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NEW SZÉCHENYI PLAN

CSM - The Second Conference of PhD Students in Mathematics

Program and Book of Abstracts

Organized by the Bolyai Institute of the University of Szeged and the Regional Committees of the Hungarian Academy of Sciences

June 28–30, 2012
Szeged, Hungary

Supported by TÁMOP-4.2.2/B-10/1-2010-0012 project

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Preface

Between the 28th and 30th of June, 2012, the Bolyai Institute of the University of Szeged and the Regional Committees of the Hungarian Academy of Sciences organizes the Conference for PhD Students in Mathematics, which is the second conference in a series. The conference is held in parallel with the Eighth Conference for PhD Students in Computer Science, with common invited speakers and common social programs.

The main aim of our conference is on the one hand to provide opportunities to PhD students and to young researchers to present their field of interest and their results in 20-minute talks in English. On the other hand, students can learn about the most recent research areas from the plenary talks given by renowned invited speakers. An equally important aspect is to provide a social experience to future mathematicians.

The topics of interest of the conference are mathematics and related fields, including theory and applications, e.g. algebra, analysis, differential equations, geometry, numeric and symbolic computation.

The contributed talks will have a length of 20 minutes with additional 5 minutes for discussion. The participants of the conference are invited to submit new research paper for publication in a special issue of *Acta Scientiarum Mathematicarum (Szeged)*.

Szeged, June 2012
The Scientific and Organizing Committee

Program

Thursday

09:00–11:00 **Registration**

11:00–11:50 **Plenary talk: András Kornai**
Language death in the digital age

11:50–12:00 **Break**

12:00–12:20 **Szilárd Szabó**
Spectral curves and roots of Kac–Moody algebras

12:25–12:45 **Amir Mosavi**
Optimal design of the NURBS curves and surfaces utilizing multiobjective optimization and decision making algorithms of RSO

13:00–14:30 **Lunch at "Roosevelt téri Halászcsárda"**

14:30–15:30 **Bolyai Institute Colloqium: Horst R. Thieme**
Global attractors, stability, and population persistence

15:30–15:50 **Coffee break**

15:50–16:10 **Tamás Mészáros**
Shattering extremal set systems

16:15–16:35 **Eszter Rozgonyi**
Additive representation functions

16:40–17:00 **Break**

17:00–17:20 **Franciska Petényi**
Character expansiveness in finite groups

17:25–17:45 **András Szántó**
Quasi-orthogonal subalgebras of matrix algebras

17:50–18:10 **István Szöllősi**
On short exact sequences of Kronecker modules

18:15–18:35 **Valentino Lanzone**
Binary fields on limited systems

19:00 **Reception**

20:45 **Social program**
⚽ Euro Cup 2012 Semifinal (public viewing)

Friday morning

Session 1

08:30–08:50 **Raluca Mureşan**
Uniform exponential stability for evolution families that are not exponentially bounded

08:55–09:15 **Cristina Praţa**
A version of a theorem of A.M. Lyapunov for the stability and instability of evolution families in Banach spaces

09:20–09:40 **Ábel Garab**
Global dynamics of a delayed Ricker model

09:45–09:55 **Break**

09:55–10:15 **Mónika Szűcs**
Avian-human influenza epidemic model with pulse vaccination

10:20–10:40 **Essam Awwad**
Application on the boundedness of nonlinear Volterra integral equations

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Session 2

08:30–08:50 **Ferenc Rárosi**
Comparison of statistical classification methods in radiotherapy data

08:55–09:15 **Kinga Sikolya**
Parameter estimation in a spatial linear regression model

09:20–09:40 **Renáta Vas**
Joint asymptotic normality of kernel type density estimator for spatial observations

09:45–09:55 **Break**

09:55–10:15 **Edit Lázár**
How to use the Rash-modell

10:20–10:40 **Ádám Hulmán**
Statistical visualisation tool development in Mathematica for longitudinal data analysis

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10:40–11:00 ☕ **Coffee break**

Friday morning

11:00–11:50 **Plenary talk: Horst R. Thieme**
Apparent paradoxes in disease models with horizontal and vertical transmission

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Session 1

12:00–12:20 **László Székely**
Disease regulated population in a SEIR model with delay

12:25–12:45 **Attila Dénes**
Global dynamics for the spread of ectoparasite borne diseases

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Session 2

12:00–12:20 **Balázs Bárány**
Slicing the Sierpinski gasket

12:25–12:45 **Gábor Lovics**
A method to approximate the whole Pareto-optimal set of a linearly constrained convex multiobjective optimization problem

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13:00–14:00 **Lunch at "Roosevelt téri Halászcsárda"**

Friday afternoon

Session 1

14:00–14:20 **Yikihiko Nakata**
Global analysis for spread of an infectious disease via human transportation

14:25–14:45 **Diána H. Knipl**
Modelling the spread of infectious diseases via global airline transportation

14:50–15:10 **Kyeongah Nah**
Mathematical models for P. vivax malaria and the dilution effect of the domestic animal population on the malaria transmission

15:15–15:25 ☕ **Coffee break**

15:25–15:45 **László Csizmadia**
On the stability regions of the inverted pendulum

15:50–16:10 **András Szijártó**
Observation problems posed for the Klein–Gordon equation

Session 2

14:00–14:20 **Claudia Zaharia**
A characterization of the completeness of random normed spaces

14:25–14:45 **Anca Farcaş**
On some modified Szász–Mirakjan operators

14:50–15:10 **Péter Kórus**
Uniform convergence of double trigonometric series with general monotonic coefficients

15:15–15:25 ☕ **Coffee break**

15:25–15:45 **Attila Szalai**
Characterization of stability of contractions

15:50–16:10 **György Gehér**
Positive operators arising from contractions

17:40 **Social program**
Bus trip, Museum, Gala dinner

Saturday

08:30–08:50 **Illés Horváth**
On different versions of sector conditions

08:55–09:15 **András Némedy Varga**
Statistical properties of the system of two falling balls

09:20–09:40 **Péter Nándori**
Lorentz process with shrinking holes in a wall

09:45–09:55 **Break**

10:00–10:20 **Tamás T. Szabó**
Change detection in INAR(p) processes and continuous branching models

10:25–10:45 **Kristóf Körmendi**
Parameter estimation for critical, symmetric 2-type Galton–Watson processes

10:50–11:10 **Péter Kevei**
Diminishing processes

11:15–11:35 **Coffee break**

11:35–11:55 **Manuela Wiesinger-Widi**
Gröbner bases by matrix triangularization

12:00–12:20 **Gergely Kiss**
Linear functional equations

12:25–12:45 **Mirjana Mikalački**
On doubly biased Maker-Breaker games

13:00–14:00 **Lunch at "Roosevelt téri Halászcsárda"**

14:00–14:20 **Gábor V. Nagy**
A simple combinatorial proof of Shapiro's Catalan convolution

14:25–14:45 **Balázs Udvari**
The number of convex polygons determined by a point set

14:50–15:10 **László Ozsvárt**
Graphs with ordered vertices

15:15–15:35 **Róbert Vajda**
On attacking Zolotarev polynomial approximation problems with quantifier elimination and Gröbner bases

15:40 **Closing and coffee**

Abstracts of plenary talks

Global attractors, stability, and population persistence

Horst R. Thieme
Arizona State University, Phoenix, Arizona, USA

Global compact attractors, in combination with Lyapunov functions and the Laplace transform, can be used to prove the global stability of extinction and persistence equilibria. A linearized stability analysis can be bypassed. For illustration, two models are considered: one for bacteria and phages in a chemostat and one for the spread of an infection in a spatially distributed population. To large parts, this is a joint work with Hal L. Smith.

Apparent paradoxes in disease models with horizontal and vertical transmission

Horst R. Thieme
Arizona State University, Phoenix, Arizona, USA

The question as to how the ratio of horizontal to vertical transmission depends on the coefficient of horizontal transmission is investigated in host-parasite models with one or two parasite strains. In an apparent paradox, this ratio decreases as the coefficient is increased provided that the ratio is taken at the equilibrium at which both host and parasite persist. Moreover, a completely vertically transmitted parasite strain that would go extinct on its own can coexist with a more harmful horizontally transmitted strain by protecting the host against it. Several stability results are presented for the coexistence equilibrium (host and two parasite strains). Under standard incidence, undamped oscillations may occur.

Abstracts of contributed talks

Application on the boundedness of nonlinear Volterra integral equations

Essam Awwad
University of Pannonia, Veszprém, Hungary

For linear and nonlinear Volterra integral equations with time delay, we obtain some results on the boundedness of the solutions. We also apply our results on Volterra integral equations to prove the BIBO (bounded input bounded output) stability of nonlinear control system with time delay.

This is a joint work with Prof. Győri István, and Prof. Hartung Ferenc, Department of Mathematics, Faculty of Information Technology, University of Pannonia, Veszprém, Hungary.

Keywords: Boundedness, Volterra integral equations, Bounded input bounded output (BIBO) stability, Differential equations with delays.

Slicing the Sierpiński gasket

Balázs Bárány
Technical University of Budapest, Hungary

Denote by $\Lambda \subset \mathbb{R}^2$ the *right-angle Sierpiński gasket*. Precisely, Λ is the attractor of the Iterated Function System (IFS)

$$\left\{ S_0(x,y) = \left(\frac{x}{2}, \frac{y}{2}\right),\ S_1(x,y) = \left(\frac{x}{2}+\frac{1}{2}, \frac{y}{2}\right),\ S_2(x,y) = \left(\frac{x}{2}+\frac{1}{4}, \frac{y}{2}+\frac{\sqrt{3}}{4}\right) \right\}.$$

It is well known since S_0, S_1, S_2 are contractions that the Λ set is the unique non-empty compact set satisfying

$$\Lambda = \bigcap_{n=1}^{\infty} \bigcup_{i_1,\dots,i_n=0}^{2} S_{i_1} \circ \cdots \circ S_{i_n}([0,1]^2) \text{ and } \Lambda = S_0(\Lambda) \cup S_1(\Lambda) \cup S_2(\Lambda).$$

We investigate the dimension of intersections of the Sierpiński gasket with lines. Let us denote the Hausdorff dimension of a bounded set V by $\dim_H V$ and respectively the box dimension by $\dim_B V$. Denote the s-dimensional Hausdorff measure by $\mathcal{H}^s$. It is well known that $s := \dim_H \Lambda = \dim_B \Lambda = \frac{\log 3}{\log 2}$ and $0 < \mathcal{H}^s(\Lambda) < \infty$.

Define proj_θ as the projection onto the line through the origin making angle θ with the x-axis. Our goal is to analyze the dimension theory of the slices $E_{\theta,a} = L_{\theta,a} \cap \Lambda$,

where $L_{\theta,a} = \{(x,y) \,:\, \mathrm{proj}_\theta(x,y) = a\} = \{(x, a + x\tan\theta) \,:\, x \in \mathbb{R}\}$ for $a \in \mathrm{proj}_\theta(\Lambda)$. Denote by ν the natural self-similar measure of Λ. That is, $\nu = \frac{\mathcal{H}^s|_\Lambda}{\mathcal{H}^s(\Lambda)}$. In this case, ν satisfies that

$$\nu = \sum_{i=0}^{2} \frac{1}{3} \nu \circ S_i^{-1}.$$

Denote by ν_θ the projection of ν by angle θ. That is, $\nu_\theta = \nu \circ \mathrm{proj}_\theta^{-1}$. Similarly, let Λ_θ be the projection of Λ.

Theorem 1. *Let us suppose that* $\tan\theta \in \mathbb{Q}$ *and* $\theta \in (0, \frac{\pi}{2})$. *Then there exist constants* $\alpha(\theta), \beta(\theta)$ *depending only on* θ *such that*

(1) $\alpha(\theta) := \dim_B E_{\theta,a} = \dim_H E_{\theta,a} < s - 1$ *for Lebesgue almost all* $a \in \Lambda_\theta$,

(2) $\beta(\theta) := \dim_B E_{\theta,a} = \dim_H E_{\theta,a} > s - 1$ *for* ν_θ*-almost all* $a \in \Lambda_\theta$.

If $\tan\theta \in \mathbb{Q}$ then proj_θ satisfies a dimension conservation formula, that is,

$$\beta(\theta) + \dim_H \{a \in \Lambda_\theta : \dim_H E_{\theta,a} = \beta(\theta)\} = s.$$

Moreover, we provide a multifractal analysis for the set of points in the projection for which the associated slice has a prescribed dimension. We describe the function

$$\chi(\delta) = \dim_H \{a \in \Lambda_\theta : \dim_H E_{\theta,a} = \delta\}$$

for every $\alpha(\theta) \le \delta \le b_{\max}$, where $b_{\max}$ is a constant depending on θ and $\tan\theta \in \mathbb{Q}$. We will prove that the function χ is decreasing, continuous and concave.

The talk is based on [1] which is a joint work with Károly Simon and Andrew Ferguson.

The results discussed above are supported by the grant TÁMOP - 4.2.2.B-10/1–2010-0009.

References

[1] B. Bárány, A. Ferguson, K. Simon: Slicing the Sierpiński gasket, to appear in Nonlinearity, (2012).

On the stability regions of the inverted pendulum

László Csizmadia
University of Szeged, Hungary

Let a be a step-function defined by

$$a(t) = \begin{cases} A_1, & \text{if } k(T_1+T_2) \le t < (k+1)T_1 + kT_2; \\ -A_2, & \text{if } (k+1)T_1 + kT_2 \le t < (k+1)(T_1+T_2),\ k \in \mathbb{Z}, \end{cases}$$

where $T_1 > 0,\ T_2 > 0,\ A_1 > 0,\ A_2 > g$ with $A_1T_1 = A_2T_2$, and g denotes the constant of the gravity. Consider the second order differential equation

$$\ddot{x} - \frac{g + a(t)}{l}x = 0,$$

which describes the small vibrations of the l-length pendulum around the upper equilibrium $x = 0$, provided that the suspension point of the pendulum is vibrated vertically by the $T_1 + T_2$-periodic acceleration $a(t)$.

We give a sufficient condition guaranteeing stability for the upper equilibrium. We apply this condition to the classical case $T_1 = T_2$, $A_1 = A_2$, and draw a *global* stability map on the $\varepsilon - \mu$ plane, where

$$\varepsilon^2 := \frac{1}{8}\frac{AT^2}{l}, \quad \mu^2 := \frac{g}{A}.$$

The map is global in the sense that ε and μ are not supposed to be small.

We are intrested in the case $T_1 \neq T_2$. If $\gamma := T_1 - T_2,\ T = \frac{T_1+T_2}{2},\ \kappa = \frac{g}{A_2}$ then what can we say about stability? Numerical experiments have been performed with Wolfram Mathematica.

Global dynamics for the spread of ectoparasite borne diseases

Attila Dénes
University of Szeged, Hungary

A mathematical model is introduced to simultaneously study the dynamics of ectoparasite infestation and infectious diseases spread by those ectoparasites.

The system has four potential equilibria. We identify three reproduction numbers that determine whether the infectious or the non-infectious parasites can invade the population, and whether a population already infested by non-infectious parasites can be invaded by the infection. By using Lyapunov functions and persistence theory, we show that the solutions always converge to one of the equilibria, depending on those three reproduction numbers. Hence the global dynamics is completely characterized by the reproduction numbers.

On some modified Szász–Mirakjan operators

Anca Farcaş
Babeş-Bolyai University, Cluj, Romania

The main purpose of the paper is to give an estimation of the rate of convergence for a new class of positive and linear operators of Szász–Mirakjan type . Furthermore, a Voronovskaja type result is proved for these modified operators using both uniform and statistical convergence.

Global dynamics of a delayed Ricker model

Ábel Garab
University of Szeged, Hungary

Consider the following discrete population model

$$x_{n+1} = x_n e^{a - x_{n-k}},$$

where a is a positive parameter and k is a positive integer. This model is a delayed version of the original Ricker model, which was introduced by Bill Ricker in 1954. S. Levin and R. May conjectured in 1976 that for the delayed Ricker model, local stability of the nontrivial equilibrium implies global stability. Based on rigorous, computer aided calculations and some analytical tools, we prove the conjecture for the case when k=1. This is joint work with Ferenc Bartha and Tibor Krisztin

Positive operators arising from contractions

György Pál Gehér
University of Szeged, Hungary

Let $\mathcal{H}$ be a complex Hilbert space. If $T \in \mathcal{B}(\mathcal{H})$ is a contraction i.e..: $\|T\| \leq 1$, then the sequence $\{T^{*n}T^n\}_{n=1}^{\infty}$ of positive operators is decreasing, so it has a positive limit in the strong operator topology (SOT):

$$A_T = \lim_{n\to\infty} T^{*n}T^n.$$

We say that A_T *is induced by* T, or A_T is the *asymptotic limit* of T. The subspace

$$\mathcal{N}(A_T) = \mathcal{H}_0(T) := \{x \in \mathcal{H} \colon \lim_{n\to\infty} \|T^n x\| = 0\}$$

is hyperinvariant for T and it is called the *stable subspace* of T. The subspace

$$\mathcal{N}(A_T - I) = \mathcal{H}_1(T) := \{x \in \mathcal{H} \colon \lim_{n\to\infty} \|T^n x\| = \|x\|\}$$

is the largest invariant subspace where T is an isometry.

We will describe those positive operators that arises from a contraction in such a way. Moreover we can ensure uniform convergence, and expect from the case when $0 < \dim \mathcal{H}_1 < \aleph_0$ we can choose a co-stable contraction i.e.: $\mathcal{H}_0(T^*) = 0$. After that we give some sufficient condition for two contraction, having the same asymptotic limit.

Keywords: Hilbert space contraction, asymptotic limit, positive operators.

Modelling the spread of infectious diseases via global airline transportation

Diána H. Knipl
University of Szeged, Hungary

National boundaries never hindered infectious diseases to reach distant territories; however, the speed at which an infectious agent now can spread around the world has significantly increased in the last 50 years. We introduce an SEAIR-based model for long distance travel networks, which describes the dynamics of a pandemic on regions connected by air transportation. Due to the high connectedness of several distant places, we include the possibility of transmission of the disease during travel, which is modeled by an age structured system where age is the time elapsed since the start of the travel. The model is equivalent to a large system of delay differential equations, we examine fundamental properties of the system. We detail the method of the calculation of the reproduction number, and parametrize the model with influenza and real air traffic data.

Keywords: influenza modelling, long distance travel networks, delay differential equations.

On different versions of sector conditions

Illés Horváth
Budapest University of Technology and Economics, Hungary

The theory of central limit theorems for additive functionals of ergodic Markov processes via martingale approximation was initiated in 1986 by Kipnis and Varadhan.

We investigate the following question: given a stationary and ergodic Markov process $\eta(t)$ on the probability space $(\Omega, \mathcal{F}, \pi)$ and a function $f : \Omega \to \mathbb{R}$, what conditions on η and f guarantee central limit theorem/invariance principle for the integral

$$N^{-1/2} \int_0^{Nt} f(\eta(s))\,\mathrm{d}s \tag{1}$$

as $N \to \infty$?

Kipnis and Varadhan originally proved efficient martingale approximation and central limit theorem for the reversible case with no assumptions other than the necessary ones. Since then, the theory have been extended by Varadhan and others to include processes with a varying degree of non-reversibility. Applications include the tagged particle in simple exclusion, persistent random walk in random environment, the myopic self-avoiding walk, self-repellent Brownian polymer etc.

Some of the sufficient conditions for martingale approximation are abstract and not easy to verify for particular applications. So-called sector conditions were introduced in order to provide conditions that can be checked more directly. The (strong) sector condition was introduced by Varadhan in 1996, and was later further generalized by the graded sector condition (by Sethuraman, Varadhan and Yau, 2000) and the relaxed sector condition (by H., Tóth, Vető in 2012).

We state and prove a newer version of the graded sector condition; apart from the conditions being less restrictive than in previous formulations, the proof is also less technical.

This is joint work with Bálint Tóth and Bálint Vető.

The results discussed above are supported by the grant TÁMOP - 4.2.2.B-10/1–2010-0009.

Statistical visualisation tool development in Mathematica for longitudinal data analysis

Ádám Hulmán, János Karsai, Tibor A. Nyári, Ádám G. Tabák, Daniel R. Witte
University of Szeged, Hungary

The translation of statistical models into a clinical setting is usually not simple when the model includes interactions and nonlinear associations between time-varying variables. We introduce the problem with a case-study from diabetes epidemiology. A large longitudinal dataset was analysed with multilevel models to assess fasting and 2h plasma glucose trajectories during 18 years of follow-up, with adjustment for time-varying covariates. An interactive application was developed in Wolfram Mathematica to enhance the interpretation of the results.

Diminishing processes

Gergely Ambrus, **Péter Kevei**, Viktor Vígh
University of Szeged, Hungary

Let $\Xi_0 = [-1, 1]$, and define the segments Ξ_n recursively in the following manner: for every $n = 0, 1, \ldots$, let $\Xi_{n+1} = \Xi_n \cap [a_{n+1} - 1, a_{n+1} + 1]$, where the point a_{n+1} is chosen randomly on the segment Ξ_n with uniform distribution. For the radius ρ_n of Ξ_n we prove that $n(\rho_n - 1/2)$ converges in distribution to an exponential law, and we show that the centre of the limiting unit interval has arcsine distribution. We also consider the higher-dimensional analog of the problem.

Linear functional equations

Gergely Kiss, Miklós Laczkovich
Eötvös University Budapest, Hungary

We investigate the structure of the solutions of the equation

$$\sum_{i=1}^{n} a_i f(x + b_i y) = 0 \qquad (x, y \in \mathbb{C})$$

and formulate some general conjectures concerning the solutions. In some special cases we describe every solution of the equation. Our method is to apply the theory of spectral synthesis and analysis on discrete Abelian groups.

Parameter estimation for critical, symmetric 2-type Galton–Watson processes

Kristóf Körmendi, Pap Gyula
University of Szeged, Hungary

We introduce the 2-type Glaton–Watson process, and define it's parameters. Conditional least square estimators for the parameters will be given. We will prove the asymptotic properties of these estimators. We will discuss the problems arising, when trying to generalize the results for the non symmetric case.

Uniform convergence of double trigonometric series with general monotonic coefficients

Péter Kórus
University of Szeged, Hungary

The basic theorem in the theory of uniform convergence of sine series is due to Chaundy and Jolliffe from 1916. They proved that if $\{c_k\}_{k=1}^{\infty}$ is a nonnegative, monotonically decreasing sequence, then the series $\sum_{k=1}^{\infty} c_k \sin kx$ converges uniformly in x if and only if $kc_k \to 0$. Since 1916, a number of papers were published to extend this theorem by enlarging the class of monotonic coefficients, while keeping the condition $kc_k \to 0$ still necessary and sufficient. The largest class defined for the appropriate extension is class SBVS_2 introduced by P. Kórus in 2009. In that year, M. Dyachenko and S. Tikhonov proved a theorem for the uniform convergence of cosine series with β-general monotone coefficients. That theorem leads to the following. If $\{c_k\}_{k=1}^{\infty}$ is from SBVS_2, then the conditions $\sum_{k=1}^{\infty} c_k$ converges and $kc_k \to 0$ are sufficient for the uniform convergence of the series $\sum_{k=1}^{\infty} c_k \cos kx$, and these conditions are necessary in case $\{c_k\}_{k=1}^{\infty}$ is also nonnegative.

For the uniform regular convergence of the double sine series

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \sin jx \sin ky,$$

Žak and Šneider proved in 1966 that if $\{c_{jk}\}_{j,k=1}^{\infty}$ is nonnegative, monotonically decreasing (in a two dimensional sense), then the regular convergence of the previously defined double sine series is uniform if and only if $jkc_{jk} \to 0$ as $j + k \to \infty$. This result was extended in 2011 to the coefficient class SBVDS_1 (the sufficiency part was even proved for a larger class SBVDS_2). Now we give sufficient conditions for the uniform regular convergence of the sine-cosine series $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \sin jx \cos ky$ and the double cosine series $\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{jk} \cos jx \cos ky$ with coefficients from SBVDS_1 which are necessary in case $\{c_{jk}\}_{j,k=1}^{\infty}$ is also nonnegative.

Binary fields on limited systems

Valentino Lanzone, Gábor Péter Nagy
University of Basilicata, Potenza, Italy

Motivated by applications to modern cryptography, we describe some simple techniques aimed at performing computations over binary fields using systems with limited resources.

The primes p for an efficient implementation of arithmetic in prime fields are chosen be either a Mersenne prime of the form $p = 2^n - 1$, or a pseudo-Mersenne prime of the form $p = 2^n - r$ with the smallest possible integer r. These primes allow an efficient modular reduction by using the replacement $a2^n \equiv ar \pmod{p}$, repeating it as necessary.

From the observation of classical algorithms with this particular p, we have developed similar simple algorithms for binary fields using an appropriate representation of bit sequences by suitable integers and presented an implementation of the arithmetic in $\mathrm{GF}(2^t)$ with polynomial basis. We keep in mind the isomorphism: $\mathrm{GF}(2^t) \simeq \mathrm{GF}(2)[x]/p(x)$, where $p(x) = x^t + r(x)$ is an irreducible polynomial of degree t over $\mathrm{GF}(2)$ with few terms and the smallest possible degree of $r(x)$. Usually $p(x)$ is an irreducible polynomial with three or five terms (trinomials and pentanomials, respectively). The operations between t-bits binary sequences are identified with the operations between polynomials of degree $t-1$ modulo $p(x)$.

By using an appropriate representation of binary numbers as integers, we are able to access the bits representing the coefficients of the polynomials with appropriate functions and statements in terms of integers.

The algorithms described in the present work provide an increased efficiency in computations, when compared to the usual arithmetic over prime fields. We can observe that our algorithms on binary fields have execution times falling in a limited range and therefore are more efficient and suitable, while, in particular, the operation of inversion on prime fields has an execution time much larger than on binary fields, and this grows very rapidly.

We can expect that our algorithms are very useful for cryptography, such as for elliptic curve cryptosystems, based on binary fields using simple algorithms that are designed to work in limited systems such as microcontrollers, smart cards, rfids, etc. with very low use of memory.

How to use the Rash-modell

Edit Lázár
University of Debrecen, Hungary

The topic is not new. We use questionnaires/surveys at weekdays and in the teaching too, to measure students knowledge. Here I want to look at only its connection with maths and maths education. How can you prepare a good, well-structured test? How can you choose the right/best questions? All of the questions are necessary or can you leave some of them? To response these and similar questions is the classical test-theory. Unfortunately this is not without errors and it is not cover all of the questions e.g. in this case all of the questions have equal importance for the summary. For the better approximation nowadays there are new ramifications of test-theory e.g. Rash-modell. This modell is used in the pedagogy. In my essay I want to show this modell and its use by an SPSS example in the math-teaching. I want to analyse a real test (only using the results data without showing/knowing the original questions).

A method to approximate the whole Pareto-optimal set of a linearly constrained convex multiobjective optimization problem

Tibor Illés, **Gábor Lovics**
Budapest University of Technology and Economics, Hungary

In economic and engineering application of mathematics sometimes we need to optimize more than one objective function at the same time. In this type of problems we need to find solutions, where one of the objectives can not be improved without worsen the other. These solutions are called Pareto-optimal solutions, and in since 1950's such methods are known that compute one of the Pareto-optimal solutions. Recently, for unconstrained mulitobjective optimization problems such algorithm has been developed by Oliver Schütze at al. (2003) that try to approximate the whole set of the Pareto-optimal solutions at the same time. In this talk we generalize the subdivision algorithm of Schütze and others for linearly constrained multiobjective problem. The objective functions in our case need to be differentiable convex functions. Further generalization of the more general class of problems (convex constrained and convex objective function for mineralization problem) seems to be possible. The main idea of the method to find joint decreasing direction, for the all objective function at the same time.

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Shattering extremal set systems

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A basic problem in mathematics is to characterize objects given by local properties. In general it is a hard task to verify such local properties, what makes the study of these structures difficult.

We say that a set system $\mathcal{F}$ on $\{1, 2, ..., n\}$ shatters a given set S if all of its subsets can be obtained by intersecting it with a set from $\mathcal{F}$. It can be proved that a set system $\mathcal{F}$ shatters at least $|\mathcal{F}|$ sets. Here we study shattering-extremal set systems, those, which shatter exactly $|\mathcal{F}|$ sets.

When considering the elements of $\mathcal{F}$ as characteristic vectors, one can define the ideal of polynomials in n variables vanishing on $\mathcal{F}$. It is worth investigating this ideal $I(\mathcal{F})$ instead of $\mathcal{F}$, because several algebraic tools can be of great help, among others Gröbner bases and the standard monomials for different term orders. This point of view yields an efficient algorithm for testing the shattering-extremality of a set system ([1]) and also a possible way to generalize shattering-extremality for any finite sets of vectors.

Another standard way of looking at a set system in general is to embed it into the Hamming graph $\{0,1\}^n$, and study the properties of the resulting inclusion graph. Several interesting results can be obtain for the inclusion graphs of shattering-extremal families (e.g. [2]), together with my supervisor Lajos Rónyai (BME, SZTAKI) we studied the inclusion graphs of shattering-extremal families of small Vapnik-Chervonenkis dimension.

References

[1] L. Rónyai, T. Mészáros: Some Combinatorial Applications of Gröbner bases, Proc.CAI 2011, Lecture Notes in Computer Science, Vol. 6742

[2] G. Greco, Embeddings and trace of finite sets, Information Processing Letters, Vol. 67 (1998), 199–203

On doubly biased Maker-Breaker games

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Let V be a finite set and let $\mathcal{F} \subseteq 2^V$ be the family of its subsets. Positional game is a pair $(V, \mathcal{F})$, where V is referred to as *board*, and the sets of $\mathcal{F}$ is referred to as the *winning sets*. In the $(a : b)$ *Maker - Breaker* game, a type of positional game, two players called *Maker* and *Breaker* take turns in claiming previously unclaimed elements of the board. Maker claims a elements in each move and *Breaker* claims b elements in each move. The game ends when all the elements of the board are claimed. Maker aims to claim all the elements of some $F \in \mathcal{F}$ and Breaker wants to prevent him from doing that, i.e. he wants to put at least one element in each winning set. The game is *"Maker's win"* if Maker has a strategy to win against any strategy of Breaker. Otherwise, the game is *"Breaker's win"*.

When $a = b = 1$ the game is called *unbiased*, and in many $(1 : 1)$ Maker – Breaker games, Maker wins quite easily. Thus, to "even out the odds" and increase Breaker's chances to win, the *biased* games are introduced. The question that comes naturally is to determine the winner of $(1 : b)$ Maker - Breaker game, when b is greater than one, and to increase b until the game becomes more balanced.

We study $(a : b)$ Maker - Breaker games played on the edge set of the complete graph on n vertices, $E(K_n)$, where n is sufficiently large integer and both a and b can be greater than one. The winning sets we look at are various graph theoretic properties like spanning tree, Hamilton cycle, etc. For each $a = a(n)$, we want to determine $b_0(a, n) = b_0(a)$, so that for $b < b_0(a)$ the game is Maker's win, and for $b > b_0(a)$ the game is Breaker's win. We refer to $b_0(a)$ as the *threshold bias for* a.

Keywords: Maker - Breaker games, biased games.

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References

[1] V. Chvátal and P. Erdős, Biased Positional Games, *Annals of Discrete Math.* 2:221–228, 1978.

[2] H. Gebauer and T. Szabó, Asymptotic random graph intuition for the biased connectivity game, *Random Structures & Algorithms* 35:431–443, 2010.

[3] Y.O. Hamidoune, M. Las Vergnas, A solution to the Box Game, *Discrete Mathematics* 65:157-171, 1987.

[4] D. Hefetz, M. Mikalački, M. Stojaković, Doubly biased Maker - Breaker Connectivity Game, *The Electronic Journal of Combinatorics* 19(1):P61, 2012.

[5] M. Krivelevich, The critical bias for the Hamiltonicity game is $(1 + o(1))n/\ln n$, *Journal of the American Mathematical Society* 24:125–131, 2011.

Optimal design of the NURBS curves and surfaces; utilizing multiobjective optimization and decision making algorithms of RSO

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A reliable optimal design process for the Non-uniform rational B-spline (NURBS) curves and surfaces is considered as one of the foundational concepts in CAGD, CAD and image processing. Yet the involved mathematical modeling of the optimal design and parameters tuning is a complicated, highly non-linear and multiobjective optimization (MOO) problem which cannot be handled as such by the traditional single objective optimization algorithms. The complexity of the problem is even increased when the criteria of product beauty is included to the evaluation process. More on the problem, its applications, and previous approaches, is available in [1, 3, 6], where the usage of MOO algorithms enhances the design process by enabling optimization of several design objectives at once. In this article the design objectives include surface elastic energy, surface area, approximation error and surface beauty. For solving problems as such, with a high level of complexity, modeling the true nature of the problem is of great importance. For this reason, in this study, a considerable amount of efforts is devoted into modeling phase. Furthermore, as an alternative to the previous approaches the robust and interactive MOO algorithm of RSO [2] is proposed in order to efficiently optimize all the design objectives at once including the criteria of beauty in which could not be completely considered in the previous attempts. In this framework the quality of the surface, similar to the previous research workflows, is measured using a set of certain functions. Then the model is integrated to the optimizer via advanced interfaces to the RSO algorithm and its brain-computer evolutionary multiobjective optimization implementation and visualization. Utilizing the

RSO algorithm; the problem is actually treated by reducing the dimensionality and the dataset size [4, 7], multi-dimensional scaling, clustering, and visualization tools [2, 5]. At the end the application of learning and intelligent optimization as well as reactive business intelligence approaches in improving the process of such complex optimization problems is further discussed.

References

[1] A. Mosavi, On Engineering Optimization the Splined Profiles, in Proceedings of International modeFRONTIER Users' Meeting, Trieste, Italy, 2010.

[2] R. Battiti, M. Brunato, Reactive Business Intelligence. From Data to Models to Reactive Search Srl, Italy, February 2011.

[3] R. Goldenthal, M. Bercovier, Design of Curves and Surfaces by Multiobjective Optimization, Mathematical Methods for Curves and Surfaces, 2004.

[4] A. Mosavi (2010), Multiple criteria decision-making preprocessing using data mining tools. IJCSI International Journal of Computer Science Issues, V7, Issue 2, No 1.

[5] A.Mosavi, M.Azodinia, Kasun N. Hewage, Abbas S. Milani, M.Yeheyis, Reconsidering the Multiple Criteria Decision Making Problems of Construction Workers Using; Grapheur, Poster presented at EnginSoft International Conference, Verona, Italy 20-21 Oct.

[6] Amir Mosavi, The Multiobjective Optimization Package of IOSO; Applications and Future Trends, Conference of PhD Students in Computer Science, Szeged, Hungary, June 29 - July 2, 2010

[7] A. Adejuwonand, A. Mosavi, Domain Driven Data Mining - Application to Business, IJCSI International Journal of Computer Science Issues, Vol. 7, Issue 4, No 2, July 2010.

Uniform exponential stability for evolution families that are not exponentially bounded

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In the present paper we give a characterization of the uniform exponential stability for evolution families that are not exponentially bounded in terms of the admissibility of the following pairs of function spaces $(L^1(X), L^\infty(X))$ and $(L^p(X), L^q(X))$.

A simple combinatorial proof of Shapiro's Catalan convolution

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Using generating functions, Shapiro proved the following elegant convolution formula involving Catalan numbers of even index:

$$\sum_{k=0}^{n} C_{2k}C_{2n-2k} = 4^n C_n.$$

We give a simple combinatorial proof of this formula.

In addition, we show bijectively that it is equivalent with the alternating convolution formula of central binomial coefficients. Our key observation is a non-standard interpretation of the Catalan number C_{2k}.

Mathematical models for P. vivax malaria and the dilution effect of the domestic animal population on the malaria transmission

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According to the estimation on the incubation period of P. vivax malaria in Korea, the incubation periods fall into two categories with short- and long-term incubation periods. We compare several possible mathematical models having different expression for the incubation periods.

With model having two exposed compartments for having short and long-term incubation period, K. Nah et al (2010) studied the dilution effect of animals on the transmission of P.vivax malaria.

Global analysis for spread of an infectious disease via human transportation

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We investigate the global dynamics of epidemic models describing disease transmission dynamics between multiple regions. We generalize the model developed in [1] to consider different characters of each region such as population size and transportation rate. We analyze the global dynamics in terms of the basic reproduction number. We illustrate the stability regions of equilibria in a parameter plane and discuss how human transportation between regions influences the spread of the disease. This is a joint work with Gergely Röst.

References

[1] J. Liu, J. Wu, and Y. Zhou, *Modeling disease spread via transport-related infection by a delay differential equation*, Rocky Mountain J. Math., 38 (2008), pp.1525–1540.

Lorentz process with shrinking holes in a wall

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The periodic Lorentz process is a fascinating non-linear, chaotic dynamical system that has been deeply investigated in the last decades. The model is very simple: a massless point particle moves freely in the plane (or in our case, in a strip) until it hits one of the periodically situated smooth convex scatterers, when it is reflected. The limit of the diffusively scaled trajectory of the particle is known to be the Brownian motion. Further, if the particle is restricted to a half strip, then the scaling limit is going to be the so-called reflected Brownian motion. Here we introduce a *time-dependent scatterer configuration* (by adding a vertical wall with a shrinking hole) that almost confines the particle to the half strip in such a way that the scaling limit is a *quasi-reflected Brownian motion*. This process is Markovian but not strong Markovian and is a natural generalization of both the Brownian motion and the reflected Brownian motion. Local time results for the periodic Lorentz process, having independent interest, are also found and used.
In this talk, we present the results of the preprint P. Nándori, D. Szász: Lorentz Process with shrinking holes in a wall (to appear in *Chaos: An Interdisciplinary Journal of Nonlinear Science*). The results discussed above are supported by the grant TÁMOP - 4.2.2.B-10/1–2010-0009.

Statistical properties of the system of two falling balls

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I would like to talk on joint results with Péter Bálint and Gábor Borbély (see [1]) on a billiard type model with intermittent behavior. The system of two falling balls, introduced by Wojtkowski, describes the motion of two point particles of mass m_1 and m_2 that move along the vertical half-line (bounded from below by the ground), subject to constant gravitational force, and collide elastically with each other and the floor.

We consider the case when the lower ball is heavier (i.e. $m_1 > m_2$) which corresponds to ergodic and hyperbolic dynamics; however, hyperbolicity is not uniform related to arbitrary long series of bounces of the lower ball on the floor before colliding with the upper ball. We present a detailed analysis of the discrete time model and prove that, for an open set of mass ratios, the correlations decay, modulo logarithmic factors, as $\mathcal{O}(1/n^2)$. This rate is summable, accordingly, the central limit theorem is also proved; that is, the system exhibits normal diffusion.

In addition to presenting these results I would like to mention some ongoing work concerning the continuous time model, which can be regarded as a suspension flow, and the structure of the singularity set of the dynamics, which is highly relevant for the statistical properties.

The work reported in this talk has been developed in the framework of the project "Talent care and cultivation in the scientific workshops of BME" project. This project is supported by the grant TÁMOP - 4.2.2.B-10/1–2010-0009.

References

[1] Péter Bálint, Gábor Borbély and András Némedy Varga: *Statistical properties of the system of two falling balls*, to appear in Chaos, avialable at `http://www.math.bme.hu/ nemedy/Research`

Graphs with ordered vertices

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We call $G = (V, E, <)$ a vertice-ordered graph (vograph), if (V, E) is a simple graph, and $<$ is a linear ordering on the set V. By fixing G, we can study vographs that does not contain a subgraph on which the vertice ordering is the same as on G. We will be presenting this problem on a few examples.

Character expansiveness in finite groups

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Halasi, Maróti, Sidki and Bezerra in [1] studied *conjugacy expansive* and *normal conjugacy expensive* groups.

In a similar way one can define *character expansive* groups. A finite group G is *character expansive* if for any complex character α and irreducible character χ the number of irreducible constituents of the product $\alpha\chi$ (counting without multiplicity) is at least the number of irreducible constituents of α (again counting without multiplicity). For example, Abelian groups are character expansive.

We examined whether a character expansive group can always be written as a direct product of simple or Abelian groups. We note that the other direction of this problem is false, since not every simple group is character expansive. Unfortunately until now we were unable to solve the above problem completely.

Instead of character expansiveness we can consider a weaker notion. We say that G is *normal character expansive* if for any normal subgroup N and any irreducible character χ of G we have $n(1_N^G) \leq n(1_N^G \cdot \chi)$. Character expansiveness clearly implies normal character expansiveness.

We conjecture that a group is normal character expansive if and only if it is a direct product of simple or Abelian groups.

The talk is based on a joint work with Zoltán Halasi, Attila Maróti and László Héthelyi.

The research has been carried out in the framework of the project "Talent care and cultivation in the scientific workshops of BME", supported by the grant TÁMOP - 4.2.2.B-10/1-2010-0009.

References

[1] Halasi, Z.; Maróti, A.; Sidki, S.; Bezerra, M. Conjugacy expansiveness in finite groups. To appear in *J. Group Theory.*

A version of a theorem of A.M. Lyapunov for the stability and instability of evolution families in Banach spaces

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In the present paper we give some characterizations for the stability and instability of the evolution families by using Lyapunov function, in Banach spaces.

These results are a generalization of those obtained by N.U. Ahmed in [1] for the case of C_0-semigroups.

References

[1] N.U. Ahmed; *Semigroups Theory with Applications to Systems and Control*, Pittman Research, Notes Math. 1991.

[2] J.L. Daleckij, M.G. Krein; *Stability of Differential Equations in Banach Space*, Amer. Math. Soc., Providence, R.I., 1974.

[3] R. Datko; *Extending a Theorem of Lyapunov to Hilbert Spaces*, J. Math. Anal. Appl. 32, 610–616 (1970).

[4] C. Preda, P. Preda; *Lyapunov operator inequalities for exponential stability of Banach space semigroups of operators*, Applied Mathematics Letters, vol. 25(3), 401–403 (2012).

Comparison of statistical classification methods in radiotherapy data

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Radiotherapy is an effective treatment for breast cancer, but it can bring significant late morbidity, particularly to the heart.
For the „gold standard" decision about the preferable treatment position, series of CT scans and therapy plans are needed in both positions. This method is expensive (technology, work of physicians) and means extra radioactive dose to the patients.
Our goal is to compare different classification methods and set up a model based classifier method that identifies the better setup using not too many predictors and only one CT slice. We were searching for a method which can be used later in the medical practice.
The data base consisted of 138 patients of breast cancer measured in both positions. The radiation dose to the heart may individually vary in the supine versus the prone position in most cases. Indicator of the risk is the dose to left anterior descendent coronary artery (LAD). The LAD mean dose difference is a continuous dependent variable. We predicted not just the preferable position, but the estimated dose difference also, using different regression models and classification methods and three predictors.
We compared several classification methods (multiple logistic regression: hierarchical and non hierarchical logistic models, discriminant analysis, decision tree, neural networks, perceptron models) in this real database. The performance of the methods was compared by ROC (receiver operating characteristics) analysis and by the percentage of correctly classified cases. We used a random cross-validation method and we estimated not just the proportion of misclassified patients, but the distribution of the "misclassified dose" also.

With a defined "grey zone" (15% of the patients) we achieved a good classification of the cases using regression based method. Sensitivity: 89.2%, specificity: 87.5%.

Additive representation functions

Csaba Sándor, **Eszter Rozgonyi**, Sándor Z. Kiss
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Let $\mathbb{N}$ be the set of nonnegative integers. For a given set $\mathcal{A} \subset \mathbb{N}$ the representation functions $R^{(1)}_{h,\mathcal{A}}(n)$, $R^{(2)}_{h,\mathcal{A}}(n)$ and $R^{(3)}_{h,\mathcal{A}}(n)$ are defined as the number of solutions of the equation $a_{i_1} + \cdots + a_{i_h} = n$, $a_{i_1}, \ldots, a_{i_h} \in \mathcal{A}$ without any condition and with condition $a_{i_1} < \cdots < a_{i_h}$ and $a_{i_1} \leq \cdots \leq a_{i_h}$.

In 1978, Nathanson proved, if $\mathcal{A}$ and $\mathcal{B}$ are distinct nonempty sets of integers such that $R^{(1)}_{2,\mathcal{A}}(n) = R^{(1)}_{2,\mathcal{B}}(n)$ for all sufficiently large n then $\mathcal{A} = F_{\mathcal{A}} \cup S$ and $\mathcal{B} = F_{\mathcal{B}} \cup S$, where $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S have some nice properties. We extend this result to the $R^{(1)}_3(n)$ representation function. In this case the construction of the sets $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S are a bit difficult but still nice.

In 2011, Yang posed the next problem in his article: if $p \geq 3$ is a prime and $\mathcal{A}$ is a set of nonnegative integers, then does there exist a set of nonnegative integers $\mathcal{B}$ with $\mathcal{A} \neq \mathcal{B}$ such that $R^{(1)}_{p,\mathcal{A}}(n) = R^{(1)}_{p,\mathcal{B}}(n)$ for all sufficiently large n? We solved this problem in a generalized way. We proved that for arbitrary $h \geq 2$, $h \in \mathbb{Z}$ there exist $\mathcal{A}, \mathcal{B} \subset \mathbb{N}$, such that $R^{(i)}_{h,\mathcal{A}}(n) = R^{(i)}_{h,\mathcal{A}}(n)$ for all sufficiently large n and for $i = 1, 2, 3$.

In the proofs we use generating functions and some other theorem from algebraic and analytic number theory.

In the talk I will try to give a short summarize about the results, which is joint work with Sándor Z. Kiss and Csaba Sándor.

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Parameter estimation in a spatial linear regression model

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Let $[a, c] \subset (0, \infty)$ and $b_1, b_2 \in (a, c)$, let $\gamma_{1,2} : [a, b_1] \to \mathbb{R}$ and $\gamma_0 : [b_2, c] \to \mathbb{R}$ be continuous, strictly decreasing functions and let $\gamma_1 : [b_1, c] \to \mathbb{R}$ and $\gamma_2 : [a, b_2] \to \mathbb{R}$ be continuous, strictly increasing functions with $\gamma_{1,2}(b_1) = \gamma_1(b_1) > 0$, $\gamma_2(b_2) = \gamma_0(b_2)$, $\gamma_{1,2}(a) = \gamma_2(a)$ and $\gamma_1(c) = \gamma_0(c)$. We consider the problem of estimating the parameters of a linear regression model $Z(s,t) = m_1 g_1(s,t) + \cdots + m_p g_p(s,t) + U(s,t)$

based on observations of Z on the set G which contains the points bounded by the functions γ_0, γ_1, γ_2 and $\gamma_{1,2}$, where the driving process U is a Gaussian random field and $g_1, \ldots, g_p$ are known functions. Using a discrete approximation we obtain explicit forms of the maximum-likelihood estimators of the parameters in the cases when U is either a Wiener or a stationary or nonstationary Ornstein–Uhlenbeck sheet. In the case when U is a standard Wiener sheet we have a generalization of the results of Arató, N. M. [1] and Baran *et al.* [4]. We also consider the cases when the driving process U is a stationary and a zero start Ornstein–Uhlenbeck sheet and generalize the results of Arató, N. M. [2] and Baran *et al.* [3].

Moreover, we present some simulation results to illustrate the theoretical ones where the driving Gaussian random sheets are simulated with the help of their Karhunen-Loève expansions (see e.g. [5]).

References

[1] Arató, N. M., Mean estimation of Brownian sheet. *Comput. Math. Appl.* (1997) **33**, 13–25.

[2] Arató, N.M., Mean estimation of Brownian and Ornstein–Uhlenbeck sheets. *Teor Veroyatnost i Primen.* (1997) **42**, 375–376

[3] Baran, S., Pap, G., Zuijlen, M., Estimation of the mean of stationary and nonstationary Ornstein-Uhlenbeck processes and sheets. *Comput Math Appl.* (2003) **45**, 563–579

[4] Baran, S., Pap, G., Zuijlen, M., Parameter estimation of a shifted Wiener sheet. *Statistics.* (2011) **45**, 319–335

[5] Deheuvels, P., Peccati, G. and Yor, M., On quadratic functionals of the Brownian sheet and related processes. *Stochastic Process. Appl.* (2006) **116**, 493–538.

Spectral curves and roots of Kac–Moody algebras

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We study the following question: given a finite number of conjugacy classes in $\mathfrak{gl}_r(\mathbb{C})$ with generic eigenvalues when does there exist a matrix from each class whose sum is equal to 0. The problem can be translated into a purely combinatorial one. We give a geometric / invariant theoretic proof of a theorem of Crawley–Boevey that a solution exists if and only if the associated combinatorial data is a root of the corresponding Kac–Moody algebra. This is joint work with Olivier Biquard.

Characterization of stability of contractions

Attila Szalai
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The talk is based upon a joint work with my supervisor László Kérchy. We characterize those sequences of bounded analytic functions $\{h_n\}_{n=1}^{\infty} \subset H^{\infty}$ which have the property that an absolutely continuous contraction T is stable (that is the powers T^n converge to zero) exactly when the operators $h_n(T)$ converge to zero in the strong operator topology. Our result is extended to polynomially bounded operators, too.

Quasi-orthogonal subalgebras of matrix algebras

Dénes Petz, **András Szántó**
Budapest University of Technology and Economics, Hungary

The idea of complementarity in quantum measurement theory and information theory has led to the investigation of the subject. Subalgebras of the $M_n \otimes M_n$ complex matrix algebra corresponding to either measurements or subsystems are considered. Quasi-orthogonality is a generalization of mutually unbiasedness, and it means orthogonality of the traceless subspaces. We seek direct-sum decompositions of the full matrix-algebra to pairwise quasi-orthogonal subalgebras. We provide several results on the $n = 2$ case, and some for general n. The related notion of conditional POVM is explained as well.

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Disease regulated population in a SEIR model with delay

László Székely
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We consider a SEIR-type disease transmission model with fixed latency period, standard incidence and variable population size. It is assumed that the fertility of infected individuals is decreased, and individuals recover from the disease and acquire permanent immunity with probability f, and dies from the disease with probability $1 - f$. Two threshold parameters are found which determine whether the disease dies out or remains endemic and whether the size of the population tends to zero, remains

finite or grows exponentially. In addition, for the proportional form of the model, in the particular case when infected individuals are assumed to be unable to give birth, we give a complete classification of the equilibria by a novel application of the envelope method. Joint work with Gergely Röst, Shao Yuan Huang, Róbert Vajda and Maoxing Liu.

Observation problems posed for the Klein–Gordon equation

András Szijártó
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We study several observation problems posed for the Klein–Gordon equation. These problems are similar to some question of control theory, where one search the initial state of the equation with the help of the later state of the system. We examine the cases, when we know the partial state of the system in two distinct time instants, and we construct the initial functions with the help of Fourier expansions under some suitable conditions.

On short exact sequences of Kronecker modules

István Szöllősi
Babeş-Bolyai University, Cluj-Napoca, Romania

Let $K : 1 \underset{\beta}{\overset{\alpha}{\leftarrow\!\!\leftarrow}} 2$ be the *Kronecker quiver* and κ an arbitrary field. The path algebra κK over the Kronecker quiver is the *Kronecker algebra*. We will consider the category of finite dimensional right modules over this algebra, the category of *Kronecker module*s (a Krull–Schmidt category). The category of Kronecker modules can be identified with the category of the finite dimensional κ-representations of the Kronecker quiver.

In our talk we deal with the explicit description of the middle terms in various short exact sequences of Kronecker modules, revealing some interesting combinatorial properties. Kronecker modules correspond to *matrix pencil*s in linear algebra and we show how our results can lead to the explicit solution (in some special cases for now) of the *matrix subpencil problem,* an important open problem with applications in control theory and engineering.

Avian-human influenza epidemic model with pulse vaccination

Mónika Szűcs
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Avian influenza has high virulence for birds, and sometimes it spreads from birds to humans. Avian influenza cannot be transmitted among humans yet. However, the influenza virus easily mutates, and it is possible that in the future a mutant variant of avian influenza will be able to spread among humans. As a control measure, for the human population we apply vaccination at discrete times with equal intervals. Our mathematical model interprets the spread of avian influenza and mutant avian influenza with pulse vaccination. We construct the eradication solution of the impulsive system, and obtain conditions on the parameters for the local and global stability of this infection-free solution.

Change detection in INAR(p) processes and continuous branching models

Gyula Pap, **Tamás T. Szabó**
University of Szeged, Hungary

The p-th order INAR(p) process was introduced by Alzaid and Al-Osh in 1987 for $p = 1$, and by Du and Li in 1991 in the case when p is an arbitrary positive integer. It is a nonnegative integer-valued analogue of the AR(p) process and is widely used in time series analysis. The model has several parameters which influence the evolution of the process. If these parameter s change during our observations, then the parameter estimations and forecasts based on the data will be wrong. Therefore it is important to detect changes in these parameters. In an earlier paper we introduced a test process for change detection in the case when the INAR(p) process has a unique stationary distribution. The main result is that the test process converges in distribution to a Brownian bridge. The consistency of the test and some properties of the change-point estimator have also been established. In this talk we aim to extend these some of these results to a special continuous-state branching process, which will be obtained as the solution of a stochastic differential equation. The result is the first step in a project to construct the correct analogues of existing discrete-time change detection methods for continuous-state branching processes.

The number of convex polygons determined by a point set

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Let $\mathcal{P}$ be a set of points in general position in the plane. Erdős and Szekeres proved in 1935 that for every k there exists a number $ESz(k)$ such that when $|\mathcal{P}| \geq ESz(k)$ there is a convex k-gon in $\mathcal{P}$. (That is, there are k points of $\mathcal{P}$ which are the vertex set of a convex k-gon.) For general k, $ESz(k)$ is unknown; Erdős and Szekeres conjectured that it is $2^{k-2}+1$. The conjecture has been validated for $k \leq 6$.

A related question is the following: at least how many convex k-gons are there in a point set with no three collinear points? Even the first non-trivial special case, $k = 4$, is unsolved. Let $\square(\mathcal{P})$ be the number of convex quadrilaterals in the general point set $\mathcal{P}$ and $\square(n) = \min_{|\mathcal{P}|=n} \square(\mathcal{P})$. Then the best known bounds are: $0.37553\binom{n}{4} + O(n^3) \leq \square(n) \leq 0.3807\binom{n}{4} + O(n^3)$.

In the talk, we will show classical results on both topics and we will present a new method for determine $\square(n)$.

On attacking Zolotarev polynomial approximation problems with quantifier elimination and Gröbner bases

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The classical Zolotarev approximation problem is a real polynomial approximation problem. Given a polynomial p of degree n, one asks for the best uniform approximation of p on a closed bounded interval by a polynomial q of degree $n-1$ or less. We show how this minimax problem can be formulated as a real quantifier elimination problem in different ways. Moreover, we explore how additional theoretical knowledge about the optimal approximating polynomial can lead to equational constraints and therefore to more effective solution to the problem. We slightly generalize the problem setting as well. The problem family is also important to benchmark existing quantifier elimination software packages. All the explorations and computations have been done by the author using Wolfram Mathematica.

Joint asymptotic normality of kernel type density estimator for spatial observations

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The Central Limit Theorem is considered for m-dependent random fields. The random field is observed in a sequence of irregular domains. The sequence of domains is increasing and at the same time the locations of the observations become more and more dense in the domains. The Central Limit Theorem is applied to prove asymptotic normality of kernel type density estimators. It turns out that the covariance structure of the limiting normal distribution can be a combination of those of the continuous parametric and the discrete parametric results. Numerical evidence is presented.

References

[1] Fazekas, I. and Chuprunov, A. (2004) A central limit theorem for random fields. *Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis,* **20** (1), 93–104, www.emis.de/journals/AMAPN.

[2] Fazekas, I. and Chuprunov, A. (2006) Asymptotic normality of kernel type density estimators for random fields. *Stat. Inf. Stoch. Proc.* **9**, 161–178.

[3] Karácsony, Zs. and Filzmoser, P. (2010) Asymptotic normality of kernel type regression estimators for random fields. *Journal of Statistical Planning and Inference*, **140**, 872–886.

[4] Lahiri, S.N. (1999) Asymptotic distribution of the empirical spatial cumulative distribution function predictor and prediction bands based on a subsampling method. *Probab. Theory Related Fields*, **114** (1), 55–84.

[5] Lahiri, S.N., Kaiser, M.S., Cressie, N. and Hsu, N.J. (1999) Prediction of spatial cumulative distribution functions using subsampling. *J. Amer. Statist. Assoc.* **94** (445), 86–110.

[6] B.U. Park, T.Y. Kim, T.-S. Park, S.Y. Hwang (2009) Practically Applicable Central Limit Theorem for Spatial Statistics. *Math. Geosci.* **41**, 555–569.

Gröbner bases by matrix triangularization

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Using results by Hermann and Dubé we show how we can compute a Gröbner basis of a set F of polynomials over a field by building, in a first step, one big matrix of shifts of the polynomials in F and, then, triangularizing this matrix. We give a bound for the size of the matrix that does not depend on the term ordering. It depends on the highest degree of the polynomials in F, the number of variables and the number of polynomials in F. By the same approach, we also give a criterion for how to determine solvability of the system $F = 0$ with a bound for the matrix which is smaller than the bound used for Gröbner bases computation.

A characterization of the completeness of random normed spaces

Claudia Zaharia
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By considering several concepts of boundedness in random normed spaces, we find suitable conditions under which the completeness of a RN-space is a consequence of the probabilistic stability for the additive Cauchy functional equation.

References

[1] B. Lafuerza Guillén, J. A. Rodríguez Lallena, C. Sempi - A study of boundedness in probabilistic normed spaces, J. Math. Anal. Appl. 232 (1999), 183–196

[2] D. Miheţ, V. Radu - On the stability of the additive Cauchy functional equation in random normed spaces, J. Math. Anal. Appl. 343 (2008), 567-572

[3] J. Schwaiger - Remark 12, in: Report on the 25th Internat. Symp. on Functional Equations, Aequationes Math. 35 (1988), 120–121

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