MATHEMATICALl BASES OF ElLECTROMECHANICAL ACTUATORS MODELING

Az elektromechanikus aktuátorok modellezésének matematikai alapjai

Prof Dr. Dusan Maga
PhD, kutató
Edutus Főiskola/Edutus College

Abstract:

Modelling of electromechanical actuators is a discipline as old as the actuators themselves. Computer based technologies are less or more successfully used from 20th century (following the boom of power and price of available computational hardware and software). Nowadays, in early 21st century, there is a bunch of low price (very often even free) and relatively high quality software available, together with high computer power. This usually leads to very easy, simply and user friendly operating of offered software products. While these attributes have many advantages, there is a huge amount of potential possible risk: almost under any conditions the software responses to users! This, without sufficient knowledge of physical background, should lead to fatal mistakes and misunderstandings. Based on personal experience of the author, the same important role in the design process must also be devoted to mathematical background.

Összefoglaló: Az elektromechanikus aktuátorok modellezése olyan régi, mint a működtetők maguk. Számítógép-alapú technológiák többé-kevésbé a 20. századtól sikeresen alkalmazhatóak (követve az számítógép és szoftver ár, illetve az erő fellendülését). Napjainkban, a 21. század elején, egy csomó alacsony áron (sokszor akár ingyenes is), és viszonylag jó minőségű szoftver rendelkezésre áll, együtt a számítógép erős teljesítményével. Ez általában nagyon könnyen kezelhető egyszerű szoftver a kínálathoz vezet. Bár ezeknek a tulajdonságoknak számos előnye van, emellett van egy hatalmas mennyiségű potenciális kockázati lehetősége is: Szinte minden körülmény között a számítógép válaszol a felhasználónak! Ez, a fizikai háttér elegendő ismerete nélkül, végzetes tévedésekhez és félreértésekhez vezethet. A szerző személyes tapasztalatai alapján, ugyanazt a fontosságot kell fordítani a matematikai háttére is a tervezési folyamat során.

Keywords: Numerical models; Electromagnetic fields; FDM.

Introduction

The designer's role (especially in technically oriented branches) is to find a system to be able to characterize, design and verify the properties of constructed machinery. The beginning of all these activities is joined with Maxwell's equations (for electromagnetic fields) or to similar equations sets (for different technical problems; chosen parameters can be seen in Table 1). The behavior of wanted results is there describe by set(s) of differential equations. These can (almost) ***exactly*** be solved by analytical methods; nevertheless, the analytical approach has a major shortcoming: the equations can be solved only for very simple geometries and simply distribution of other input date (typically: field sources).

Table 1 – Analogous quantities

Quantity	Magnetostatic	Temperature	Gravitational
Potential	Potential Ω	Temperature	Newtonian
Flux density	Flux density B	Temperature gradient	Gravitation force
Constant (parameter) of medium	Permeability μ	Thermal conductivity	Gravitation constant (reciprocal)

Another important factor is the complexity of time domain. Based on these, the investigated tasks could be divided into following categories:

a, time independent,

b, time dependent, with unneglectable but not significant influence of time domain,

c, time dependent with significant influence of the time domain.

The analytical solutions can be applied for all of the mentioned cases; however, in cases b, and c, the non-harmonic inputs and their responses may completely disable the application of analytical approach. Therefore, the numerical methods (coarse and ***unexact***) can and must be used. There is a wide range of numerical methods; even though, they are based on one of the following three ones:

- Finite Difference Method (FDM),
- Finite Element Method (FEM),
- Boundary Element Method (BEM).

While the first two of the mentioned are built on differential principles, the BEM is based on integral calculus. The major attention in this paper will be paid to a very simple FDM, completed with brief overview on numerical methods' general principles. Backgrounds of the finite elements, as well as the principles of the boundary elements will be presented in author's future work).

Numerical Field Solution Methods – General Principles

The process of technical problems modeling can be described by the following scheme (Figure 1). Role of user and computer (software) in different steps is indicated within the figure. A lot of different and sometimes complicated factors has to me implemented into the presented system:

- geometry of the model, including the possible simp0lifications: e. g. regularities and mirroring,
- description of field sources (e. g. permanent magnets, current densities, currents, …), if any,
- description of used materials properties,
- description of boundary conditions (often could replace the external field sources).

Anyway, the used software and the hardware are typically very precise and do not do any significant mistakes. The central source of possible mistakes is always the user!

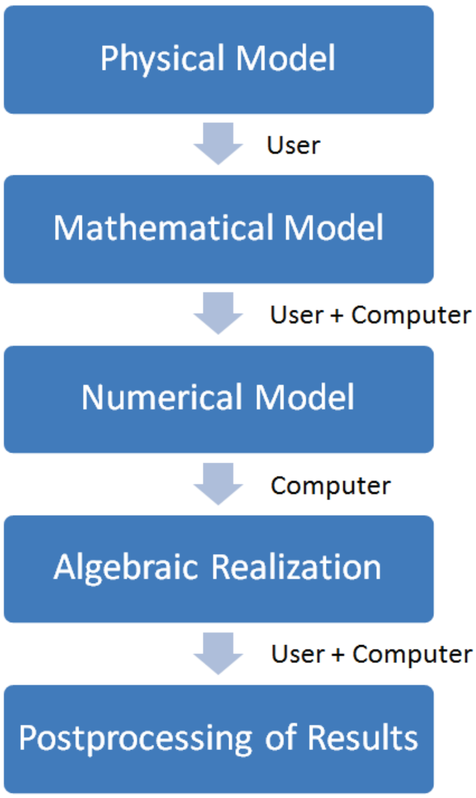


Fig. 1 General Scheme of the Numerical Analysis

The Finite Difference Method

This method has been successfully used to evaluate the field distribution of any type of magnetic field. This very simply method has its bases in early 60-ties of 20th century. Even today are available different upgrades of its scheme, as well as the original principle supported by user friendly environment and powerful computational hardware. The required solution is a solution of Poisson's type equation (1), continuous in area of interest. The solution can be obtained in very easy way – when required not for the complete area but only for a set of chosen points (nodes). These must be chosen very carefully – to represent the geometrical and other properties of the solved problem. This, so called discretization scheme, can be seen in Figure 2. It can be seen, that the step between nodes in x-direction is unique and equal to h, while step in y-direction is equal k. Generally, $h \neq k$.

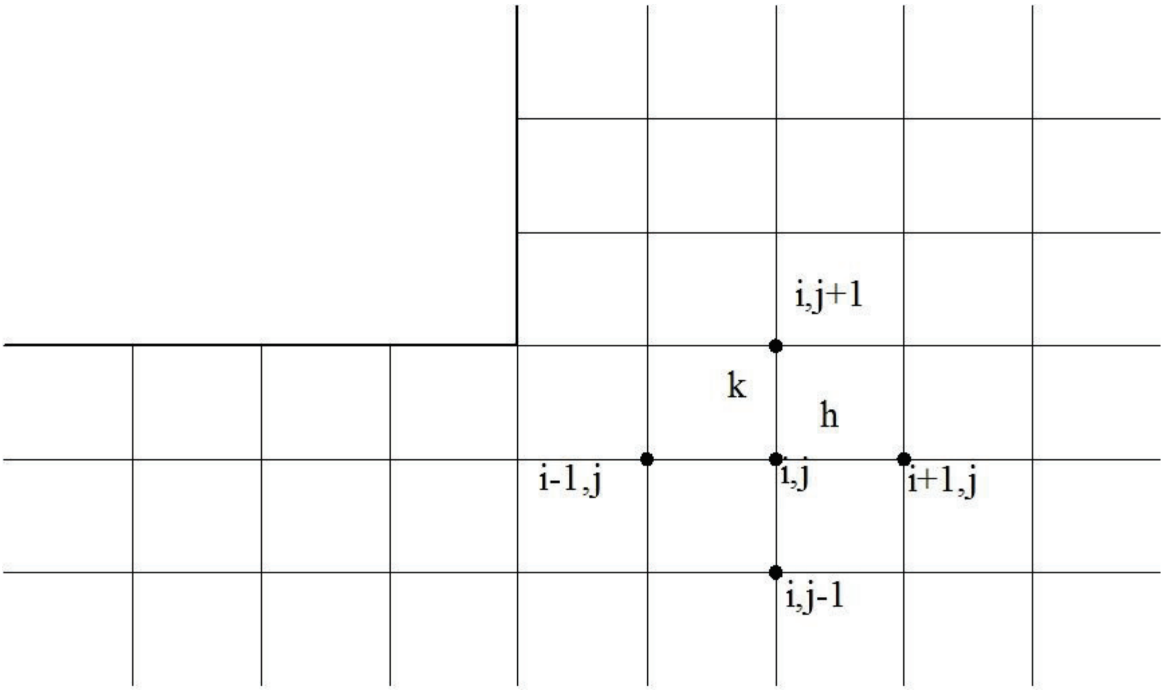


Fig. 2 Discretization scheme of the FDM in 2D [1]

The Poisson's equation for 2D electrostatic fields can be the following:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -\frac{Q}{\kappa} \qquad (1)$$

where U is the unknown field potential, Q represents the field sources and κ material properties.

Taylor's theorem (successfully implemented by Joseph-Loise Lagrange) can be used to express the second order derivates in (1)

$$U(x+h)=U(x)+hU'(x)+\frac{1}{2}h^2U''(x)+\frac{1}{6}h^3U'''(x)+\dots \quad (2)$$

$$U(x-h)=U(x)-hU'(x)+\frac{1}{2}h^2U''(x)-\frac{1}{6}h^3U'''(x)+\dots \quad (3)$$

It can be seen, that the equation (2) and (3) are expressing the derivate in x-direction. It is necessary to use the same procedure (with parameter *k*) to express the derivative in y-direction. Comparing the (1) and (2) to Fig. (2) it is easy to replace the coordinates with node-indexes:

$U(x) = U_{i,j}$

$U(x+h) = U_{i+1,j}$

$U(x-h) = U_{i-1,j}$

Addition of (1) and (2), including the coordinate substitution with above presented rules, will lead to the following:

$$U''_{i,j}=\frac{U_{i+1,j}-2U_{i,j}+U_{i-1,j}}{h^2} \quad (4)$$

The equation (4) represents partial derivative from (1) in x-direction. It is obvious, that a part of the original equation(s) is missing in (4). Nevertheless, the missing part consists (based on Taylor's theorem) of derivatives of order 4 and higher. Because of the general mathematical and physical principles, this neglecting should (will) not lead to significant errors. Using the same procedure, the partial derivative for y-direction is expressed:

$$U''_{i,j}=\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{k^2} \quad (5)$$

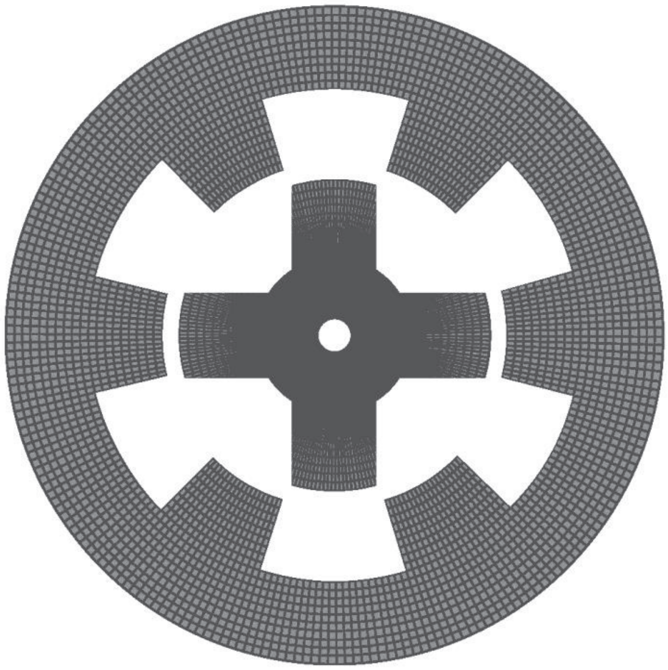
Completing the (4) and (5) into (1) the following expression appears:

$$\frac{1}{h^2}\left(U_{i+1,j}-2U_{i,j}+U_{i-1,j}\right)+\frac{1}{k^2}\left(U_{i,j+1}-2U_{i,j}+U_{i,j-1}\right)+Q=0 \quad (6)$$

Assigning the (6) to each node from the area of interest, followed by inclusion of boundary conditions, the set of equations is obtained where the number of unknowns is equal to number of equations. So the set has a unique solution.

The Examples

A comparison of FDM and FEM results, discussion of these methods accuracy, etc., has been realized by author in [5]. A simple 6/4 switched reluctance machine has been used to demonstrate the methods principles. The FD model of the machine (resp. of the machine magnetic circuit) can be seen in Fig 3.



- Fig. 3 Finite Difference Model of 6/4 SRM Magnetic Circuit [5]

- The uniformity of the mesh, described above, can easily be identified in Fig. 3; nevertheless, despite of presented, the (r, φ) uniformity in case of rotational machinery (instead of (x, y)) has been used. The magnetic field distribution, as well as the secondary results (mechanical torque) comparison, depending on mesh quality, compared to FE software results, can be seen in [5].

- Other examples, demonstrating the FDM basic mathematical principles (e. g. principal solution of field in square coaxial cable in x, y coordinates – Fig. 5), can be found in [1].

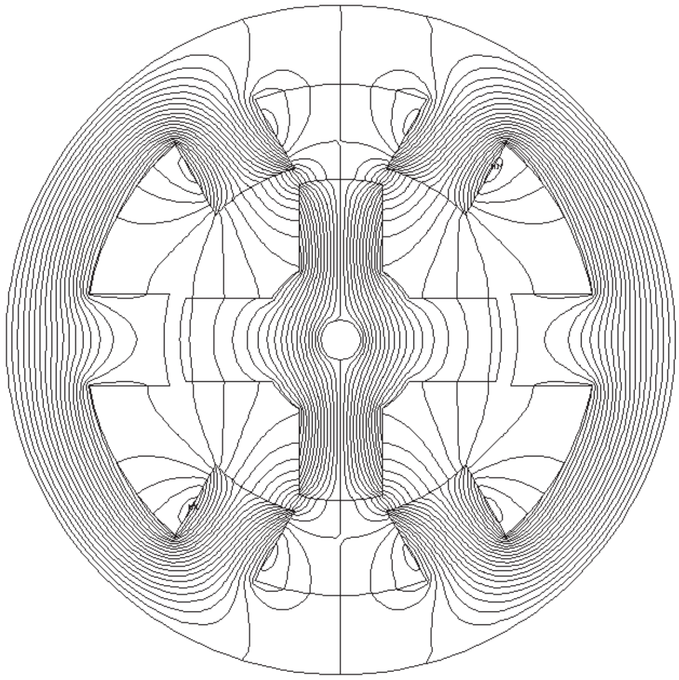
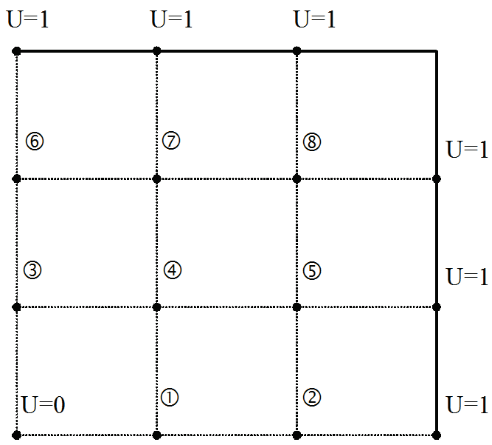


Fig. 4 Field distribution in 6/4 SRM [5]



$$\begin{bmatrix} -4 & 1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & -4 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ -2 \end{bmatrix}$$

Fig. 5. Coaxial Cable Model and Its Matrix Representation

Conclusions

This paper is the first from the series describing the field distribution solving numerical methods bases and principles. The brief overview of numerical methods is given in the beginning of the paper, then, the first of the method – the Finite Difference Method is described. The principal part of the paper is based on description of mathematical base of presented technique, the role of the given examples is primarily to illustrate that the method can really and successfully be used in magnetic (and other) field solution in rotational electromechanical machinery.

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