A Multi-Layered Phase Field Model for Extracting Near-Circular Shapes

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Introduction

Goal:

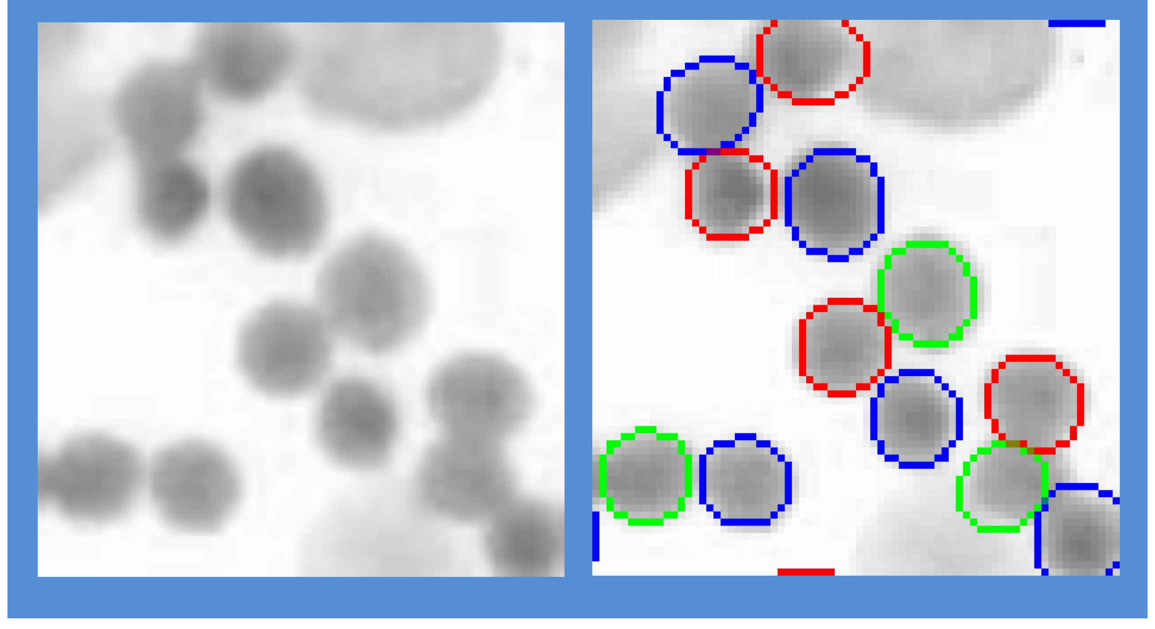
❖ Build a suitable model for the segmentation of touching or overlapping near-circular shapes

Problems:

❖ Segmented regions are subsets of the image, not the real objects: impossible to express overlaps

❖ Different degrees of overlap prevent using uniform descriptions of shapes

❖ 'Gas of circles' phase field model has a repulsive energy between nearby shapes



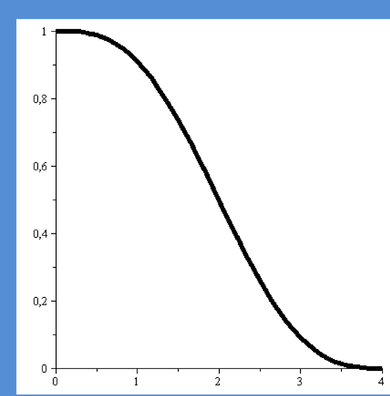
Phase field 'gas of circles' model

❖ A phase field model represents a subset $R \subset \mathcal{D}$ by a function $\phi : \mathcal{D} \to \mathbb{R}$ on the image domain , and a threshold *t*: $R = \{x \in \mathcal{D} : \phi(x) \geq t\}$.

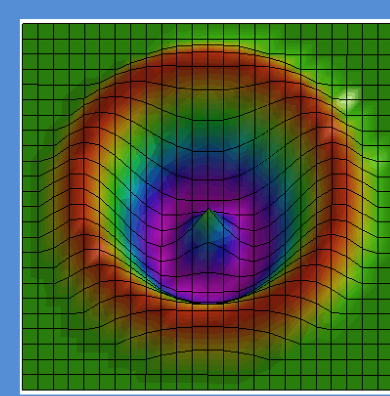
❖ The energy of the phase field model is:

$$E(\phi) = \int_{\mathcal{D}} \left\{ \frac{D_f}{2} |\nabla \phi|^2 + \alpha_f \left(\phi - \frac{\phi^3}{3} \right) + \lambda_f \left(\frac{\phi^4}{4} - \frac{\phi^2}{2} \right) \right\} d^2x - \frac{\beta_f}{2} \int_{D^2} \nabla \phi \, G(\| x - x' \|) \, \nabla \phi' \, d^2x \, d^2x'$$

❖ The single layer phase field GOC model assigns low energy to subsets of the image domain consisting of a number of near-circular regions of approximately a given radius separated by distances at least comparable to their size [1].



Interaction function $G(z)$



Geometric kernel $\nabla^2 G(z)$

Layered representation

❖ We extend the phase field model: the new model contains multiple instances of the phase field GOC model each being known as a layer:

$$\phi = \{\phi_i\}_{i \in [1..l]} : [1..l] \times \mathcal{D} \to \mathbb{R}$$

❖ The total energy of the multi-layered phase field ϕ is defined as the sum of the energies of the individual layers, plus a pairwise interlayer interaction energy penalizing overlap between foreground regions:

$$\widetilde{E}(\phi) = \sum_{i=1}^{l} E_i(\phi_i) + \frac{\kappa}{4} \sum_{i \neq j} \int_D (1 + \phi_i)(1 + \phi_j)$$

❖ κ is a new parameter controlling the strength of the overlap penalty. This is the only interaction between layers.

❖ The long-range interactions act intra-layer but not inter-layer. Thus repulsively interacting regions can 'escape' to separate layers, thereby eliminating the repulsive interaction between regions. Note that 'background' points do not generate overlap penalty.

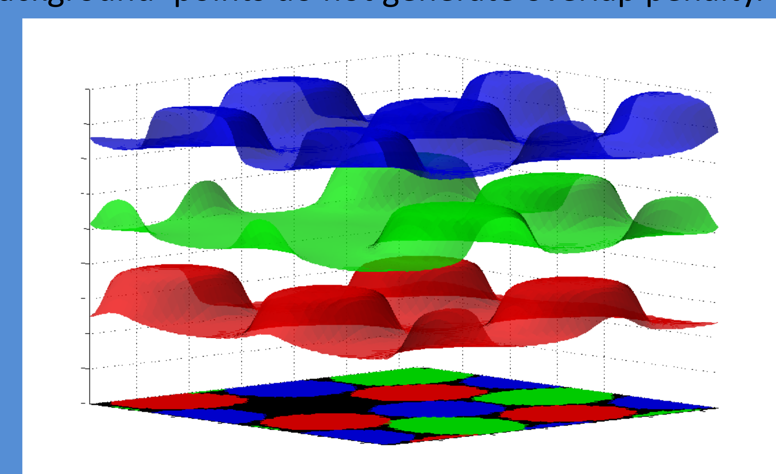


Illustration of multi-layered phase field GOC model and the corresponding segmentation

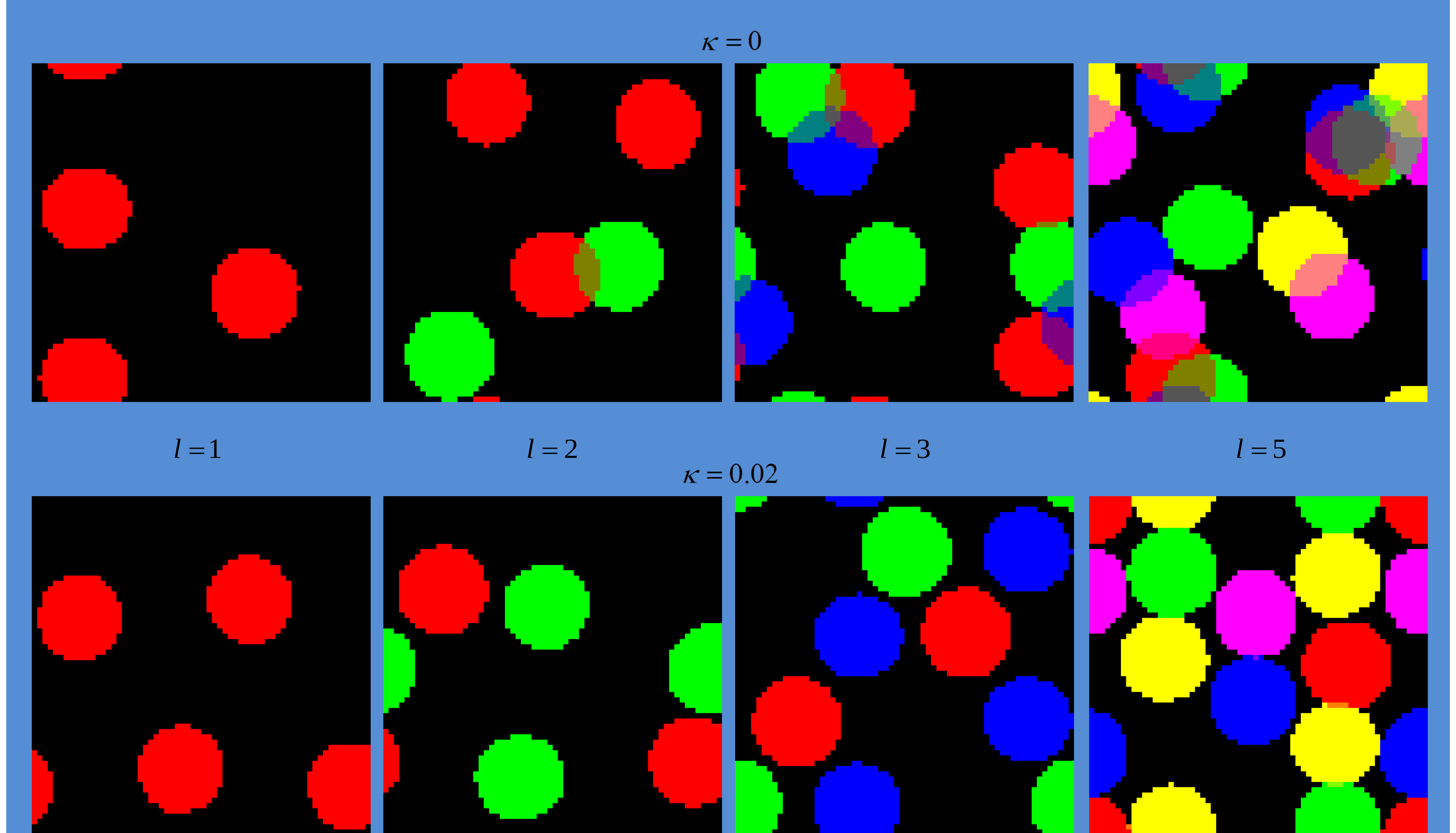
Geometric model

❖ Each individual layer has a low energy state at GOC configurations.

❖ With several layers, the model makes it possible to segment touching circles.

❖ As expected, $\kappa = 0$ yields overlapping objects, while $\kappa > 0$ prevents overlaps.

❖ If κ is too high, then either an empty configuration or unstable circles are produced.



Low energy state configurations of the proposed model

Data term

❖ For segmentation of circular shapes we can combine the GOC model as prior with a data likelihood based on intensities and image gradient :

$$E_i(I, R) = \int_{\Omega} d^2x \left\{ \gamma_1 \partial I \cdot \partial \phi_i + \gamma_2 \left[\frac{(I - \mu_{in})}{2\sigma_{in}^2} \frac{(\phi_i + 1)}{2} + \frac{(I - \mu_{out})}{2\sigma_{out}^2} \frac{(\phi_i - 1)}{2} \right] \right\}$$

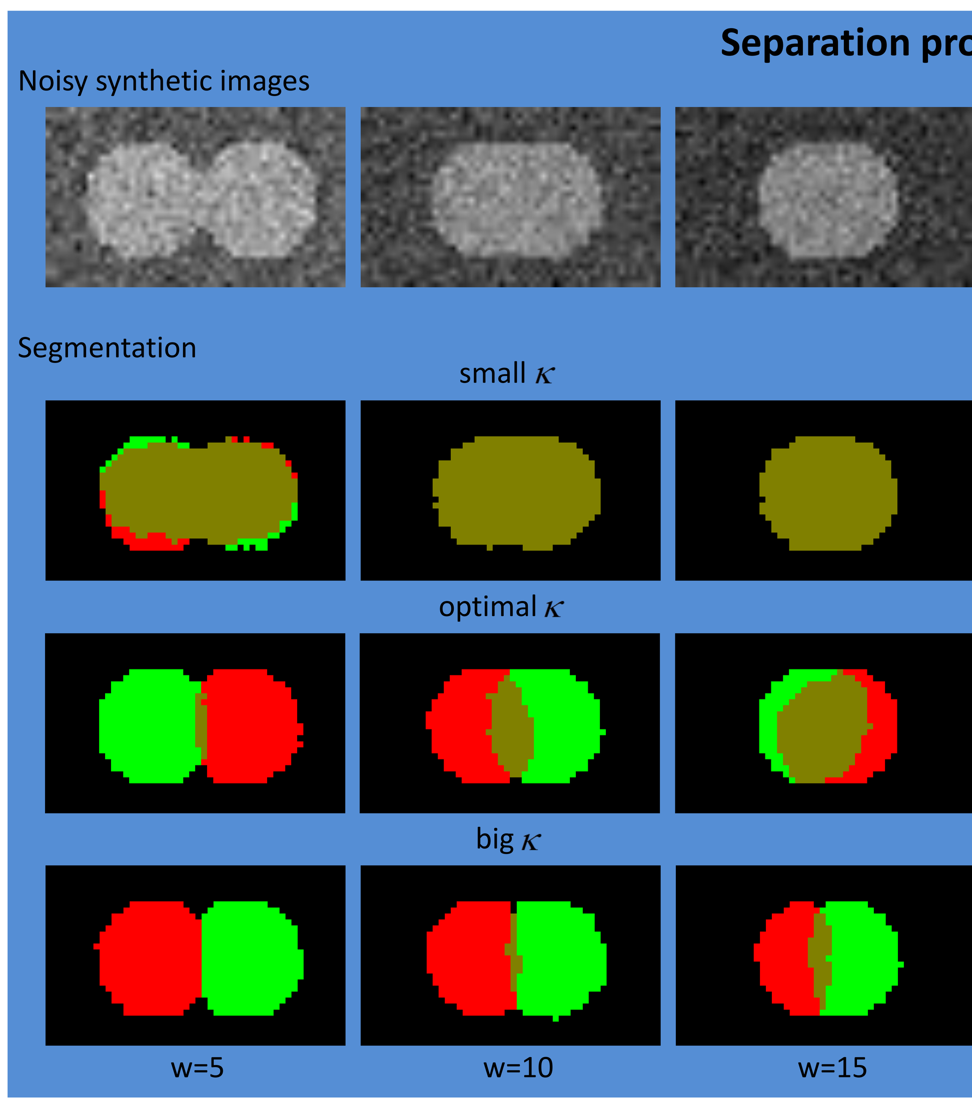
❖ $\mu_{in,out}$ and $\sigma_{in,out}$ are the parameters of Gaussian distributions modelling the intensities

❖ $\gamma_{1,2}$ are positive weights

❖ I is the image data

❖ ∂I is the image gradient

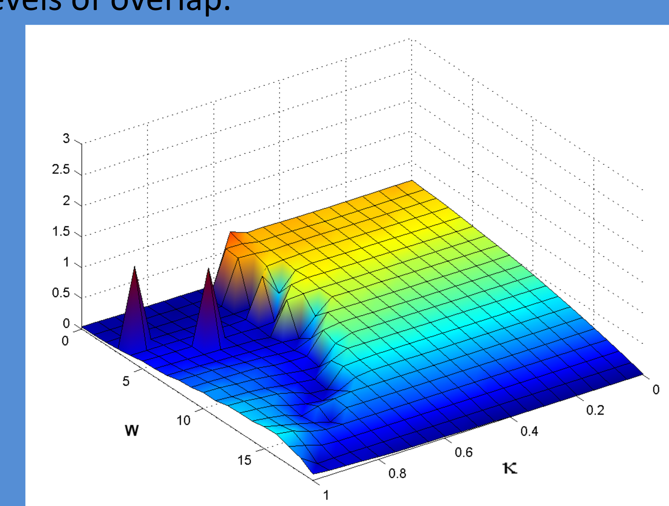
Separation properties



❖ The properties of the model in the case of two circles with different levels of overlap and different strength of penalty.

❖ There is a competition between the data term and the geometric energy. If the overlap penalty is too weak or too strong, unstable circles are produced, but an optimal segmentation of the two circles is achievable.

❖ The optimal κ value is almost the same for all levels of overlap.



Segmentation error as function of overlap w and penalty κ

Initialization

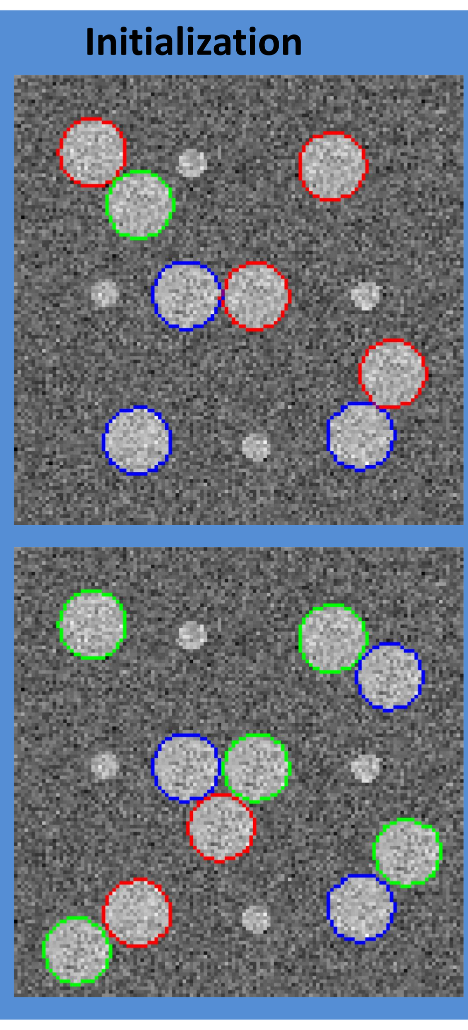
❖ The phase field energy is minimized by gradient descent.

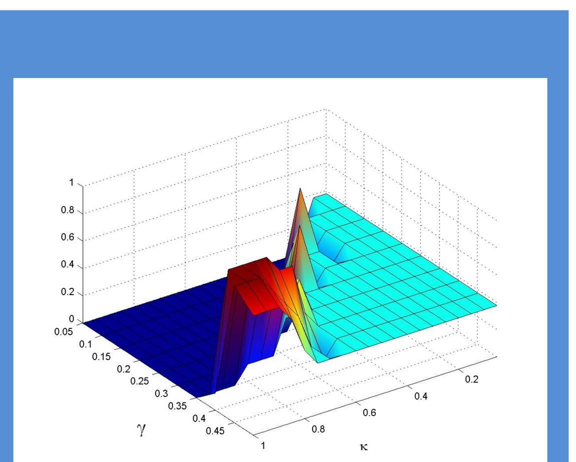
❖ The initialization of the phase field may have a strong influence on the final result.

❖ Starting from a random initialization there are several parameter pairs for which one can achieve a correct segmentation .

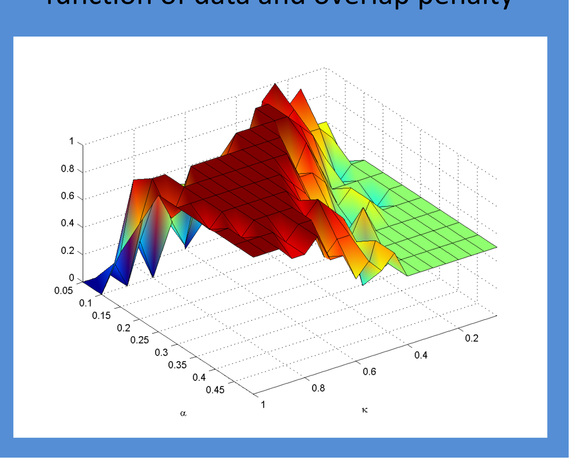
❖ In real applications, however, we can use an application specific initialization.

- *e.g.* in our biological experiments, we used a simple adaptive thresholding and connected component detection, plus random assignment of different layers to nearby initial region seeds.





Ratio of successfully detected circles as a function of data and overlap penalty

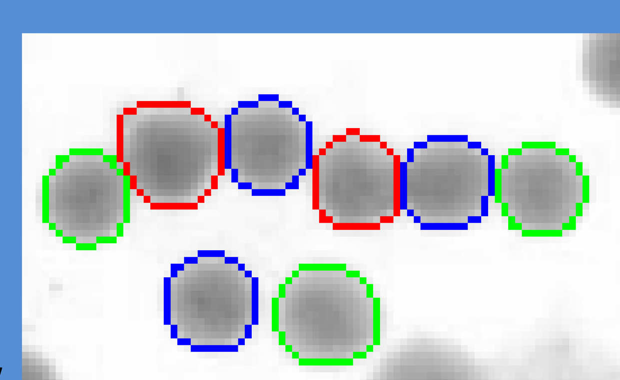


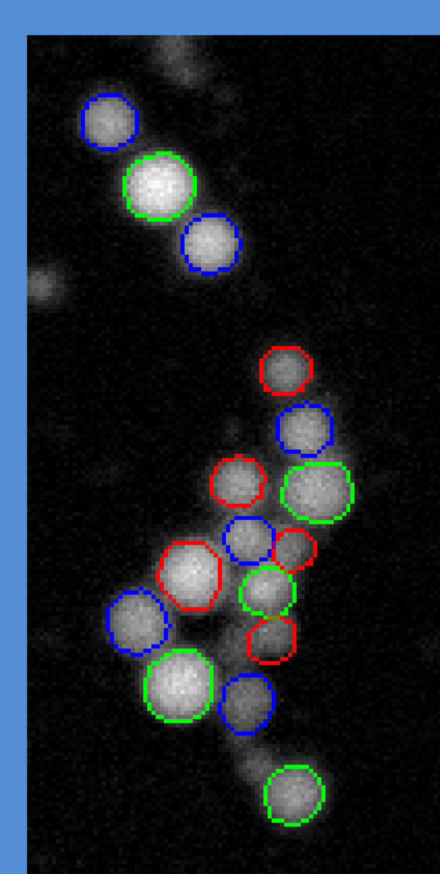
Biological application

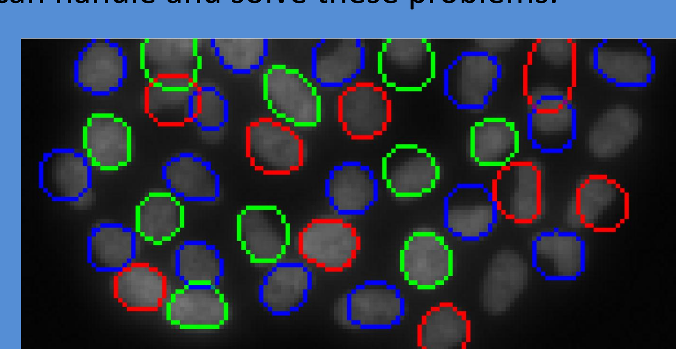
❖ In microbiology, one of the main image processing problems is to segment multiple objects, e.g. lipid droplets, cells or other sub-cellular components, that are often near-circular with many overlaps.

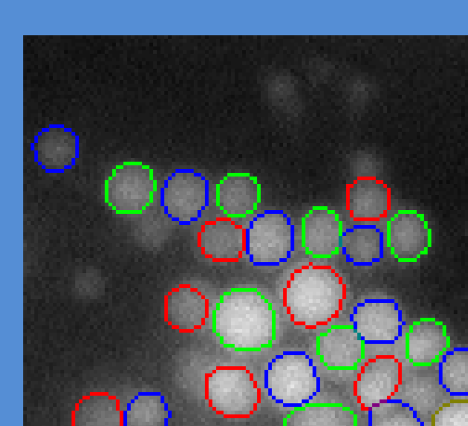
❖ The images made by light microscope tecniques are noisy, blurred and have low contrast.

❖ The results show that the proposed model can handle and solve these problems.









Conclusion

❖ The proposed multi-layer phase field GOC model is capable of representing and modelling an *a proiri* unknown number of touching or overlapping near-circular objects.

❖ The *a priori* model coupled with an appropriate data model and initialization, can efficiently extract such object configurations from synthetic and real images.

References

[1] P. Horváth and I. H. Jermyn. A new phase field model of a 'gas of circles' for tree crown extraction from aerial images. In Proc. International Conference on Computer Analysis of Images and Patterns (CAIP), Lecture Notes in Computer Science, Vienna, Austria, August 2007

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