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Empirical Studies of Reconstructing hv-Convex Binary Matrices from Horizontal and Vertical Projections

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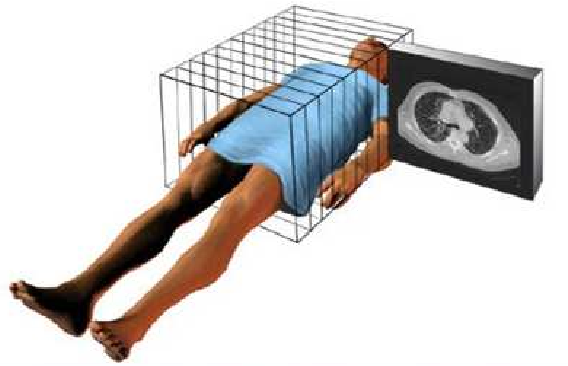
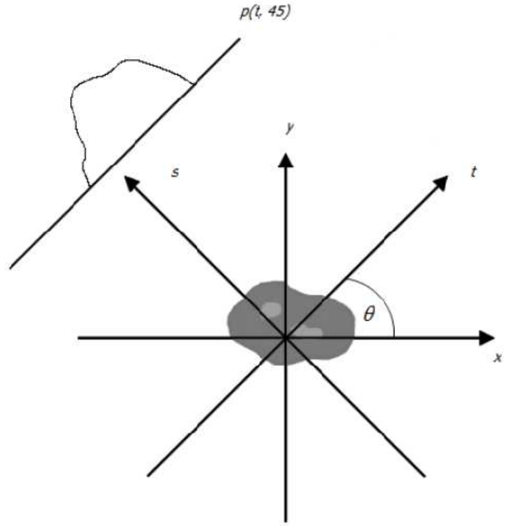
Tomography

- Task
- Applications
- Limitations
- Special cases:
  - Discrete tomography
  - Binary tomography





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Reconstruction

- Binary image represented by binary matrix
- Projections
  - Horizontal
  - Vertical

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 4 & 3 & 1 \\
\hline
1 & 0 & 1 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 1 & 0 \\
3 & 0 & 0 & 1 & 1 & 1 \\
2 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

$$\mathcal{H}(F) = [1, 4, 3, 2, 1]$$

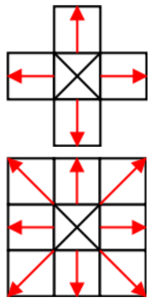
$$\mathcal{V}(F) = [1, 2, 4, 3, 1]$$

Geometrical Properties of the Binary Matrices

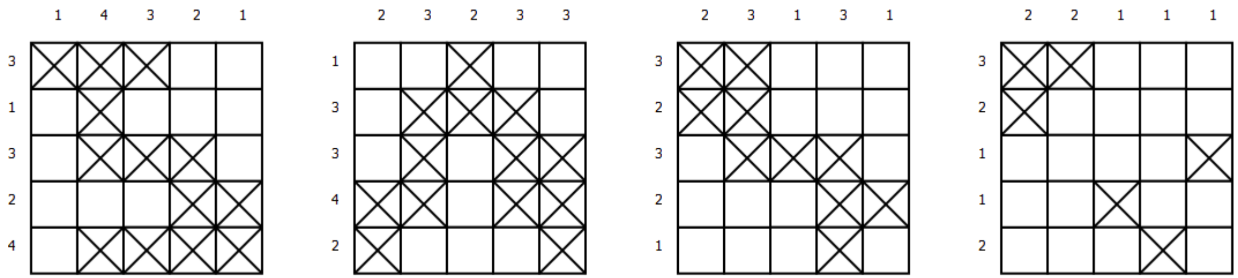
- One or more solution
- Switching components
  - Many possible solution

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Connectivity
  - 4-connected shapes
  - 8-connected shapes



- hv-convexity – all rows and columns are connected



Task of the Research

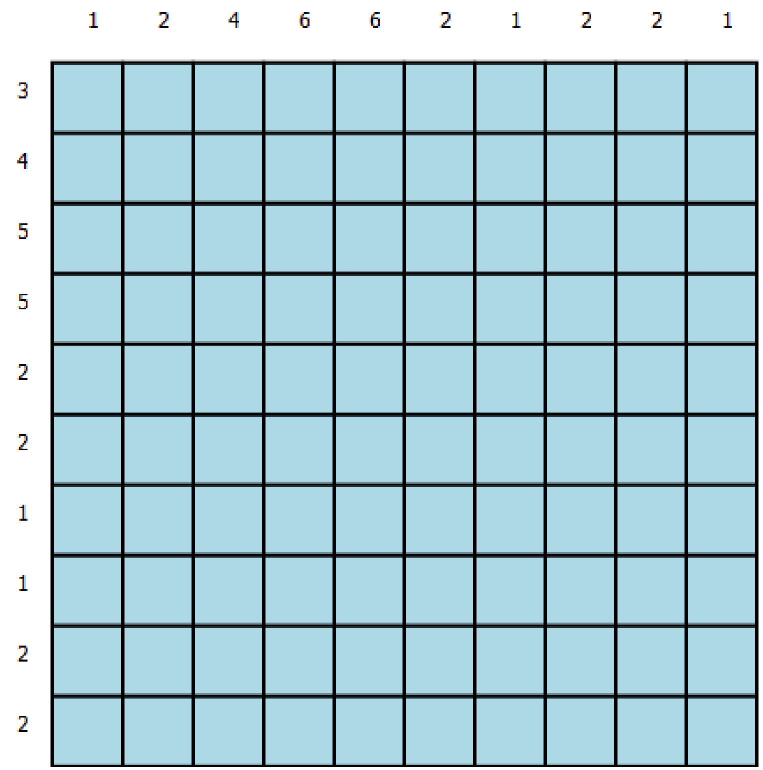
- Reconstructing hv-convex images from two projections is NP-hard, but there are heuristic algorithms for that problem
- Goal: investigate the difficulty of the problem
- Systematic study of the algorithms

Algorithms

- Core-shell algorithm
  - Array data type
  - First-Last data type
- Simulated annealing reconstruction
- Algorithm based on the location of the components

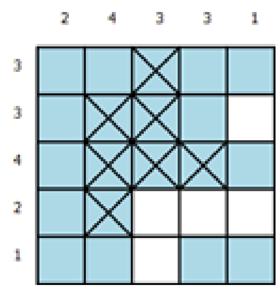
Core-shell Algorithm

- A. Kuba, 1984 – own implementation
- Two set
  - Core, increase (X)
  - Shell, decrease (blue)
- If the core cannot be increased, then use stack memory for guessing



Core-shell Algorithm

- Array data type
  - Simple implementation
  - Fast stack operations
  - Need a lots of memory
- First-Last data type
  - Complicated implementation
  - Much less, but more complicated operations
  - Slow stack operations
  - Does not need a lots of memory



$$core = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad shell = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 & 2 \end{bmatrix}$$

***Core*:**

RowFirst: 3, 2, 2, 2, -1

RowLast: 3, 3, 4, 2, -1

ColumnFirst: -1, 2, 1, 4, -1

ColumnLast: -1, 4, 3, 3, -1

***Shell*:**

RowFirst: 1, 1, 1, 1, 1

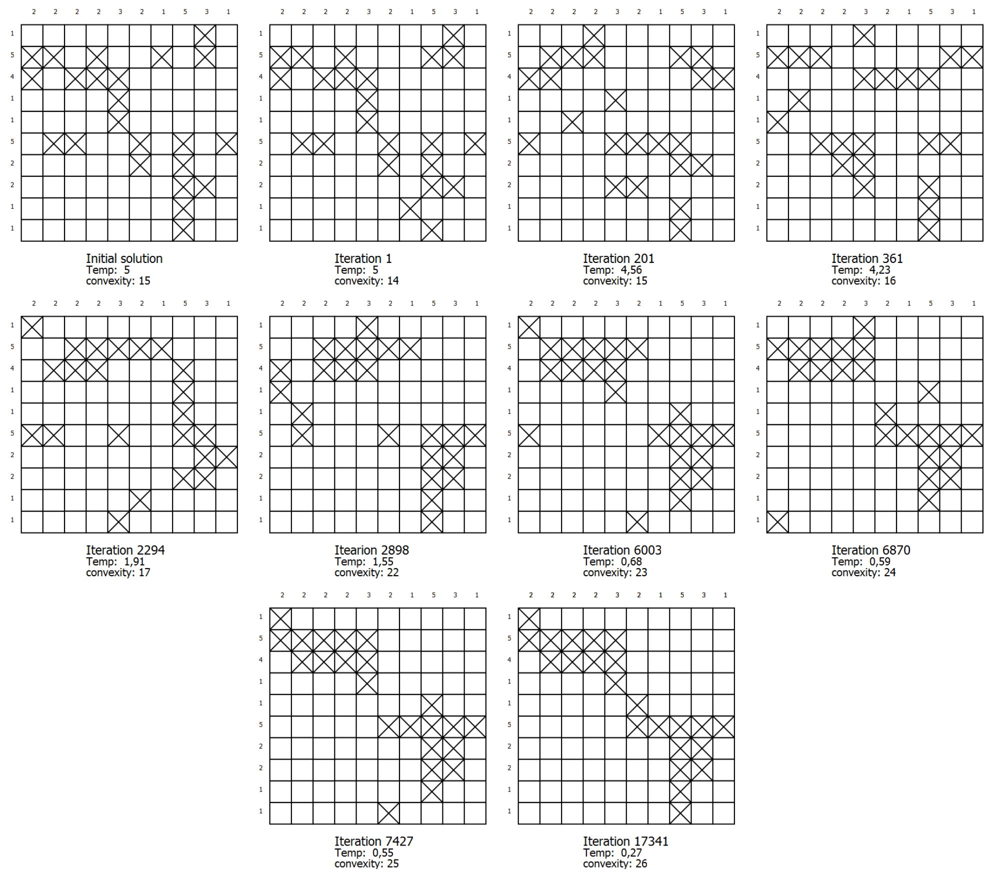
RowLast: 5, 4, 5, 2, 5

ColumnFirst: 1, 1, 1, 1, 1

ColumnLast: 5, 5, 3, 5, 5

Simulated Annealing Reconstruction

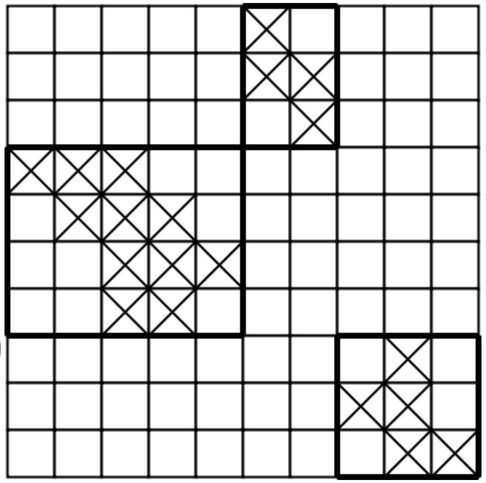
- F. Jarray and G. Tlig, 2010 – own implemetation
- Properties
  - Ryser algorithm to find initial solution
    - satisfies the horizontal and vertical projections, but hv-convexity is not guaranteed
  - Neighbor of a solution as a single switching
  - Integer programming
  - Parameters
  - Running time depends on the cooling schedule
  - Does not need lots of memory



Algorithm Based on the Location of the Components

- P. Balázs, 2008 – available implementation
- Properties
  - Seeks a set of disjoint intervals satisfying given conditions
  - Does not need lots of memory
  - Running time depends on the number of the components

$(O(m^2n^2 \cdot min\{m^3, n^3\} + min\{m, n\}^{3k}))$



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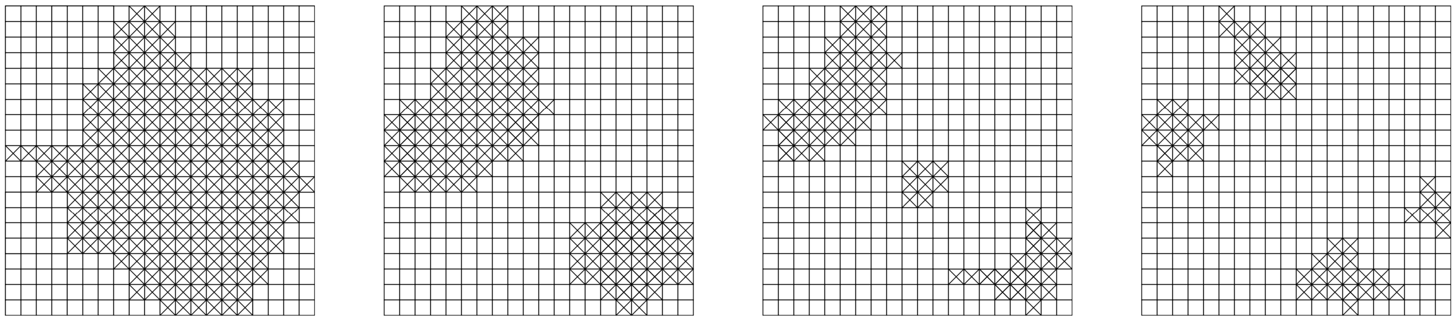
Set of Test Data

- 2000 images
  - 1-, 2-, 3-, 4-components
  - 10 x 10, 20 x 20, 30 x 30, 40 x 40, 50 x 50
  - 60 x 60, 80 x 80, 100 x 100



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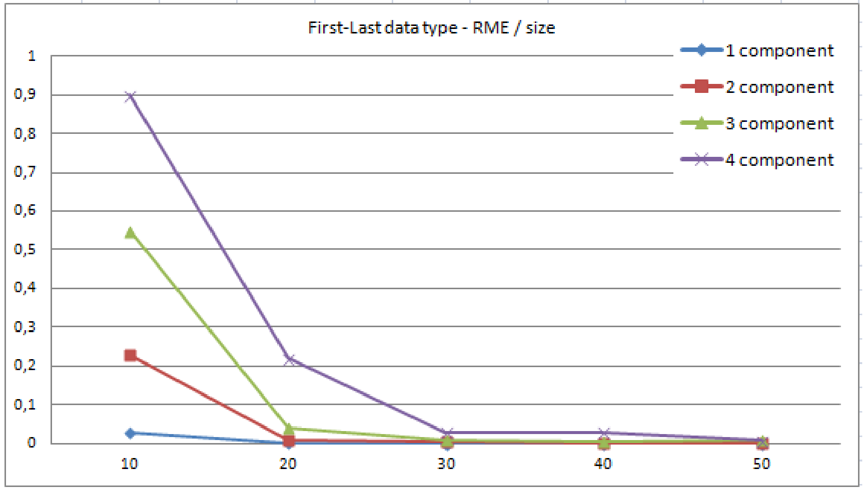
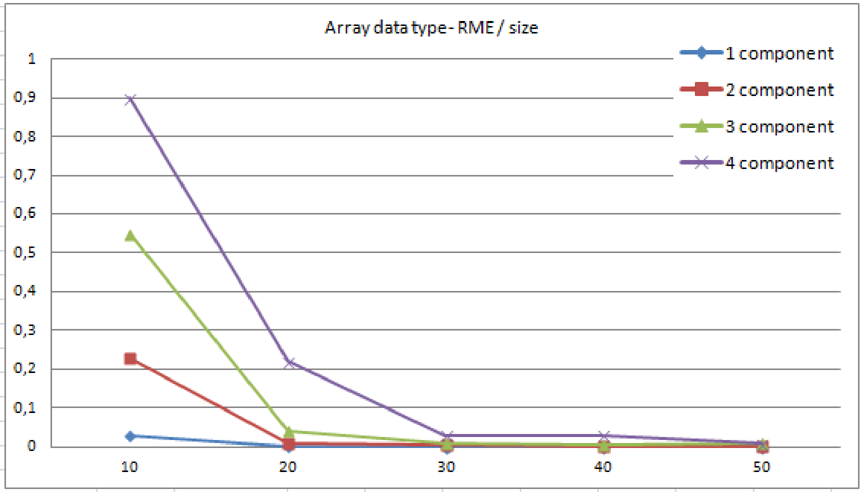
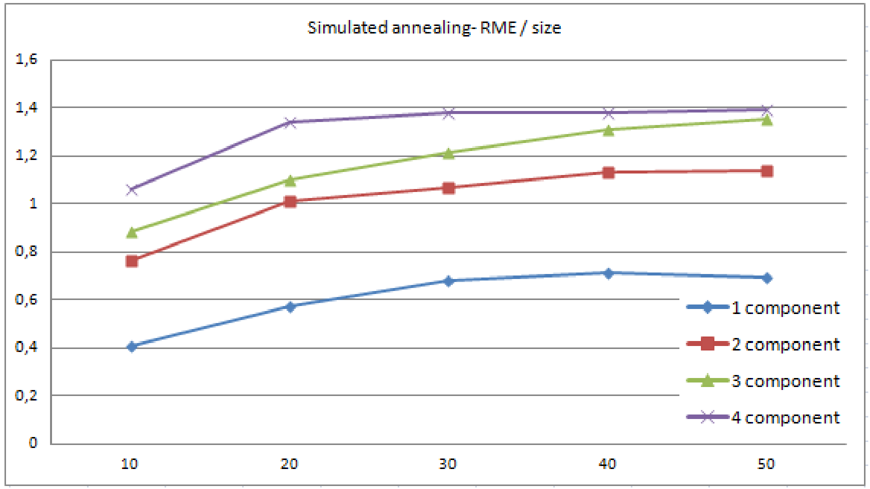
Quality of the Reconstruction

$$RME = \frac{\sum_{i,j}|m_{i,j}^{o} - m_{i,j}^{r}|}{\sum_{i,j}|m_{i,j}^{o}|}$$

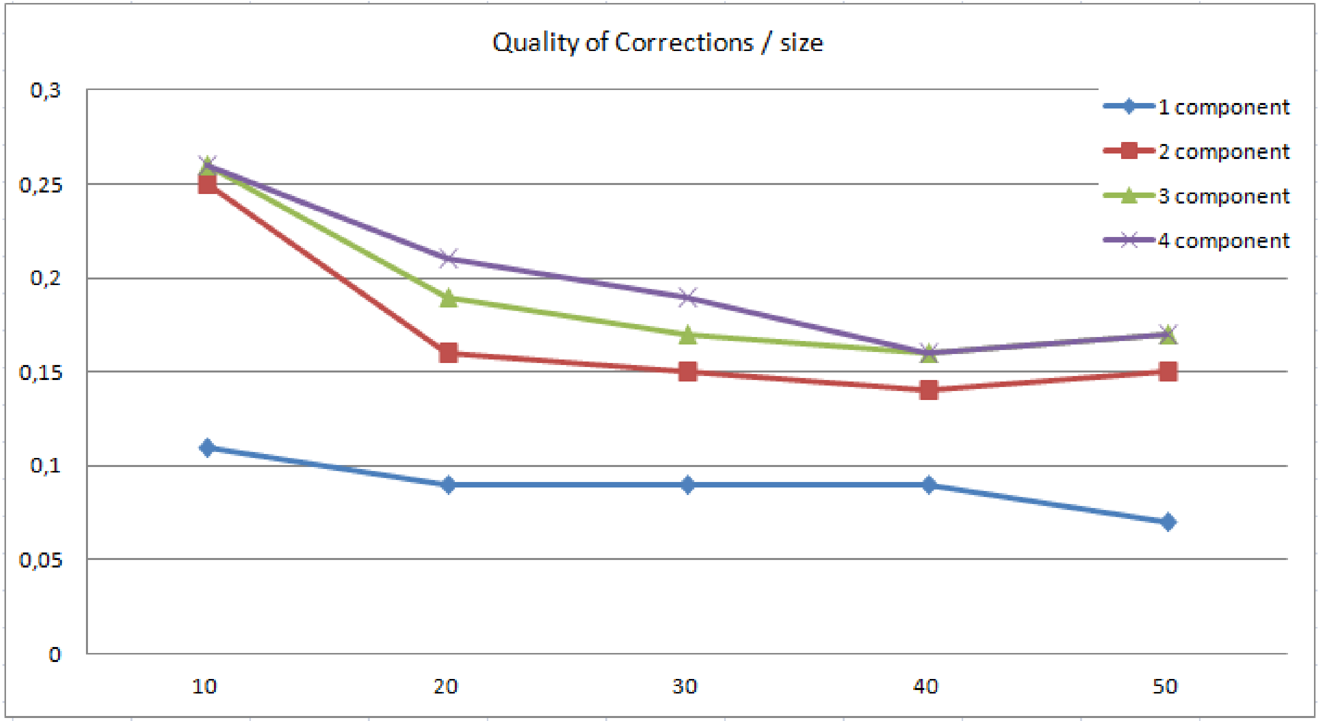






Quality of the Corrections at Simulated Annealing

$$correctionQuality = \frac{|f(R) - f(S)|}{numberOfOnes}$$



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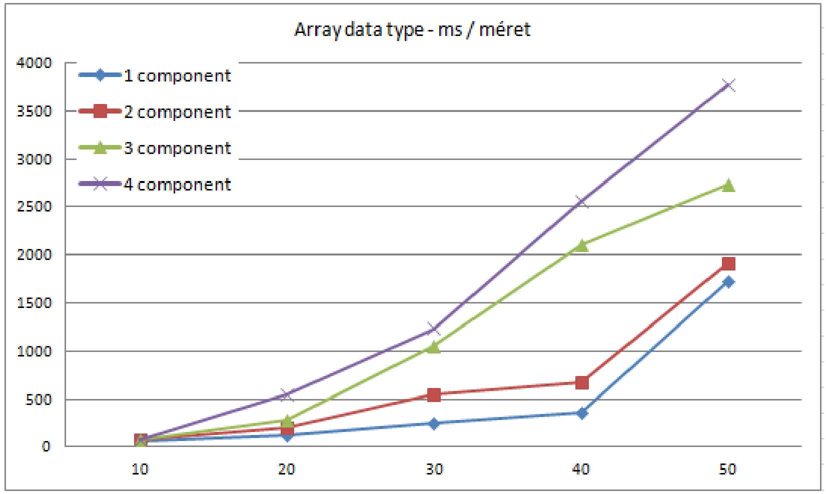
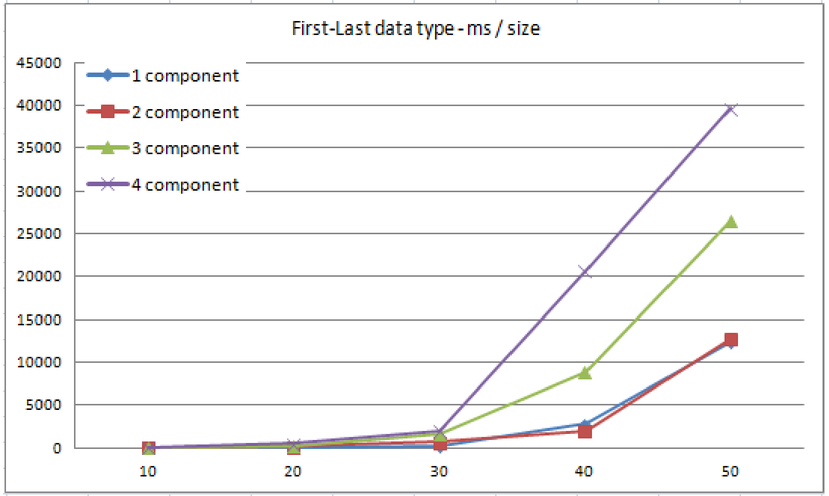
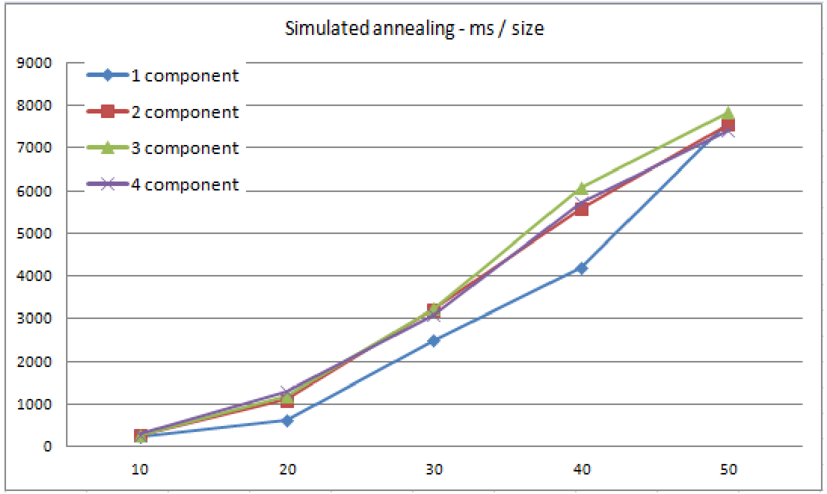
Running Time

– Omit the non-significant data







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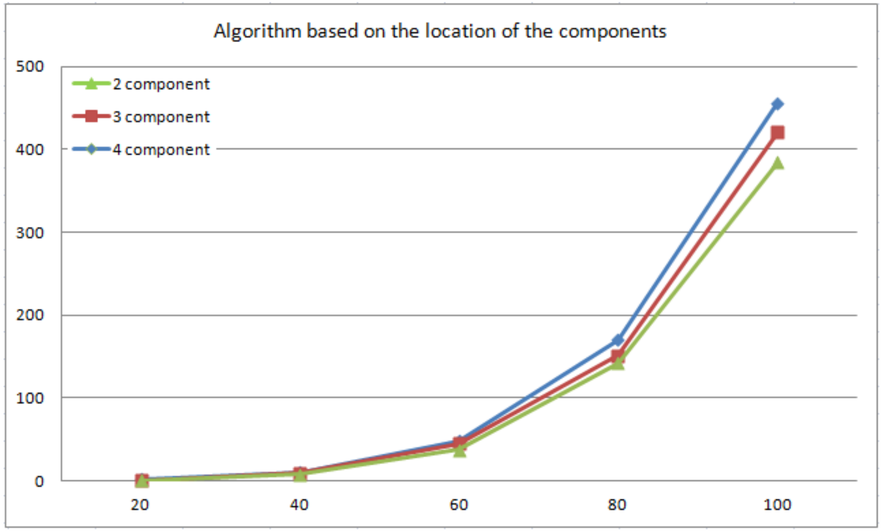
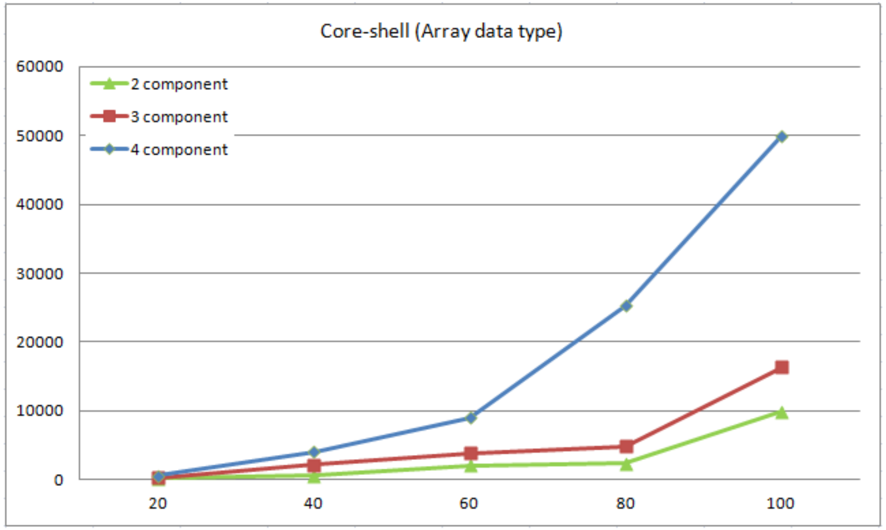
Running Time

- Core-shell algorithm with array data type and the algorithm based on the location of the components





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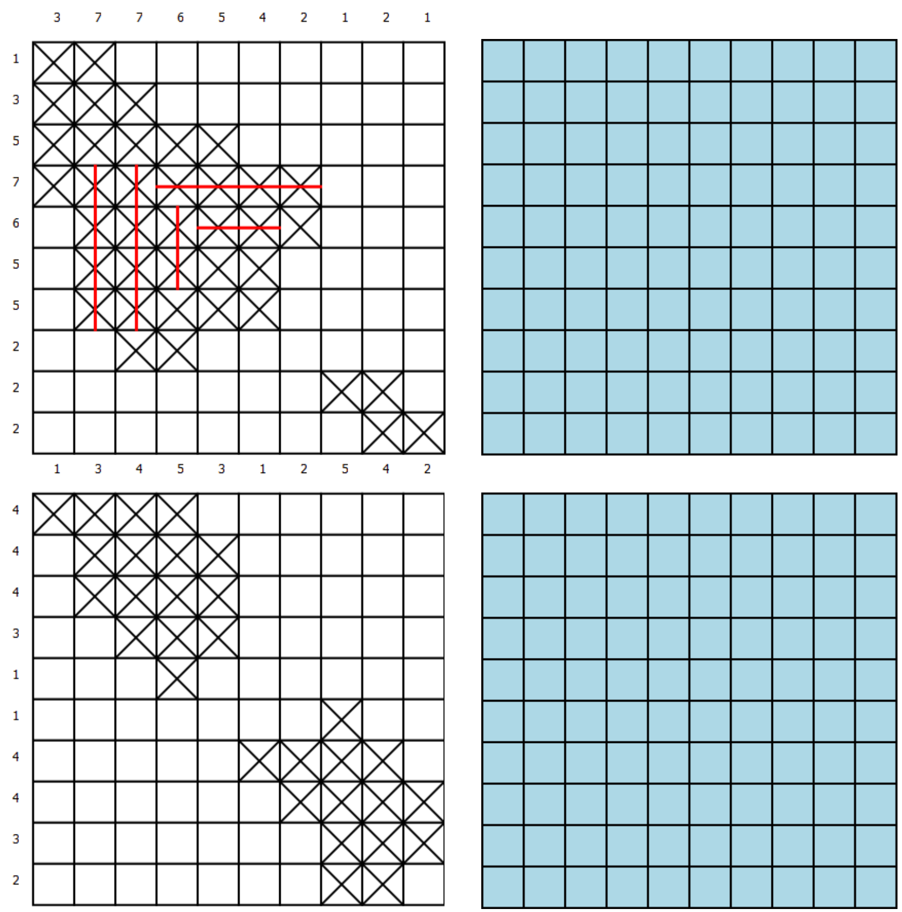
Influential Factors

- Core-shell algorithm
  - Size of the images
  - Number of the components
  - Position and size of the components
- Simulated annealing reconstruction algorithm
  - Number of switching components
- Algorithm based on the location of the components
  - Size of the image
  - Number of the components



Summary

- The difficulty of the problem depends on more than one factors
- Choose the most suitable algorithm by prior information and the projections
- Develop new algorithms

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NEW SZÉCHENYI PLAN

HUNGARY'S RENEWAL

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