between the direction of v and the direction of B. In SI units, the strength of the magnetic field is given in tesla (T). The magnitude of the magnetic field in a clinical MR device is typically 1.5-7 T, while strengths of the magnetic fields of a refrigerator magnet and the Earth are 5-10 mT and 25-65 µT, respectively.

The magnetic Lorentz force

The magnetic Lorentz force is a force exerted on an electric charge moving in a magnetic field, and it is proportional and perpendicular to the magnetic field and the velocity of the charge. When an object with charge q is moving with a velocity v through a magnetic field B, a magnetic force acts on it and the magnitude of that magnetic force is determined by

$$F = qvB \sin\theta \tag{2}$$

where θ is the angle between the direction of v and the direction of B. The direction of the magnetic force can be employed by right hand rule number 1:

1. Point the fingers of your right hand in the direction of the velocity v.
2. Curl the fingers in the direction of the magnetic field B, moving through the smallest angle.
3. Your thumb is now pointing in the direction of the magnetic force F exerted on a positive charge.

The direction of a positively charged particle will be changed perpendicularly to both the instantaneous velocity vector v and the B magnetic field according to the right-hand rule number 1 (Figure 1). In the presence of electric and magnetic fields, both of them affect the movement of a charged particle, however in a different manner. A magnetic field exerts a force only on a moving charge and changes only the direction of its movement, while an electric field affects a resting charge as well and changes both the direction and the magnitude of its velocity.

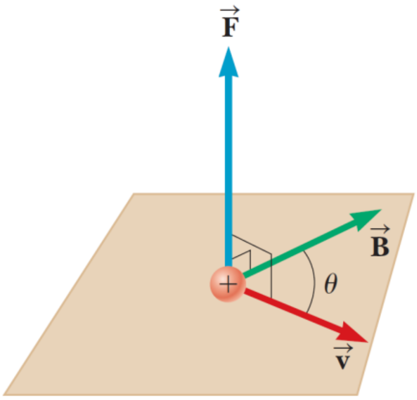

Figure 1. Direction of the force exerted by a magnetic field on a moving charged particle (source: Serway-Vuille: College Physics 9th ed.)

Mass spectrometry

As can be derived from previous sections, a charged particle accelerated in an electric field is forced on a circular track when it enters a magnetic field that is perpendicular to the direction of its movement. The equation describing the movement of the particle is based on Newton's second law and equation (2):

$$F = ma_{cp} = m\frac{v^2}{r} = qvB \tag{3}$$

where m is the mass of particle, a_{cp} is the centripetal acceleration, v is the speed of particle, r is the radius of circular track, q the charge of particle and B is the strength of magnetic field. By rearranging equation (3) the radius of the circular track is

$$r = \frac{mv}{qB} \tag{4}$$

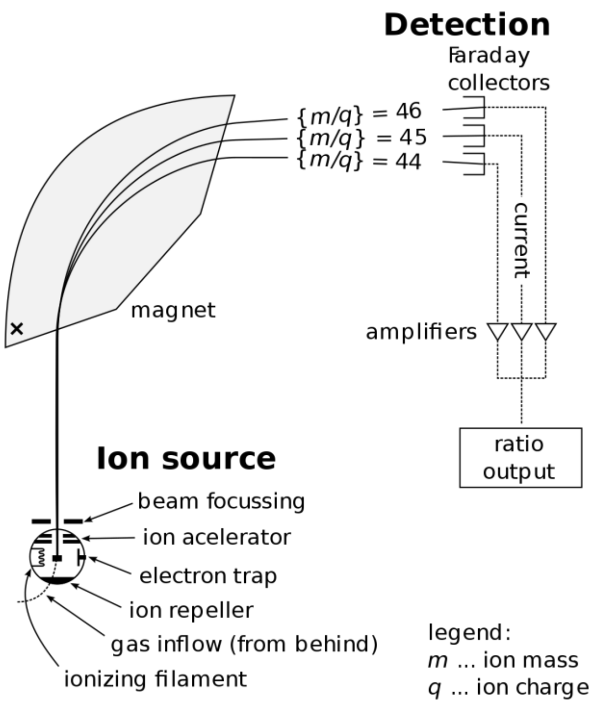

Figure 2. Setup of a mass spectrometer (source: wikipedia)

According to equation (4), radius of the track depends on the mass-to-charge ratio of particle. Thus, particles can be separated based on their ratio and recognized by the radius of their tracks. This is the principle of mass spectrometry (Figure 2).

Exercises with solutions

1. Exercise

A particle with a charge of 3.20×10^{-19} C and a mass of 5.01×10^{-27} kg moves with a speed of 1.80×10^{6} m/s on a circular track with a radius of 22 cm in a magnetic field that is perpendicular to the direction of the movement. What is the strength of the magnetic field?

By rearranging equation (3)

$$B = \frac{mv}{rq} = \frac{5.01 \times 10^{-27} \times 1.80 \times 10^{6}}{0.220 \times 3.20 \times 10^{-19}} = \mathbf{0.128\ T}$$

2. Exercise

A particle with a charge of 1.60x10^{-19} C and a mass of 3.35x10^{-27} kg moves with a speed of 2.50x10^{6} m/s on a circular track in a magnetic field of 0.230 T that is perpendicular to the direction of the movement. What is the radius of its track?

By rearranging equation (3)

$$r = \frac{mv}{qB} = \frac{3.35 \times 10^{-27} \times 2.50 \times 10^{6}}{1.60 \times 10^{-19} \times 0.230} = \mathbf{0.228\ m}$$

3. Exercise

A charged particle with a mass of 7.53x10^{-27} kg moves with a speed of 2.95x10^{6} m/s on a circular track with a radius of 32.3 cm in a magnetic field of 0.215 T that is perpendicular to the direction of the movement. What is the charge of the particle?

By rearranging equation (3)

$$q = \frac{mv}{rB} = \frac{7.53 \times 10^{-27} \times 2.95 \times 10^{6}}{0.323 \times 0.215} = \mathbf{3.20 \times 10^{-19}\ C}$$

4. Exercise

A proton moves on a circular track with a radius of 24.0 cm in a magnetic field of 0.185 T that is perpendicular to the direction of the movement. What is the speed of the proton?

The charge of a proton is 1.60x10^{-19} C, while its mass is 1.67x10^{-27} kg.

By rearranging equation (3)

$$v = \frac{rqB}{m} = \frac{0.240 \times 1.60 \times 10^{-19} \times 0.185}{1.67 \times 10^{-27}} = \mathbf{4.25 \times 10^{6}\ \frac{m}{s}}$$

5. Exercise

A charged particle moves with a speed of 2.12x10^{6} m/s on a circular track with a radius of 15.9 cm in a magnetic field of 0.279 T that is perpendicular to the direction of the movement. What is the mass-to-charge ratio of the particle?

By rearranging equation (3)

$$\frac{m}{q} = \frac{rB}{v} = \frac{0.159 \times 0{,}279}{2.12 \times 10^{6}} = \mathbf{2.09 \times 10^{-8}\ \frac{kg}{C}}$$

Exercises

1. Exercise

A particle with a charge of 3.20×10^{-19} C and a mass of 5.01×10^{-27} kg moves with a speed of 1.40×10^{6} m/s on a circular track with a radius of 13 cm in a magnetic field that is perpendicular to the direction of the movement. What is the strength of the magnetic field?

a) 0.169 T
b) 1.69×10^{-3} T
c) 2.85×10^{-3} T

2. Exercise

A particle with a charge of 1.60×10^{-19} C and a mass of 3.35×10^{-27} kg moves with a speed of 1.50×10^{6} m/s on a circular track in a magnetic field of 0.330 T that is perpendicular to the direction of the movement. What is the radius of its track?

a) 9.52 m
b) 9.52 cm
c) 1.04 cm

3. Exercise

A charged particle with a mass of 7.53×10^{-27} kg moves with a speed of 5.92×10^{6} m/s on a circular track with a radius of 1.29 m in a magnetic field of 0.108 T that is perpendicular to the direction of the movement. What is the charge of the particle?

a) 1.60×10^{-19} C
b) 4.80×10^{-19} C
c) 3.20×10^{-19} C

4. Exercise

A proton moves on a circular track with a radius of 31.0 cm in a magnetic field of 0.215 T that is perpendicular to the direction of the movement. What is the speed of the proton?

a) 6.39×10^{7} m/s
b) 1.38×10^{9} m/s
c) 1.38×10^{7} m/s

5. Exercise

A charged particle moves with a speed of 1.12×10^{6} m/s on a circular track with a radius of 17.9 cm in a magnetic field of 0.259 T that is perpendicular to the direction of the movement. What is the mass-to-charge ratio of the particle?

a) 3.13×10^{-8} kg/C
b) 4.14×10^{-8} kg/C
c) 5.15×10^{-8} kg/C

6. Exercise (true/false)

The direction of the motion of a charged particle moving in a magnetic field is always changed in response to the magnetic force acting on the particle.

false

7. Exercise (true/false)

The principle of mass spectrometry is based on the ability to separate particles using an electric field.

false

8. Exercise (true/false)

In a mass spectrometer, radius of the track of a charged particle is inversely proportional to the strength of the applied magnetic field.

true

9. Exercise (true/false)

In a mass spectrometer, radii of the tracks of particles with different masses is always different from each other.

false

10. Exercise (true/false)

In a mass spectrometer, radius of the track of a charged particle is independent from the speed of the particle.

false