Geometric model

- Each individual layer has a low energy state at GOC configurations.
- With several layers, the model makes it possible to segment touching circles.
- As expected, \( \kappa = 0 \) yields overlapping objects, while \( \kappa > 0 \) prevents overlaps.
- If \( \kappa \) is too high, then either an empty configuration or unstable circles are produced.

### Introduction

**Goal:**
- Build a suitable model for the segmentation of touching or overlapping near-circular shapes.

**Problems:**
- Segmented regions are subsets of the image, not the real objects: impossible to express overlaps.
- Different degrees of overlap prevent using uniform descriptions of shapes.
- ‘Gas of circles’ phase field model has a repulsive energy between nearby shapes.

### Phase field ‘gas of circles’ model

- A phase field model represents a subset \( \mathcal{B} \subseteq \mathbb{R}^2 \) by a function \( \phi : \mathbb{R}^2 \rightarrow [0,1] \) on the image domain, and a threshold \( t \).
- The energy of the phase field model is:
  \[
  E(\phi) = \frac{1}{2} \int_{\mathbb{R}^2} \nabla \phi \cdot \nabla \phi + \frac{\kappa}{4} \int_{\mathbb{R}^2} (\phi - \frac{1}{2})^4 \, dx 
  \]

- The single layer phase field GOC model assigns low energy to subsets of the image domain consisting of a number of near-circular regions of approximately a given radius separated by distances at least comparable to their size [1].

### Layered representation

- We extend the phase field model: the new model contains multiple instances of the phase field GOC model each being known as a layer:
  \[
  \phi = \{ \phi \}_{i=1}^{N}, \quad \int_{\mathbb{R}^2} \phi \, dx = 1 
  \]
- The total energy of the multi-layer phase field \( \mathcal{G} \) is defined as the sum of the energies of the individual layers, plus a pairwise interlayer interaction energy penalizing overlap between foreground regions:
  \[
  E(\mathcal{G}) = \sum_{i=1}^{N} E(\phi_i) + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \int_{\mathbb{R}^2} \phi_i(x) \phi_j(x) \, dx 
  \]
- \( \kappa \) is a new parameter controlling the strength of the overlap penalty. This is the only interaction between layers.
- The long-range interactions act intra-layer but not inter-layer. Thus repulsively interacting regions can escape to separate layers, thereby eliminating the repulsive interaction between regions. Note that ‘background’ points do not generate overlap penalty.

### Geometric model

- The phase field energy is minimized by gradient descent.
- The initialization of the phase field may have a strong influence on the final result.
- Starting from a random initialization there are several parameter pairs for which one can achieve a correct segmentation.
- In real applications, however, we can use an application specific initialization.
  - e.g. in our biological experiments, we used a simple adaptive thresholding and connected component detection, plus random assignment of different layers to nearby initial region seeds.

### Data term

- For segmentation of circular shapes we can combine the GOC model as prior with a data likelihood based on intensities and image gradient:
  \[
  E(\phi, t) = \frac{1}{2} \int_{\mathbb{R}^2} \nabla \phi \cdot \nabla I + \frac{1}{\sigma_x^2} \int_{\mathbb{R}^2} (I - \mu_x) \, dx + \frac{1}{\sigma_y^2} \int_{\mathbb{R}^2} (I - \mu_y) \, dy 
  \]

### Biological application

- In microbiology, one of the main image processing problems is to segment multiple objects, e.g. lipid droplets, cells, or other sub-cellular components, that are often near-circular with many overlaps.
- The images made by light microscopy techniques are noisy, blurred and have low contrast.
- The results show that the proposed model can handle and solve these problems.

### References


### Geometric kernel

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