

# Chaos Patterns in a 3 Degree Of Freedom Control with Robust Fixed Point Transformation

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**Abstract**—The controllers designed by Lyapunov’s 2nd method normally have global stability but do not concentrate on the details of the primary intent of the designer: the details of the tracking error relaxation. They have a huge number of arbitrary adaptive control parameters that –from the engineering point of view– are hard to design for prescribed detailed behavior of the controlled system. In the past few years a new method that concentrates on the design intent, easy to produce and has only a few adaptive control parameters was invented: the *Robust Fixed Point Transformation (RFPT)*. According to ample simulations it seems to be a good choice, but it has only local stability yet. Though its present form can be satisfactory for solving most of the cases, sometimes ancillary methods are needed for maintaining or restoring its convergence. It was recently discovered for one and two degree of freedom systems that when the controller quits the region of stability it still guarantees good tracking at the price of huge chattering that also was reduced and stopped. In this paper it will be shown that in the adaptive control of the 3Degree Of Freedom (DOF) system similar phenomena happen and the controller can be stabilized by similar methods.

stability have been dropped. In the great majority of simulation investigations it was found that the use of a rough and very approximate initial model is satisfactory for this purpose, however, sometimes the so designed controller quit the region of convergence. At first ancillary methods were developed for tuning only one of the three adaptive parameters of this controller to keep it within the region of convergence (e.g. [13], [14], [15], and later a far simpler approach in [16], [17]). Later it was observed in [18] in the case of the adaptive control of a van der Pol oscillator that leaving the region of convergence did not result in a “catastrophe”: at the cost of large chattering the trajectory tracking remained relatively precise, furthermore, simple possibility was found to reduce and eliminate it. The same observation was done for a 2 DOF system in [19]. In the present paper the same phenomenon, stabilizing and chattering elimination methods are investigated for a 3 DOF system, a cart+beam+hampers system.

## I. INTRODUCTION

Lyapunov in his PhD thesis in 1892 [1] investigated the stability of the motion of dynamical systems. His main finding was that though in the great majority of the classical problems the solution of the equation of motion cannot be constructed in closed analytical form, therefore the details of its behavior remain unknown, its stability can be determined by considering the behavior of a Lyapunov function. Since similar problems we have regarding the behavior of the controlled nonlinear systems, Lyapunov’s method was and is widely used in the design of nonlinear controllers from the sixties when his work was translated to English [2]. The stability of operation of the controlled system can be guaranteed in this manner (e.g. [3], [4], [5], [6], [7], [8], [9], [10]). Though the basic principle of the operation of Lyapunov’s technique is easy to be understood, its application in the practice is a complicated task that requires very good mathematical skills and the process of the controller design cannot be formulated as a simple algorithm.

An alternative approach was suggested and later investigated in [11], [12] in which the primary design intents were kept in mind at first, and the need for global (global asymptotic)

## II. THE RFPT METHOD

RFPT an adaptive control method. From the engineering standpoint it concentrates on the designer’s primary intent. Its basic equation is (1) where  $B_{n+1}$  is given in (2),  $e_{n+1}$ , which is the unit vector of the response error for the next control cycle defined in (3),  $h_{n+1}$  is the response error specified in (4). Symbol  $r^d$  denotes the “desired system’s response” (5) while the “measured response” is denoted by  $f$ .  $r^d$  can be calculated on purely kinematical basis,  $f$  is obtained as a consequence of using a “rough system model” in calculating the control forces that eventually result in the observable response  $f$ . This approach can also be regarded as a “gray box technique” since it utilizes some *partial, a priori* information on the system under control.

It worths noting that for the function  $\sigma$  in (7) any type of other sigmoid function can be used that satisfies the following restrictions:

- $\sigma$  is between  $\pm 1$
- $\sigma(0) = 0$
- $\frac{d\sigma}{dx}|_{x=0} = 1$

The *control parameters* are:

- $B_c$

- $A_c$
- $K_c$

The following equations came from [11].

$$r_{n+1} := (1 + B_{n+1})r_n + K_c e_{n+1} \quad (1)$$

$$B_{n+1} = B_c \sigma(A_c \|h_{n+1}\|) \quad (2)$$

$$e_{n+1} = \frac{h_{n+1}}{\|h_{n+1}\|} \quad (3)$$

$$h_{n+1} = f(r_n) - r_{n+1}^d \quad (4)$$

$$r^d = \begin{bmatrix} \ddot{X}^d \\ \ddot{Y}^d \end{bmatrix} \quad (5)$$

$$f = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} \quad (6)$$

$$\sigma = \frac{x}{1 + |x|} \quad (7)$$

In the present approach parameter  $A_c$  was adaptively reduced if strong chattering occurred, furthermore, the chattering was reduced by the use of the modified version of the above equations as in [19] (8)

$$\begin{aligned} \vec{h} &:= \vec{f}(\vec{r}_n) - \vec{r}^d, \quad \vec{e} := \vec{h}/\|\vec{h}\|, \\ \vec{B} &= B\sigma(A\|\vec{h}\|) \\ \vec{r}_{n+1} &= (1 + \vec{B})\vec{r}_n + K_s \sigma\left(\frac{\vec{B}K}{K_s}\right)\vec{e}. \end{aligned} \quad (8)$$

in which repeated application of the sigmoid function with the parameter  $K_s$  can reduce chattering.

### III. A 3 DOF SYSTEM

The motion equations for the 3DOF system Fig.1 is (9) . The *generalized coordinates* of the 3 Dimension of freedom system are:

- $q_1$  (rad): rotation angle of the beam,
- $q_2$  (rad): rotation angle of the hamper at the top of the beam,
- $q_3$  (m): linear displacement of the cart's body.

The *dynamic parameters* are:

- $m$  is the mass of the body, in top of the beam (kg)
- $M$  is the mass of the body of the "car" (kg)
- $L$  is the length of the beam (m)
- $\Theta$  is the moment of inertia of the hamper with respect to its own mass center point (kg · m<sup>2</sup>)

The *generalized forces* to be exerted by the controller are:

- $Q_1$  (N · m): torque at axle 1;
- $Q_2$  (N · m): torque at axle 2;
- $Q_3$  (N): force pushing the cart in the lateral direction,

furthermore  $g$  denotes the gravitational acceleration. This is just a rough initial model of the the motion since it is assumed that the hamper's mass center point is located on its axle.

For the RFPT method it is satisfactory to have some rough approximation of the dynamic parameters. Whenever the RFPT is applied for designing a "Model Reference Adaptive

Controller (MRAC)", also significant difference can be between the actual system's parameters and that of the Reference Model to be imitated by the controlled system [20]. In the simulations carried out the MRAC solution was investigated with *actual system parameters* as  $M = 30 \text{ kg}$   $m = 10 \text{ kg}$   $L = 2 \text{ m}$  ,  $\Theta = 20 \text{ kg} \cdot \text{m}^2$  ,  $g = 10 \text{ m/s}^2$  , while *the reference model* had the dynamic parameters as  $\hat{M} = 60 \text{ kg}$   $\hat{m} = 20 \text{ kg}$   $\hat{L} = 2.5 \text{ m}$  (also having effects on the dynamic behavior),  $\hat{\Theta} = 50 \text{ kg} \cdot \text{m}^2$  , and  $\hat{g} = 8 \text{ m/s}^2$  . In the simulations it was assumed that the system's response was observable as a noisy signal. (In contrast to the other methods using various model-based estimators as Kalman filters, no any special assumption was necessary for the statistical nature of this observation noise, apart from the zero mean.) The simulation results are analyzed in Section IV.

$$\begin{aligned} &\begin{bmatrix} (mL^2 + \Theta) & \Theta & mL \cos(q_1) \\ \Theta & \Theta & 0 \\ mL \cos(q_1) & 0 & (m + M) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ &+ \begin{bmatrix} -mLg \sin(q_1) \\ 0 \\ -mL \sin(q_1) \dot{q}_1^2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \end{aligned} \quad (9)$$

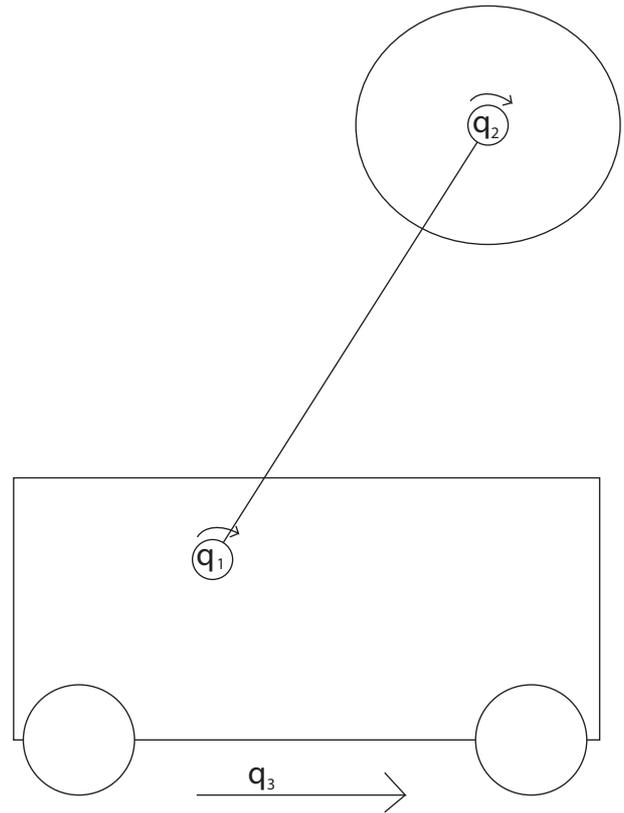


Fig. 1. Sketch of the model used for the computation

### IV. SIMULATION RESULTS AND CHAOS PATTERNS

In the simulations the following control parameter settings were used:  $K_s = 600$  ,  $K_c = 7000$  ,  $B_c = -1$  , and  $A_c$

was adaptively tuned in the case of necessity. Figures 2 and 3 display the trajectories and the phase trajectories of the controlled system revealing that the tracking in both spaces remained smooth and precise. Figure 4 reveals that besides the considerable parameter differences between the actual and the reference models significant observation disturbances were assumed. According to Figs. 5, 6, and 7 it can be stated that quite significant adaptive deformation was necessary for the imitation of the reference model but the all the occurring accelerations are very close to each other that testifies the success of the adaptive controller. Figure 7 reveals the details of the adaptation mechanism showing that the *reference* and the *recalculated* values are in each other's close vicinity, i.e. the "illusion" to be created by the MRAC controller was successful, too. Figure 6 displays an excerpt of Fig. 7 that clearly shows that the *reference* and the *recalculated* values (i.e. the cyan–yellow, the red–dark blue, and the magenta–light blue pairs) are closely in each other's vicinity. The tracking errors are displayed in Fig. 8. Figures 9–11 reveal the formation of the very much curbed chaos pattern in the exerted control forces.

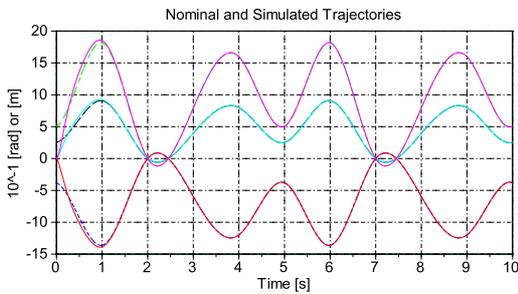


Fig. 2. The nominal ( $q_1$ : black,  $q_2$ : blue,  $q_3$ : green lines) and the simulated trajectories ( $q_1$ : cyan,  $q_2$ : red,  $q_3$ : magenta lines)

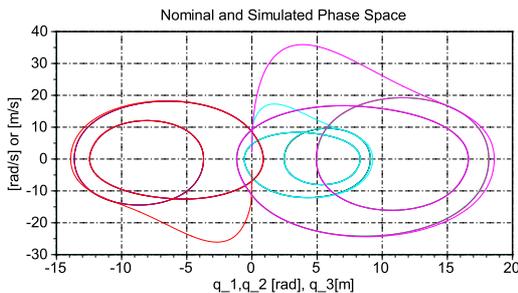


Fig. 3. The nominal ( $q_1$ : black,  $q_2$ : blue,  $q_3$ : green lines) and simulated ( $q_1$ : cyan,  $q_2$ : red,  $q_3$ : magenta lines) phase trajectories

## V. CONCLUSION

It was shown for a 3 DOF system with applied observation disturbances that the adaptive MRAC controller designed on the basis of the *Robust Fixed Point Transformation*, *chattering reduction* (implemented by a second sigmoidal function with adaptive control parameter  $K_s$ ) and *chattering elimination* (realized by tuning the adaptive control parameter  $A_c$ ) works

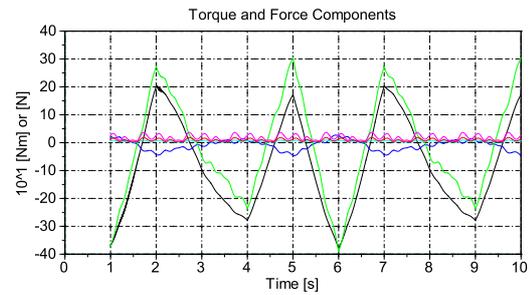


Fig. 4. The exerted control torques ( $Q_1$ : black,  $Q_2$ : blue,  $Q_3$ : green lines), and the noisy disturbance forces ( $Q_1$ : cyan,  $Q_2$ : red,  $Q_3$ : magenta lines)

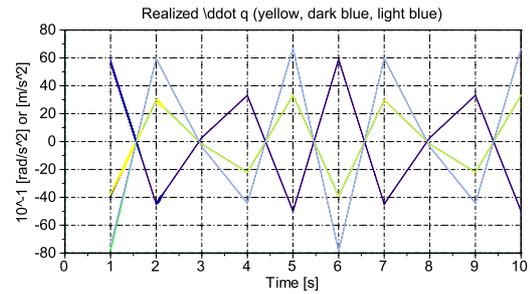


Fig. 5. The second time-derivatives of the generalized coordinates (realized:  $\ddot{q}_1$ : yellow,  $\ddot{q}_2$ : dark blue,  $\ddot{q}_3$ : light blue, kinematically desired:  $\ddot{q}_1$ : cyan,  $\ddot{q}_2$ : red,  $\ddot{q}_3$ : magenta, nominal:  $\ddot{q}_1$ : black,  $\ddot{q}_2$ : blue,  $\ddot{q}_3$ : green lines)

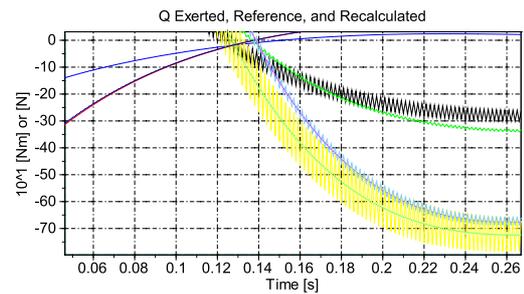


Fig. 6. The exerted ( $Q_1$ : black,  $Q_2$ : blue,  $Q_3$ : green lines), the recalculated ( $Q_1$ : yellow,  $Q_2$ : dark blue,  $Q_3$ : light blue lines), and the reference ( $Q_1$ : cyan,  $Q_2$ : red,  $Q_3$ : magenta lines) (zoomed excerpt)

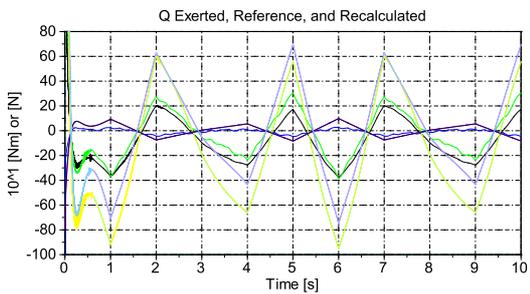


Fig. 7. The exerted ( $Q_1$ : black,  $Q_2$ : blue,  $Q_3$ : green lines), the recalculated ( $Q_1$ : yellow,  $Q_2$ : dark blue,  $Q_3$ : light blue lines), and the reference ( $Q_1$ : cyan,  $Q_2$ : red,  $Q_3$ : magenta lines)

well. The trajectories and phase trajectories are precisely tracked, though in the beginning some slight chattering can be

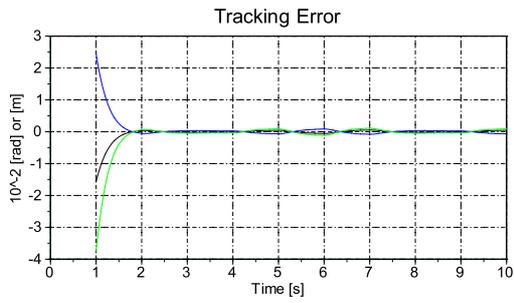


Fig. 8. The trajectory tracking error ( $q_1$ : black,  $q_2$ : blue,  $q_3$ : green lines)

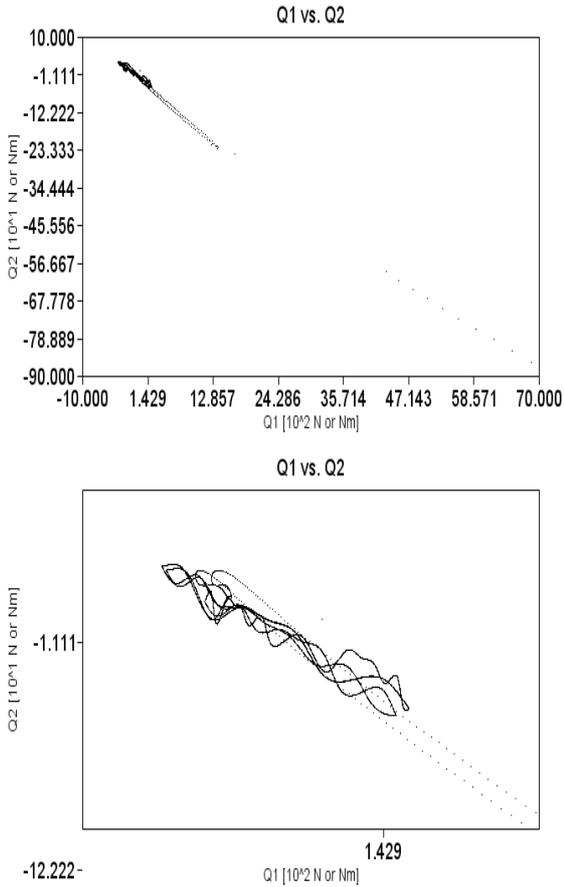


Fig. 9. The projection of the generalized forces on the  $Q_1 - Q_2$  plane with zoomed excerpts

observed. The controller successfully pulled the system into a stable range of operation. What exactly happened can be summarized as follows:

- The system started with a chaotic control signal with chattering that *ab ovo* was reduced by  $K_s$ , on this reason even the first segments of the figures show “decent behavior”, i.e. they reveal very limited fluctuation in the control signal.
- The reduction of the chaos patterns can well be traced on the  $Q_i$  vs.  $Q_j$  diagrams of the control forces that show a very sparse initial set of points that soon are begin

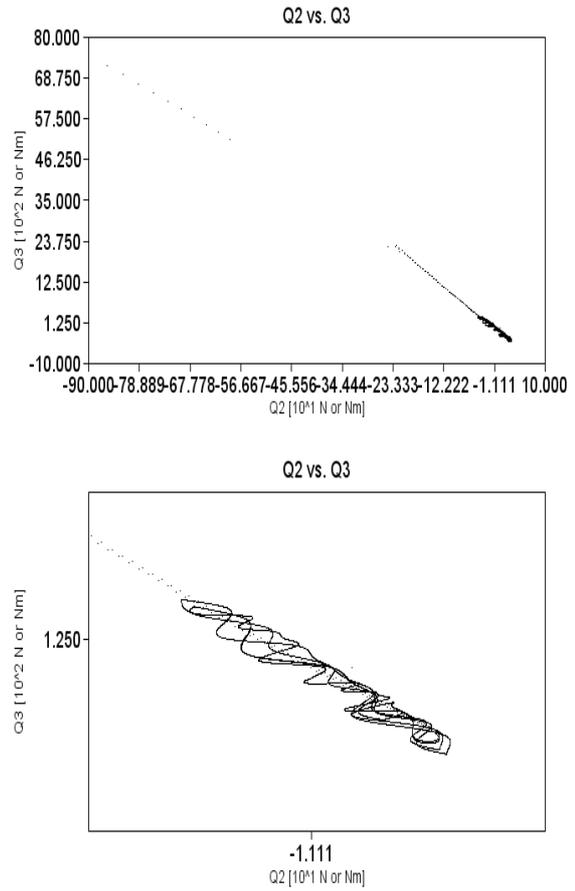


Fig. 10. The projection of the generalized forces on the  $Q_2 - Q_3$  plane with zoomed excerpts

to concentrate along some strange attractors that reveal definite structure.

- Eventually the RFPT pulled the system into stable convergence, and well stabilized the trajectory tracking and illusion generating property of the MRAC controller developed for an actual 3DOF system and a significantly different reference system.

In the future research it would be expedient to investigate the margin of the stability of the controlled system. It is also a very interesting question, how to expand the circle of usability of the RFPT-based adaptive controllers. From this point of view the variation of the properties of the *response function* depending on the actual control situation may be of great interest.

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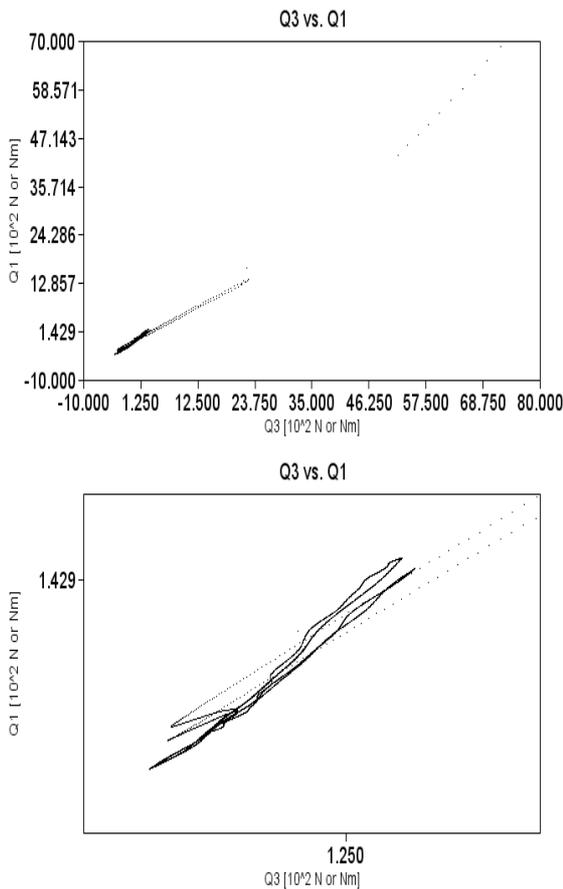


Fig. 11. The projection of the generalized forces on the  $Q_3 - Q_1$  plane with zoomed excerpts

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