Hardy's paradox and the entanglement-like structure of forward scattered waves

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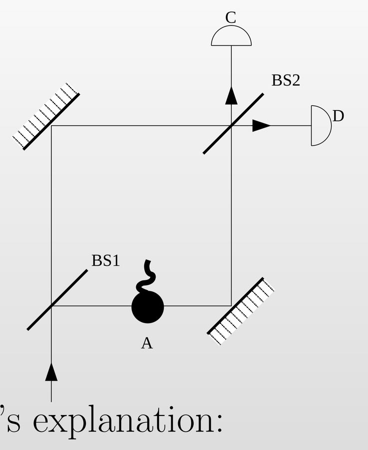
Introduction

We analyze Hardy's paradox from the point of view of scattering theory. This approach has been useful for the understanding of interaction-free measurement, which is a similar setup. We calculate the forward-scattered waves generated by the beam splitters, which are replaceable in the gedanken experiment. These two-mode waves appear to have an entanglement-like structure. In addition we also analyze a photon interferometric scenario which is directly similar to the one in Hardy's setup. The main difference between the two cases is that the annihilation of the particle-antiparticle pair which can be seen in Hardy's original setup is replaced by the interference of the two photons on a beam splitter. We discuss its relation to Hardy's paradox and as we did in the original setup we also calculate the forward scattered waves of the output beam splitters for this setup too and analyze their entanglement-like structure.

Interaction-free measurement and Forward scattering



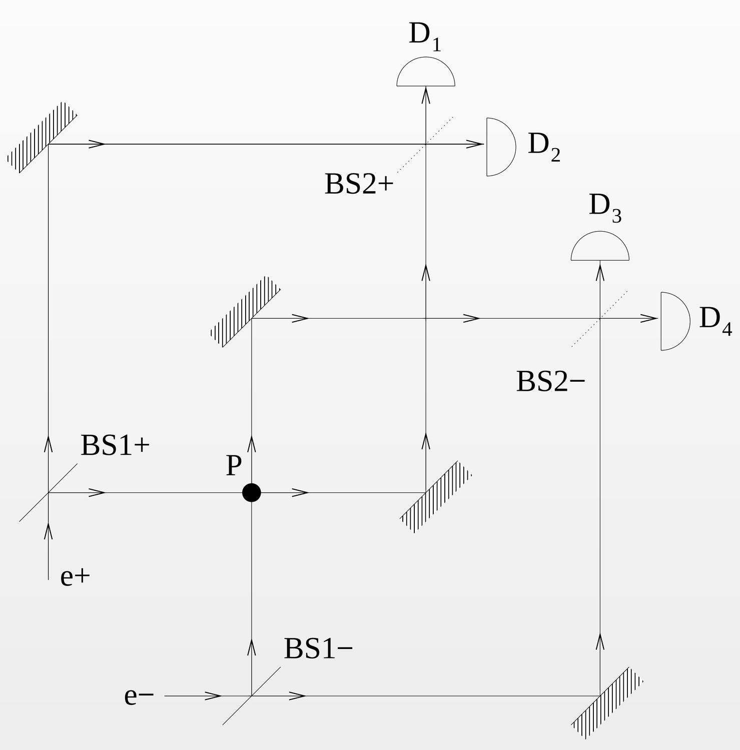
IFM:
- detector D is idle if there is no bomb,
- it can fire if there is a bomb,
- though the bomb does not explode in that case.

Geszti's explanation:

Forward Scattered Amplitude =
Amplitude with absorber - Amplitude without absorber

"The absorber extinguishes the wave field behind it by adding its negative: the so-called forward-scattered wave to it. It is just this added wave that reaches dark-port detector D in the presence of the absorber" [1]

Hardy's paradox



Hardy's gedanken experiment [2] aims at the testing of local realism without inequalities:
- The presence or absence of the beam splitters BS2± plays the role of the local setting of whether BS2+ is in its place or not.
- If none of the beam splitters is in its place, detectors C+ and C- cannot fire in coincidence.
- Local realism would require that the detection probabilities of the positron (electron) should not depend on the presence or absence of the beam splitter for the electron (positron), respectively.
- In $\frac{1}{16}$ th of the cases, the independence assumption implied by local realism leads to coincident detection at detectors C + and C - for the same situation,
- which is in contradiction with quantum mechanics.

Hardy's paradox has been a subject of experimental realization recently [3, 4].

Notation

Basis:
- $|p_0\rangle\ |e_0\rangle\ |\gamma_1\rangle = |\gamma\rangle$: there is no positron or electron in the arms because they have annihilated each other, there is a photon emitted,
- $|d_2\rangle\ |d_3\rangle\ |\gamma_0\rangle = |d_2d_3\rangle$: both the positron and the electron are in mode "d", the photon is absent,
- $|d_2\rangle\ |d_4\rangle\ |\gamma_0\rangle = |d_2d_4\rangle$: the positron is in mode "d", the electron is in mode "c", the photon is absent,
- $|d_1\rangle\ |d_3\rangle\ |\gamma_0\rangle = |d_1d_3\rangle$: the positron is in mode "c", the electron is in mode "d", the photon is absent,
- $|d_1\rangle\ |d_4\rangle\ |\gamma_0\rangle = |d_1d_4\rangle$: the positron is in mode "c", the electron is in mode "c", the photon is absent.

Arrangements:
1. BS2+ and BS2- are removed
2. BS2+ is in place, BS2- is removed
3. BS2+ is removed, BS2- is in place
4. BS2+ and BS2- are in place

The respective outgoing states are:

$$|\text{out1}\rangle = \frac{1}{2}(-|\gamma\rangle + |d_2d_3\rangle + i|d_2d_4\rangle + i|d_1d_3\rangle) \tag{1}$$

$$|\text{out2}\rangle = \frac{\sqrt{2}}{4}(-\sqrt{2}|\gamma\rangle + i|d_2d_4\rangle + 2i|d_1d_3\rangle - |d_1d_4\rangle), \tag{2}$$

$$|\text{out3}\rangle = \frac{\sqrt{2}}{4}(-\sqrt{2}|\gamma\rangle + 2i|d_2d_4\rangle + i|d_1d_3\rangle - |d_1d_4\rangle), \tag{3}$$

$$|\text{out4}\rangle = \frac{1}{4}(-2|\gamma\rangle - |d_2d_3\rangle + i|d_2d_4\rangle + i|d_1d_3\rangle - 3|d_1d_4\rangle), \tag{4}$$

(the incoming state is always $|d_2e_d\rangle$).

Forward scattering

- In Hardy's setup, the beam splitters are the replacable elements,
- hence, we calculate their forward scattered waves.

Three cases:

|fsw+⟩: BS2+ present, BS2- absent,

|fsw-⟩: BS2+ absent BS2- present,

|fsw ±⟩: both are present.

FSW-s:

$$|\text{fsw+}\rangle = |\text{out2}\rangle - |\text{out1}\rangle = \tfrac{\sqrt{2}}{4}(-\sqrt{2}|d_2d_3\rangle + i(1-\sqrt{2})|d_2d_4\rangle + i(2-\sqrt{2})|d_1d_3\rangle - |d_1d_4\rangle) \tag{5}$$

$$|\text{fsw-}\rangle = |\text{out3}\rangle - |\text{out1}\rangle = \tfrac{\sqrt{2}}{4}(-\sqrt{2}|d_2d_3\rangle + i(2-\sqrt{2})|d_2d_4\rangle + i(1-\sqrt{2})|d_1d_3\rangle - |d_1d_4\rangle) \tag{6}$$

$$|\text{fsw}\pm\rangle = |\text{out4}\rangle - |\text{out1}\rangle = -\tfrac{1}{4}(3|d_2d_3\rangle + i|d_2d_4\rangle + i|d_1d_3\rangle + 3|d_1d_4\rangle) \tag{7}$$

- These do not look product states.
- Two effective qubits at each side, the basis is

$$\{|p_c\rangle, |p_d\rangle\} \text{ and} \{|e_c\rangle, |e_d\rangle\}, \tag{8}$$

- concurrence may be calculated using Wootters's formula.

Let us denote the concurrence for the case when BS2+ is in place and BS2- is removed by C_+, and the other two concurrences by C_- and $C_\pm$. We find that:

$$C_+ = C_- = \frac{2}{3} \qquad C_\pm = 1 \tag{9}$$

These results unambiguously indicate that the normalized forward scattered wave, if it were to describe a physical system, that would be an entangled one. Moreover in the casewhen both beam splitters are present, this entanglement is maximal. It is also interesting to note that the value of concurrence of $\frac{2}{3}$ appearing in the two other cases rather special, as it is pointed out in Ref. [5]: it is the concurrence of assistance [6] of a density matrix of rank 3 in the two-qubit Hilbert-space with uniform eigenvalues.

Koniorczyk al: *Hardy's paradox and the entanglementlike structure of forward-scattered waves,* Phys. Rev. A **84**, 044102 (2011)

An optical setup

One may consider an optical setup similar to that of Hardy:
- Replace particles by photons
- Replace absorbtion by a beam splitter

In this case

$$C_1 = 1, \qquad C_2 = C_3 = 1/2. \tag{10}$$

P. Adam et al., *Forward-scattered wave analysis of an optical Hardy-like setup* Physica Scripta **t147**, 014001 (2012)

- Does it violate local realism at all?
- What is the exact condition of violation, if any, in terms of FSW-s?

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