Medical diagnostic systems

(Orvosbiológiai képalkotó rendszerek)

Fundamental concepts in acoustics

(Alapfogalmak az akusztikában)

Miklós Gyöngy

Aims

- Consider 3 methods of acoustic localisation
- Through these, learn about concepts in acoustics
    - propagation of sound
    - diffraction
    - reflection
    - scattering
    - attenuation
- Link back to diagnostic ultrasound throughout

Methods of sound localisation

1. Lightning localisation

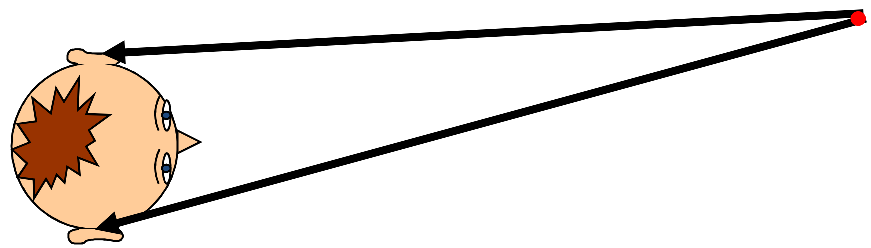
passive

(light as reference)

2. Binaural hearing

passive

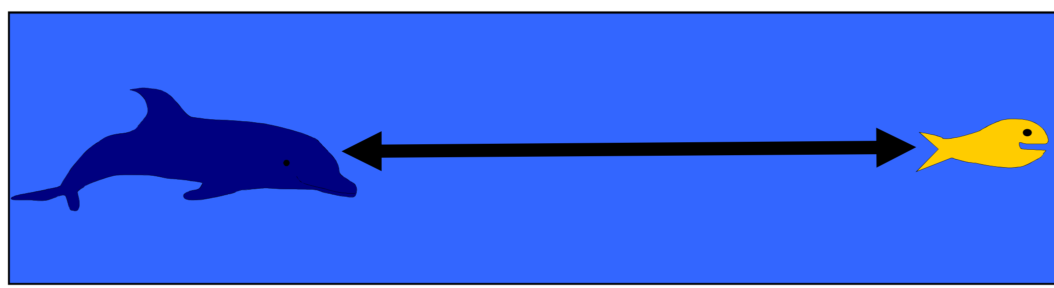
(difference in arrival times)



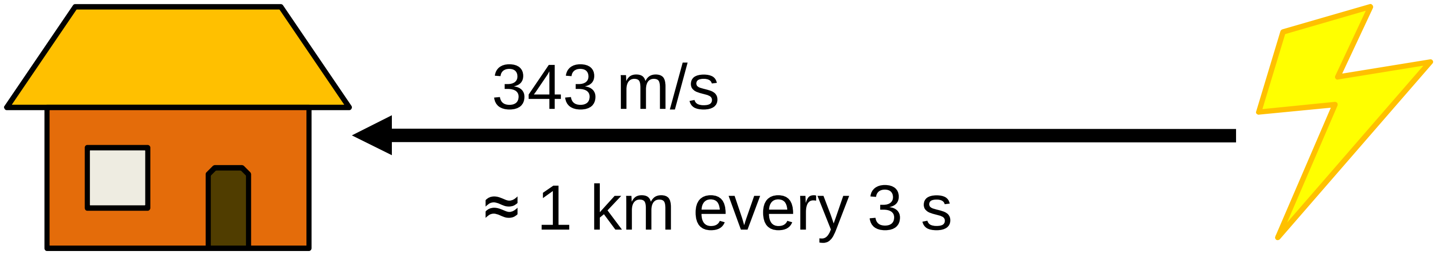
3. Echolocation

active

(pulse-echo)



1. Lightning localisation



- Passive method (with light as reference)
- Time of arrival (ToA), speed of sound (SoS) → localisation
- *Analogy with diagnostic ultrasound?*
- Where does speed of sound come from?
- What about propagation in tissue?

Analogy with diagnostic ultrasound:

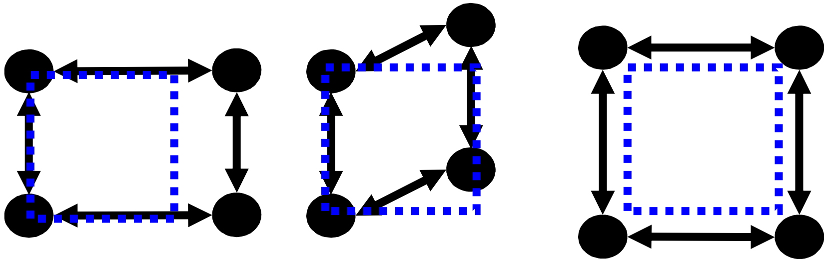
localising "flashes of lightning" – photoacoustics

- Transmit laser pulse at known time
- Optically "dark" tissue absorbs laser preferentially
- Localised heating due to laser pulse creates shock wave
- Time of arrival depends on location of emission site

http://www.ucl.ac.uk/cabi/Photoacustics/Photoacustics.html

Propagation of sound

- Mechanical vibrations cause travelling waves
- Wave can be sustained by normal stress, shear stress, and volumetric compressions
- $E\rightarrow\infty$ or $\rho\rightarrow 0$: $c\rightarrow\infty$ (block moves as one)



elastic modulus

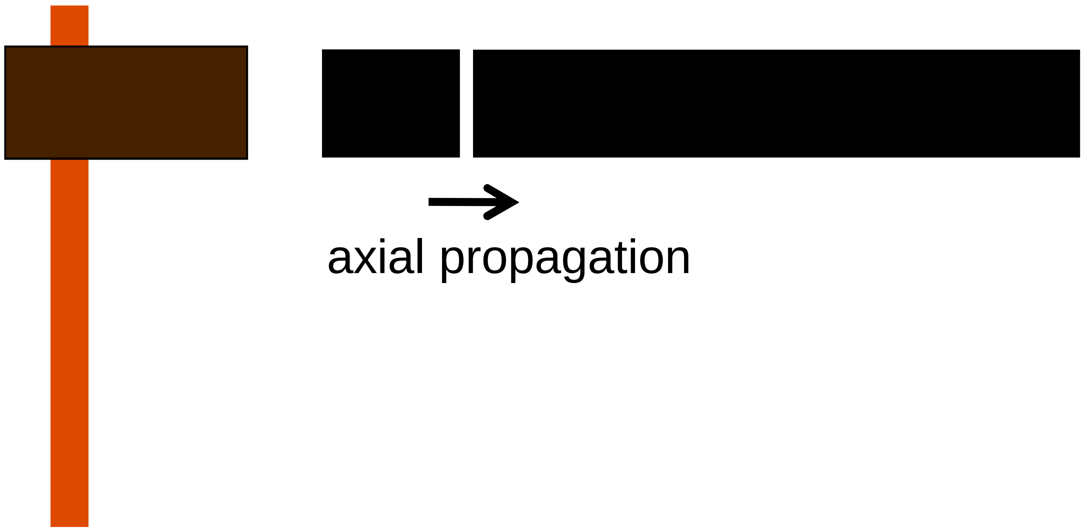
propagation speed

$$c = \sqrt{\frac{C}{\rho_0}}$$

undisturbed density

Types of elastic moduli

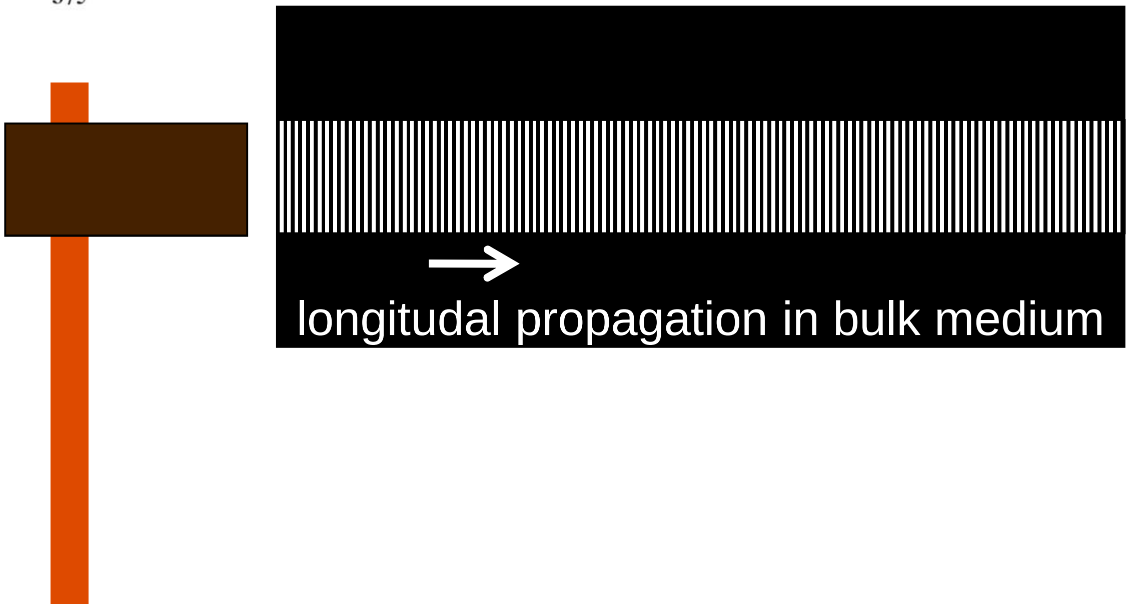
- Young's (E): axial propagation along laterally unconstrained rod
- P-wave (M): longitudal propagation, no lateral motion
- shear (G): motion transverse to direction of propagation
- bulk (K): volumetric propagation (pressure waves)
- $K=M-4G/3$: without shear, equivalent to P-wave
- $K = -V\, \partial p/\partial V$: *inverse of compressibility* κ



$$c = \sqrt{\frac{C}{\rho}}$$

- $C = E$ (Young's Modulus)
- $E = \sigma/\varepsilon$ (stress/strain)
- $c_{st.steel} = \sqrt{(216\times10^9/7800)} \approx 5300\ m/s$
- $E\rightarrow\infty$ or $\rho\rightarrow0$: $c\rightarrow\infty$ (block moves as one)

Medical diagnostic systems – Fundamental concepts in acoustics



$$c = \sqrt{\frac{C}{\rho}}$$

- $\varepsilon_{xx} = \varepsilon_{xx} = 0$
- no diffraction (high frequency)
- $C = M$ (P-wave modulus)
- $M = \sigma_{zz}/\varepsilon_{zz}$ (stress/strain)
- $c_{st.steel} = 5980\ m/s$

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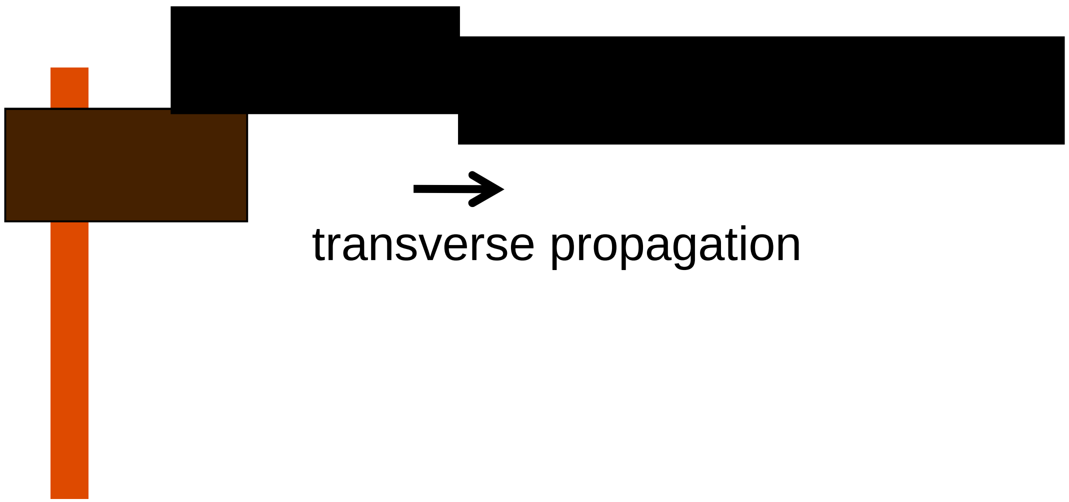
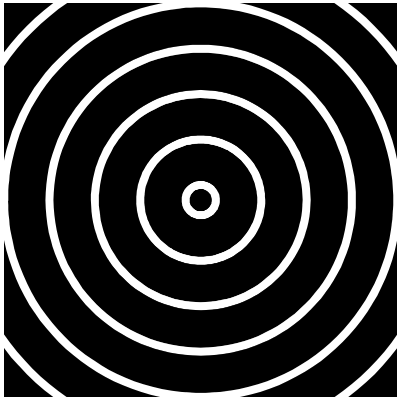


$$c = \sqrt{\frac{C}{\rho}}$$

- $C = G$ (shear modulus)
- $G = \tau/\gamma$ (shear stress/shear strain)
- $c_{st.steel} = \sqrt{(84 \times 10^9/7800)} \approx 3300\ m/s$



volumetric propagation

$$c = \sqrt{\frac{C}{\rho}}$$

- $C = K = 1/\kappa$ (bulk modulus=1/compressibility)
- $K = -V\,\partial p/\partial V$
- $K = M - 4G/3$
- K_{water}: 2.05 GPa at 1 atm → 3.88 GPa at 300 atm
- $c_{water} = \sqrt{(K/\rho)} \approx \sqrt{(2e6)} \approx$ 1400 m/s

Propagation of pressure waves

Assuming small pressure and density fluctuations

$$P = p_0 + p \text{ where } p << p$$

$$R = \rho_0 + \rho \text{ where } \rho << \rho_0$$

- a waveform retains its shape as it travels (linear propagation)
- propagation can be described by the linear wave equation

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

Solutions of linear wave equation:

- planar wave propagating in z-direction: $p = A\, g(z - t/c)$
- spherical wave: $p = A/|\boldsymbol{r} - \boldsymbol{r}_0|\, g(\mathrm{z-t/c})$

Energetics of pressure waves [Coussios 2005]

- A wave causes flow of energy without net flow of mass
- Flow of power P through area A is the acoustic intensity $I=P/A$ (W m^{-2})
- Pressure p and particle velocity v are related by impedance Z, and intensity I is given by product of two:

$$p(\mathbf{r},t) = Z(\mathbf{r})\, v(\mathbf{r},t)$$

instantaneous intensity $I_{\mathrm{inst}}(\mathbf{r},t) = p(\mathbf{r},t)\, v(\mathbf{r},t)$

acoustic intensity $I(\mathbf{r}) = p_{\mathrm{rms}}(\mathbf{r})\, v_{\mathrm{rms}}(\mathbf{r}) = \mathrm{p}(\mathbf{r})^2{}_{\mathrm{max}}/2Z(\mathbf{r})$ = &c.

(*cf*. voltage and current in electronics!)

- Using phasors to represent p, v, Z may be complex (again, *cf*. electronics)!
- **For planar waves only:**

acoustic impedance Z = characteristic impedance of medium ρc

- Intensity $I=P/A$ flows at speed c. Hence energy density $E = I/c$ (J m^{-3})

Propagation in tissue

- Is tissue a solid or a liquid?
- Can it support shear waves?
- What is propagation speed c in tissue?

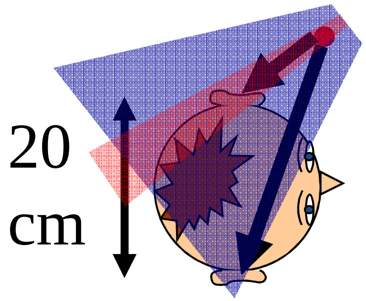
Longitudal speeds of sound [Wells 1999]

- Hard tissue (bones, teeth); $c \approx 4000$ m/s
- Soft tissue (muscle, fat); $c \approx 1540$ m/s
- Liquid tissue (blood, lymph); $c \approx 1570$ m/s
- Gas pockets (lungs, oesophagus); $c \approx 330$ m/s
- compare with steel, water and air – at 37°C!

Soft tissue

- Aqueous solution with suspension of cells and matrix of extracellular scaffolding (collagen, elastin)
- Modelled as a *viscoelastic gel*
- Solid-like elasticity and liquid-like viscosity both contribute to presence of shear waves (~3 m/s [McLaughlin and Renzi 2006])

2. Binaural hearing [Sekuler and Blake 1994]



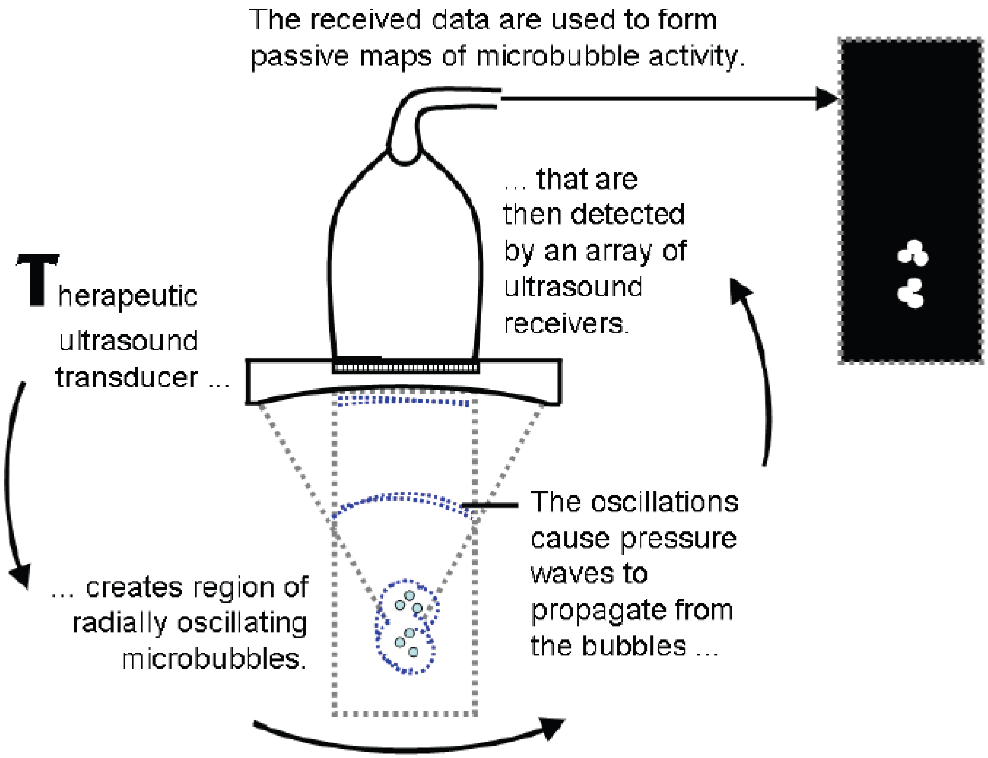
1000 Hz ⬄ 1 ms ⬄ ~34 cm

frequency *f*, attenuation coefficient *α*

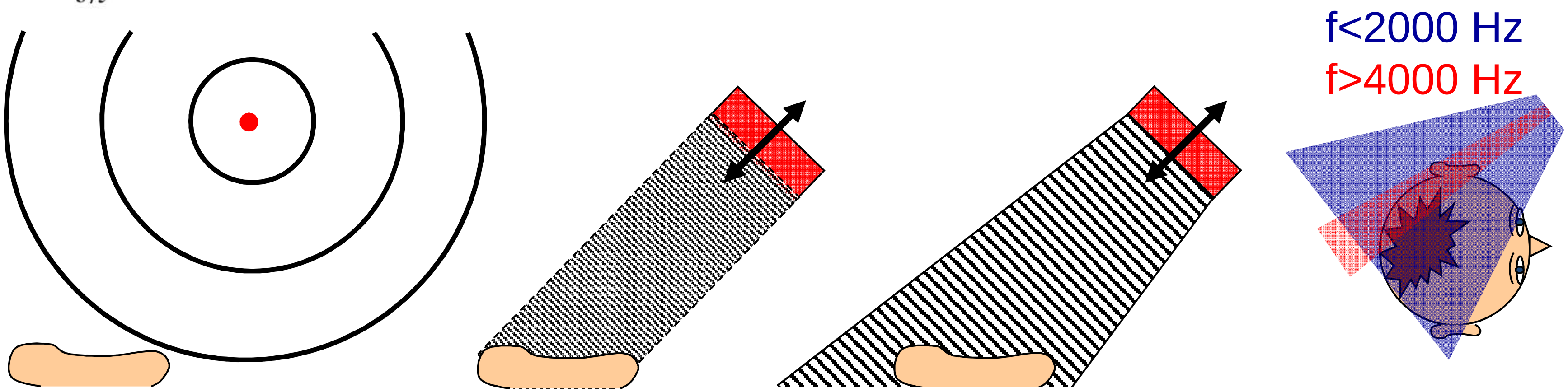
- f<2000 Hz: low α, high diffraction: interaural time difference
- f>4000 Hz: high α, low diffraction: interaural level difference
- Passive method without temporal reference
- *Time differences of arrival (TDoA) in diagnostic ultrasound?*
- What are diffraction and attenuation?
- Role of attenuation and diffraction in diagnostic ultrasond?

TDoA in medical ultrasound: Tracking popping bubbles – passive cavitation mapping

- Cavitation (bubble activity) often involved in ultrasound therapies
- Cavitation may occur at any time (no temporal reference)
- Time differences of arrival of shockwaves allows localisation of bubbles

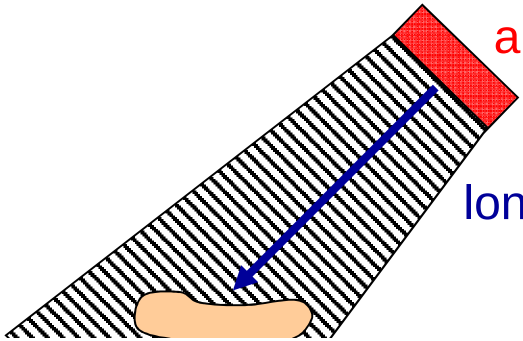


[Gyöngy 2010]



Diffraction

- Point source spreads spherically
- Set of point sources *interfere* with each other
- Continuous source region *diffracts*
  - analogous to interference of infinite point sources
- Level of diffraction decreases with frequency

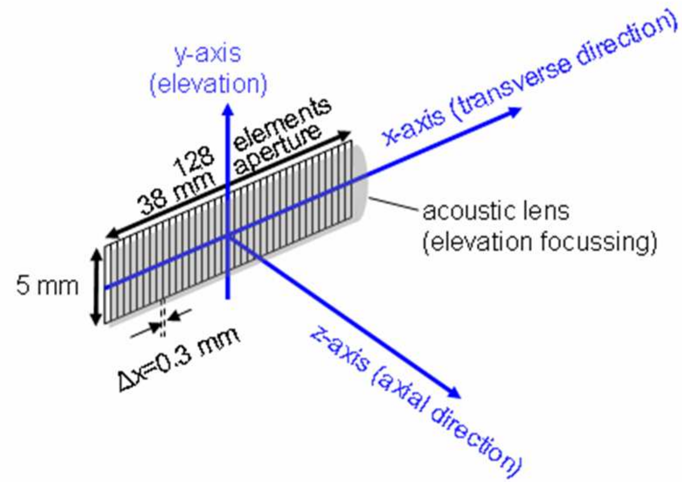


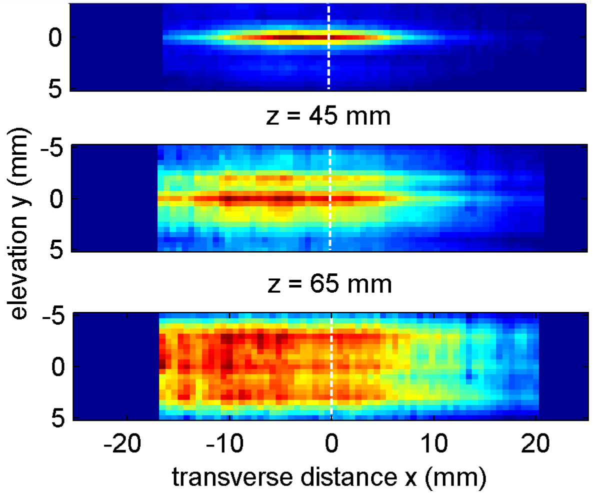
Diffraction

- Consider single frequency f
- Pressure field $p(\mathbf{r})$ expressed as complex scalar field of phasors
- Small distances $|\mathbf{r}|$ (*near-field*): $p(\mathbf{r})$ = complex interference pattern
- Large distances $|\mathbf{r}|$ (*far-field*): $|p| = \mathrm{H}(\boldsymbol{\theta})/|\mathbf{r}|$
- Transition on longitudal axis at $|\mathbf{r}|=D^2 f/4c$ [Olympus 2006]
- Transition depends on aperture D as well as frequency!

Diffraction in diagnostic ultrasound

- Typical abdominal 1D array: the L10-5 from Zonare medical systems
- Focusing in imaging plane using acoustic lens
- z=17.5 mm elevational focus
- z=60–100mm: roughly constant, ~10 mm sensitivity in elevational direction
- scattering object 5 mm out of imaging plane may be seen!





[Györgyi 2010]

Attenuation

- Consider a planar wave travelling in the z-direction
- Without any attenuation, the wave will maintain its amplitude:

$$p = A\, g(t - z/c)$$

- In reality, some of wave redirected in other direction (scattering) and some is converted to microscopic random motion – heat (absorption)
- If attenuation is uniform over distance:

$$p = A \exp(-\alpha z)\, g(t - z/c)$$

where α is attenuation coefficient in Nepers

- What if attenuation is caused by a single object?

Attenuation in diagnostic ultrasound

- For plane wave travelling in z-direction, attenuation coefficient α describes "weakening" of pressure with distance:

$$p = A\exp\{j(kz-\omega t)\}\exp(-\alpha z)$$

$$|p| = A\,exp(-\alpha z)$$

where α is in Nepers (Np for short).

- For tissue, $\alpha_{dB} \approx 1$ dB/cm/MHz [Brunner 2002]
- Therefore, at 6 MHz
  - pressure amplitude halves for every cm travelled
  - pressure received from perfectly reflecting target 10 cm deep (consider two-way propagation)?
- Exercise: show that 1 dB ≈ 0.115 Np
- What is origin of attenuation?

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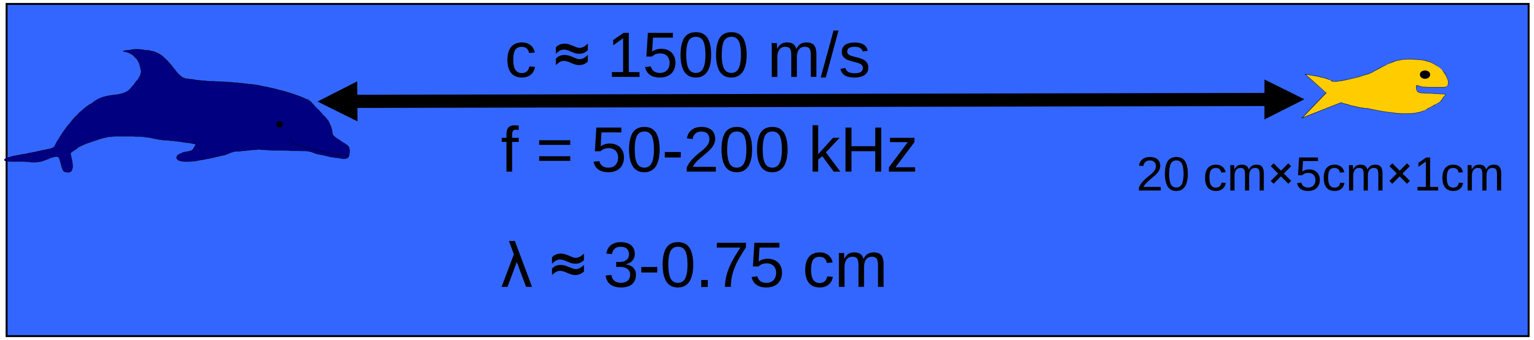
3. Echolocation [Au *et al.* 2007]



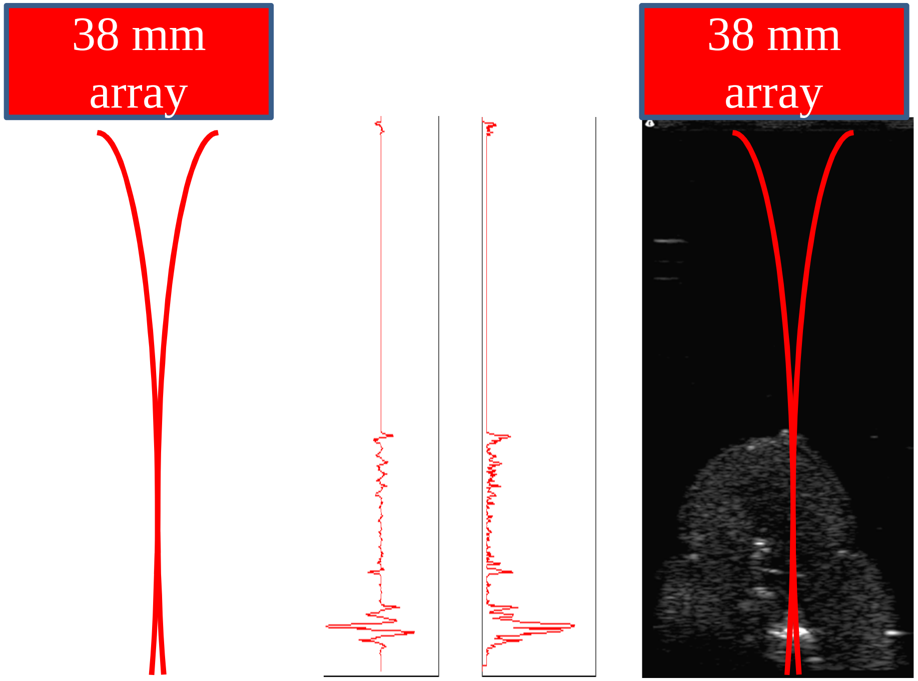
- Active method: time of transmission acts as reference
- Two-way travel time, speed of sound (SoS) → localization
- *Analogy with diagnostic ultrasound?*
- How accurate is the localization?
- How do echoes form from the fish (scattering)?

Diagnostic echolocation:

pulse-echo B-mode imaging

- Most widespread form of diagnostic ultrasound imaging
- Very simple conceptually:

1. transmit pulse along different lines
2. convert timeline of recorded echoes to distance (d=t/2c)
3. convert amplitude of echoes to brightness on a screen



Transmit and receive beamforming along thin 'line'

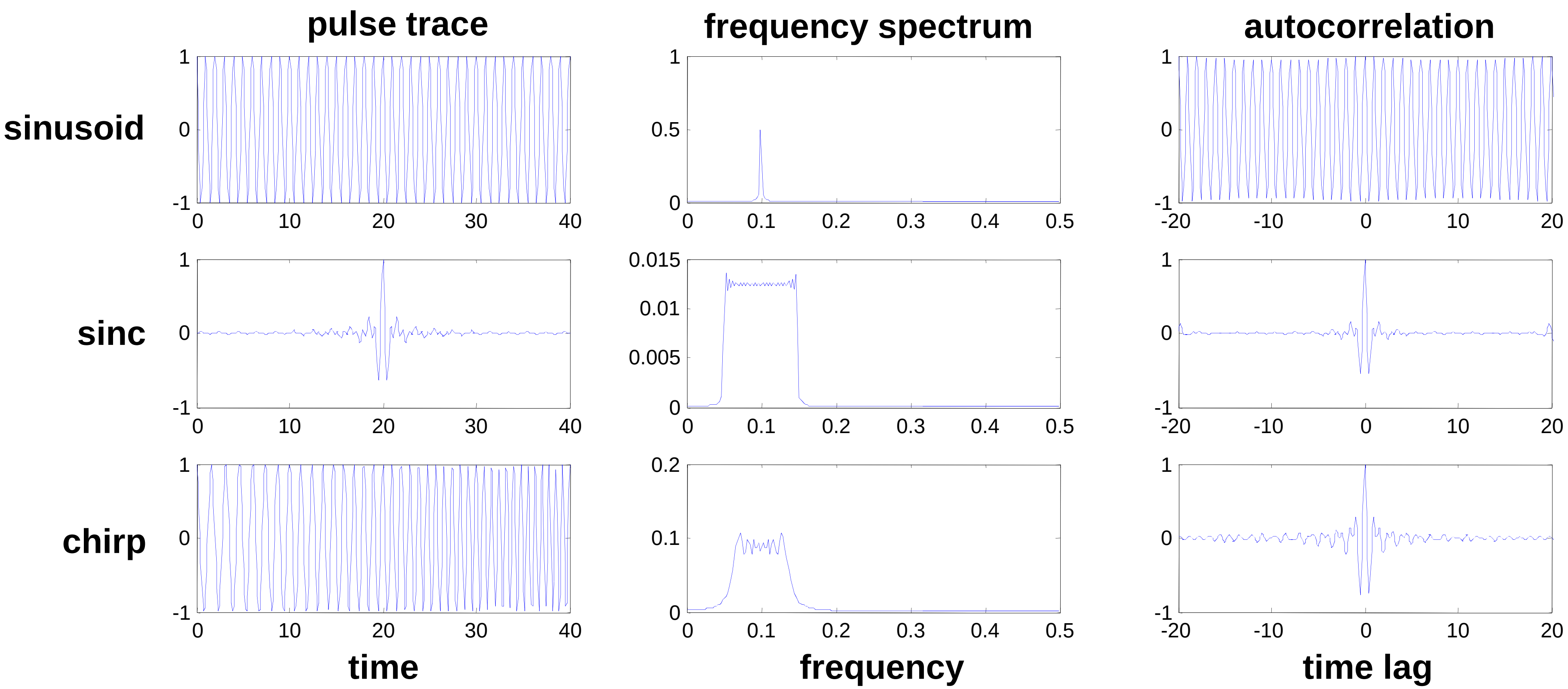
Received data amplitude (A-line)

A-line envelope

Multiple A-line envelopes create B-mode image (here, porcine liver in water imaged using Terason t2000)

Localisation accuracy

Determined by width of transmit pulse autocorrelation

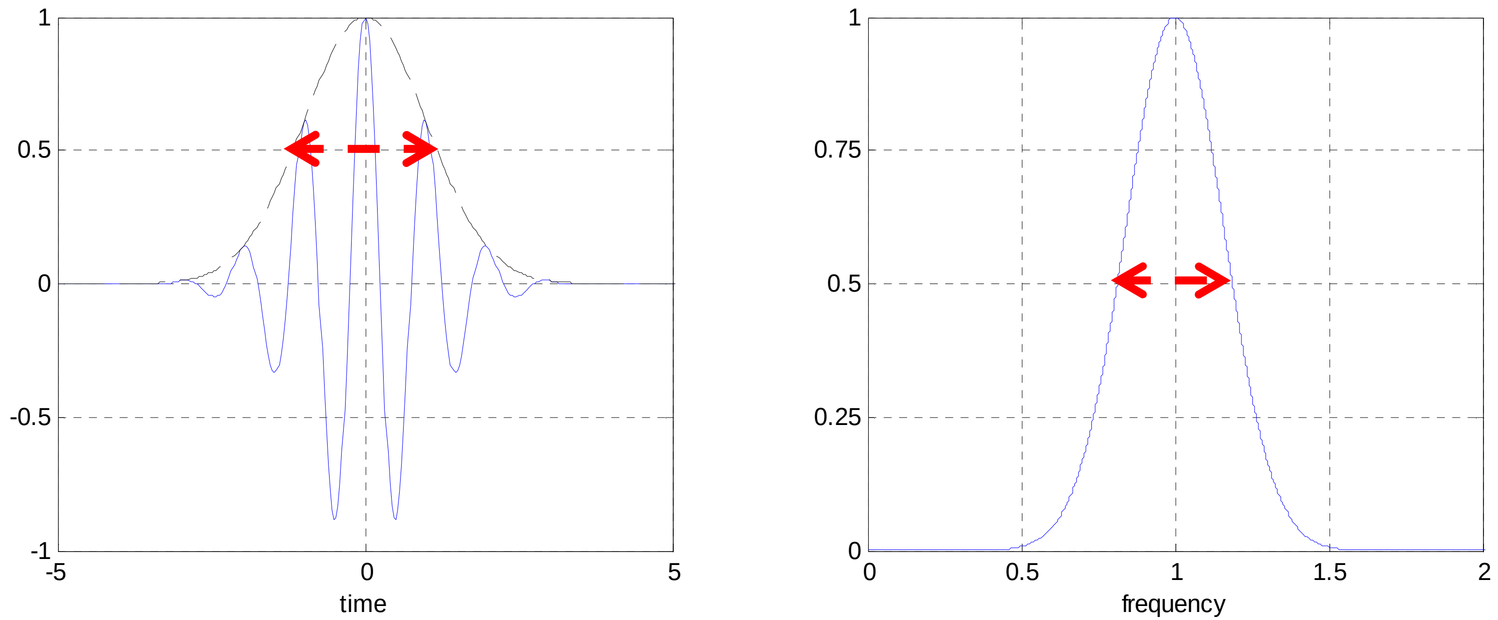


Localisation accuracy

$\Delta t\, \Delta f \approx 1$; $2.355 \times 0.375 = 0.883$

#oscillations $\approx f_0 \Delta t \approx f_0/\Delta f = Q$ $(=1/0.375=2.667)$

Approximation better for Q>>1 (underdamping)

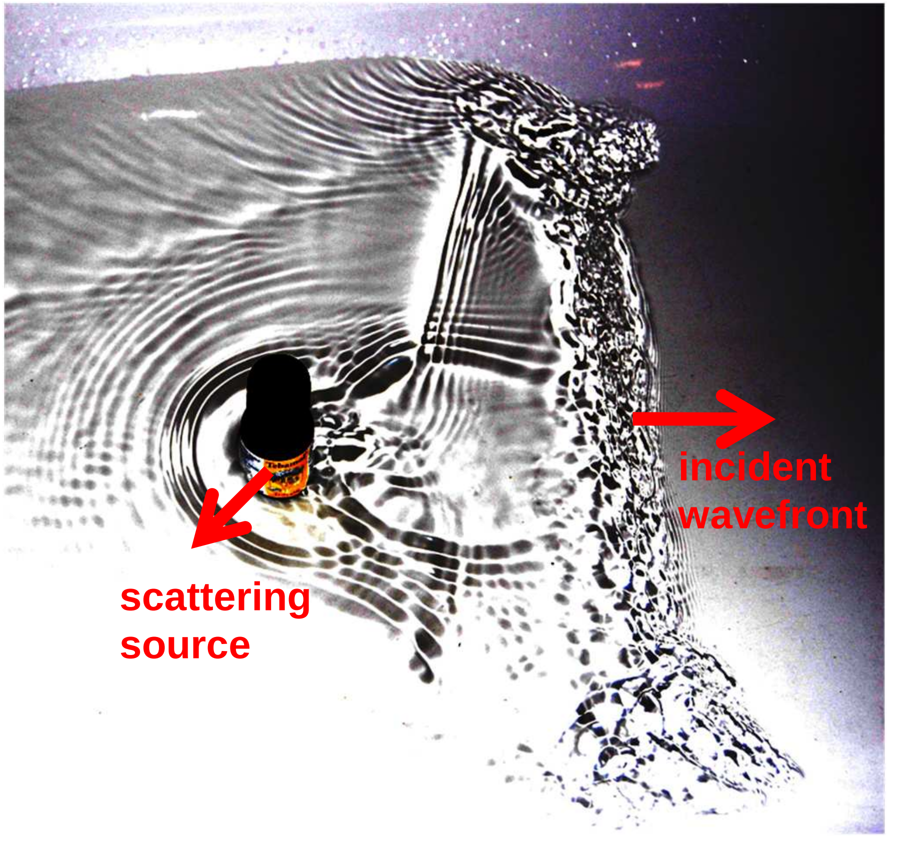


Scattering

- Caused by inhomogeneities of the medium (variations in compressibility κ and density ρ)
- Total pressure field modelled as sum of incident and scattered field:

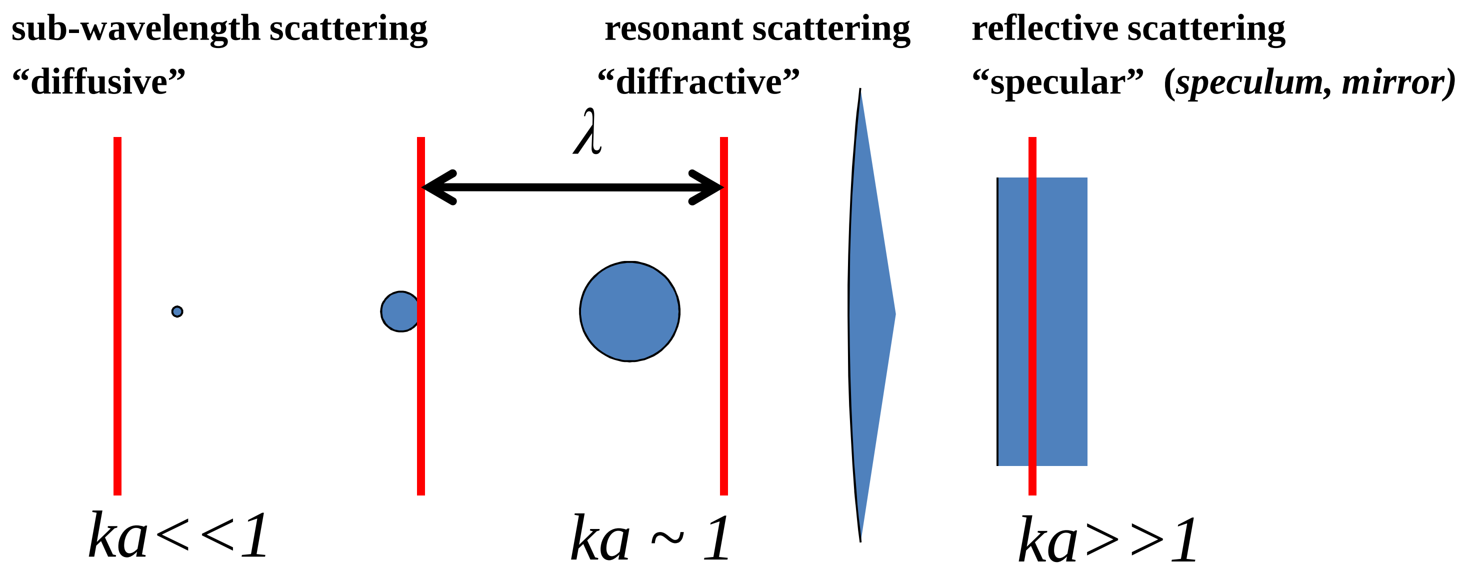
$$p(\mathbf{r},t) = p_i(\mathbf{r},t) + p_s(\mathbf{r},t)$$

- Hence, scattering creates "virtual sources"



Surface wave scattered in bath tub by 27 mm object

Regimes of echo formation (scattering):



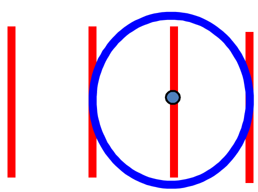
- $k = 2\pi/\lambda$: angular wavenumber
- a: characteristic size of scatterer (for sphere, equals radius)
- ka: number (dimensionless): characterises scattering behaviour
- reflection a limiting case of scattering

Sub-wavelength scattering (ka << 1) [Lighthill 2001]

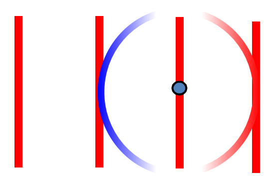
- Changes in compressibility κ and density ρ has different effects:
  - $\Delta\kappa$ causes angle-independent (monopolar) scattering
  - $\Delta\rho$ causes dipolar scattering equivalent to two opposing monopoles

$$p_s(\mathbf{r},t)\,\alpha\,\frac{\kappa_s-\kappa_0}{\kappa_0}+\frac{3(\rho_s-\rho_0)}{\rho_0+2\rho_s}\cos\theta=\{\text{fixed, incompressible}\}-1+\frac{3}{2}\cos\theta$$

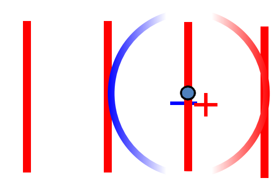
  - θ: direction relative to direction of propagation



monopolar scattering
Δ volumetric changes



dipolar scattering
Δ momentum changes



dipole ~
2 anti-phase mpoles

- Amplitude of scattered pressure increases with k and a
  - how to quantify "scattering ability" of object?

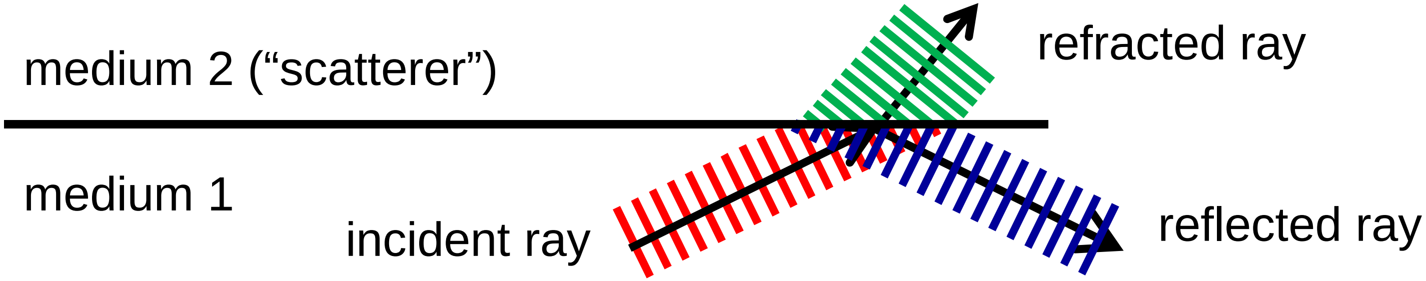
Resonant scattering (*ka* ~ 1) [Lighthill 2001]

- Incident pressure varies over object
- Interference between scattering wavefronts at different locations causes complicated scattered field
  - backscattered wavefronts from front and back of scatterer in phase →resonance
- Mode conversion at boundary (pressure wave ↔ shear wave) also causes resonance peaks
- By definition, in far-field of scatterer, pressure amplitude varies reciprocally with distance for constant angle:

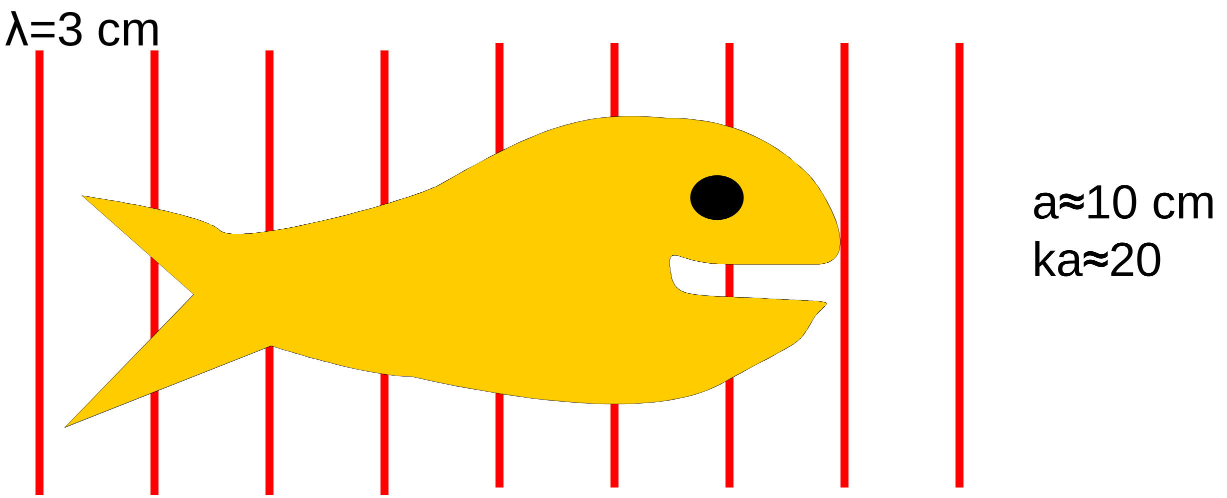
$$p_s(\mathbf{r},t) = \frac{H(\boldsymbol{\theta})}{|\mathbf{r}|}$$

Reflective scattering (*ka* >> 1) [Lighthill 2001]

- Scatterer very large: meetings of pressure wave with object boundary *independent* of each other (no phase information). *(In reality, if transmitted pulse is long enough and attenuation does not extinguish a wave before it hits a new boundary, standing waves will be set up)*
- At each boundary, mismatch in *characteristic acoustic impedance* (=ρc) creates reflection (as well as refraction)
- Laws of geometric acoustics used for *ray tracing* (*cf*. optics)
- Rays describe direction of high-frequency acoustic beams that undergo negligible diffraction or interference



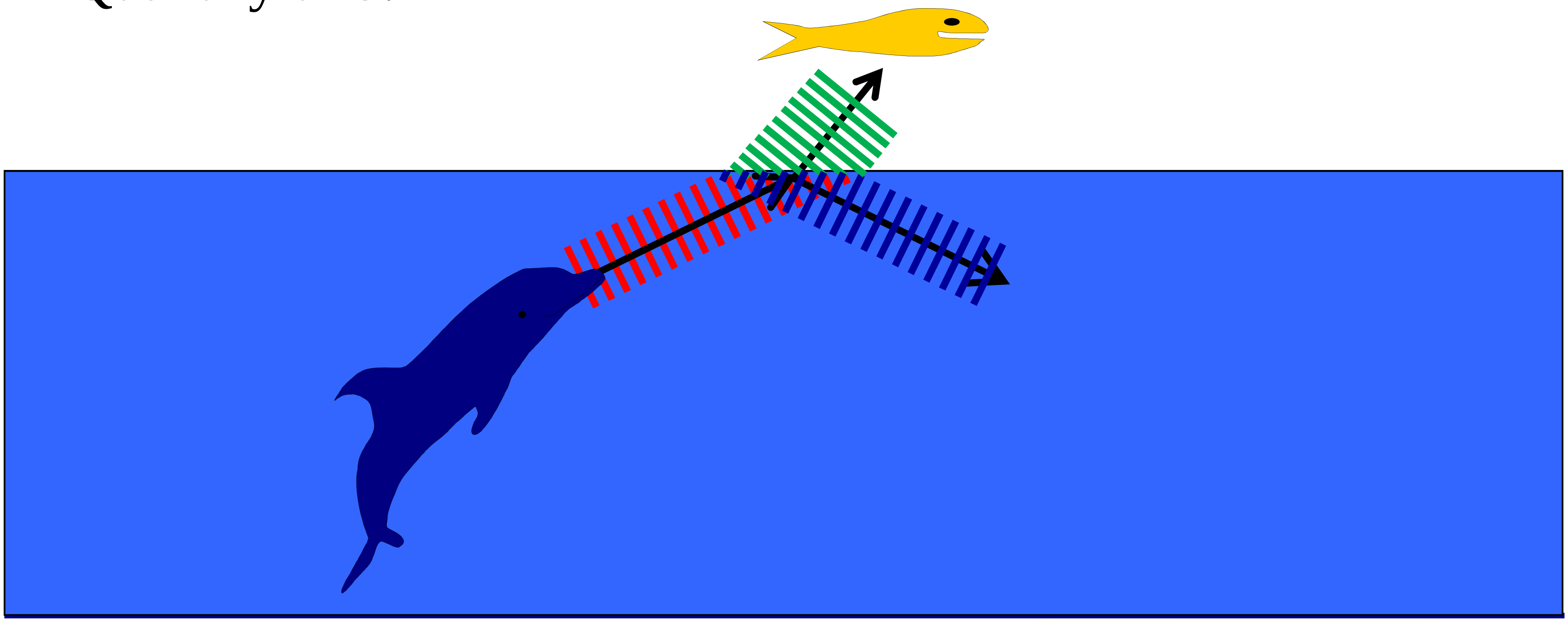
Fish as (resonant) scatterers



[Ye and Farmer 1996]	Water	Swimbladder	Fish
Mass density (kg m3)	1026	1.24	1560
Bulk modulus (MPa)	2200	0.15	2600

Echolocation of airborne objects

- Air-water boundary creates great impedance mismatch
- Most sound is reflected from boundary
- Quantify this?

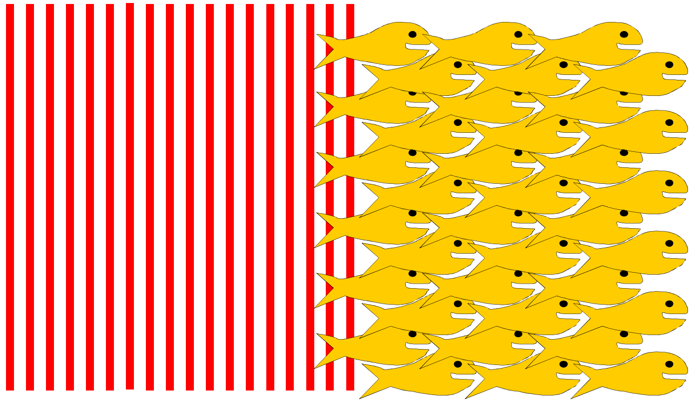


Ye *et al.* (1996): Acoustic scattering by fish in the forward direction

How does a fish school scatter?

- Multiple scattering inside fish school: diffusion of sound
- School fish as bulk inhomogeneous material: reflection
- As fish (parts) made smaller

- diffusion (causing attenuation) decreases (eventually)
- fish school becomes homogeneous medium



Acoustic concepts covered so far...
and their relevance to diagnostic ultrasound

- propagation of sound: ≈1540 m/s in soft tissue
- diffraction: focussing of mm-thick beams
- reflection and refraction: organ boundaries
- scattering: cells, collagen, elastin
- attenuation: ≈1 dB/cm/MHz

Let us review these concepts again...
and provide some additional notes

Investing in your future
New Hungary Development Plan

Propagation of pressure waves [Coussios 2005]

- Derivation of wave equation from the governing equations of acoustics:

  Eqn. of state (pressure function of density): $P(R)$

  Continuity eqn.: (mass rate of change in dV = flux in/out dV): $\partial R/\partial t = -\nabla\cdot(R\mathbf{v})$

  Momentum eqn. (Newton's second law of motion): $-\nabla P = \rho\, \partial\mathbf{v}/\partial t$

- Assuming small pressure and density fluctuations

  $$P = p_0 + p \text{ where } p << p;\ R = \rho_0 + \rho \text{ where } \rho << \rho_0$$

  Linearised eqn. of state: $p = (\kappa\rho_0)^{-1}\rho$ (compressibility $\kappa = 1/(-V\, \partial p/\partial V)$)

  Linearised continuity eqn.: $\partial\rho/\partial \mathrm{t} = -\rho_0 \nabla\cdot\mathbf{v}$

  Linearised momentum eqn.: $-\nabla p = \rho_0\, \partial\mathbf{v}/\partial t$

- Linear wave equation hence derived

$$\nabla^2 p = \nabla\cdot(\nabla p) = -\rho_0 \nabla\cdot(\partial\mathbf{v}/\partial \mathrm{t}) = \partial(-\rho_0\nabla\cdot\mathbf{v})/\partial t = \partial^2\rho/\partial t^2 = c^{-2}\, \partial^2 p/\partial t^2 \text{ where } c = (\kappa\rho_0)^{-1/2}$$

- Derivations in linear acoustics follow from these governing equations (e.g. formula on wave speed, acoustic impedance of a plane wave)

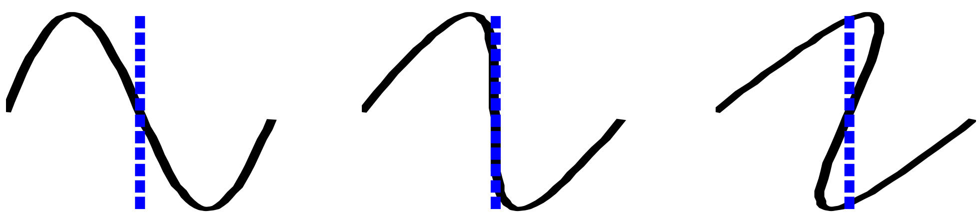
Non-linear propagation [Cobbold, pp. 228-237; Hill *et al.* 2004, pp. 34-35,*115]

- Non-linearity arises from two effects
  1. Medium non-linearity: $p = A(\rho/\rho_0) + B/2(\rho/\rho_0)^2 + \ldots$ (p non-linear function of ρ)
  2. Convective non-linearity: wave transported by particle motion
- For typical materials ($B/A>1$), both effects cause an increase of c with p:
  1. Medium non-linearity: medium less dense than expected
  2. Convective non-linearity: particle with forward motion carries pressure quicker

$$c = c_0 + \beta v = c_0 + (1+B/2A)v$$

where β is the coefficient of non-linearity (water:5.0 blood:6.3 liver:7.8 pig fat:11.1*)

- Pressure dependent wave speed causes distortion of waveform with distance
- As a result, waveform accumulates harmonics as it travels



(think of a loudspeaker placed in castor oil... what would happen as you increased the frequency?)

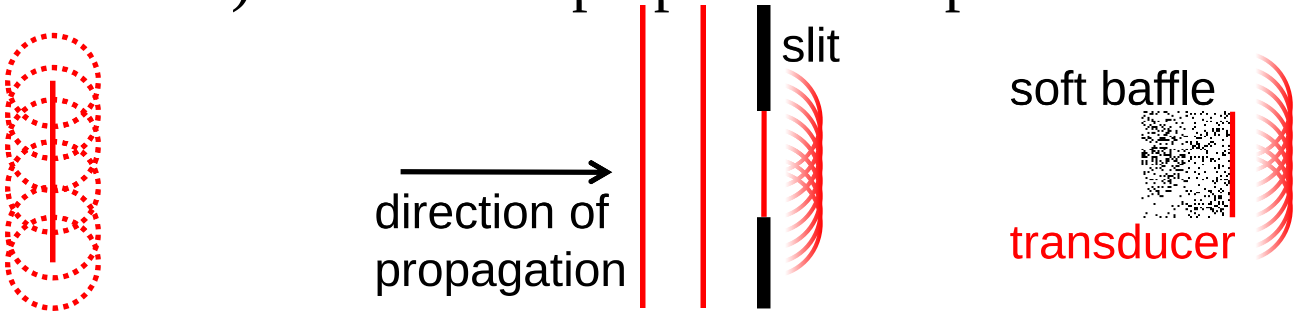
original waveform   shocked waveform   why does this not happen?

Non-linear processes in diagnostic ultrasound

- Non-linear propagation of ultrasound introduces harmonics into the wave as it propagates towards reflector/scatterer and back towards array, the degree of non-linear propagation being highest at the highest amplitude (focus)
- Pulse-echo imaging of such harmonics is called tissue harmonic imaging
- Air bubbles are highly non-linear scatterers, scattering sound at harmonics of the incident wave (for high enough amplitudes, they will scatter sound at the subharmonics, ultraharmonics and even in the broadband frequency range [Neppiras 1980])
- By introducing stabilised bubbles (ultrasound contrast agents) into bloodstream, perfusion can be imaged (contrast agent imaging)
- Harmonics can be recovered in several ways:
  - send one pulse and extract harmonic component of echo
  - send two pulses, one inverse of other, and consider difference between two echoes (pulse inversion)

Diffraction

- Huygen’s principle: each point of non-zero pressure field (such as wavefront) is itself a superposition of point sources



- But: consider a single planar source. As it spreads in two directions, the source won’t keep splitting in two!
- Modified Huygen’s principle: point sources have directivity given by *obliquity factor* (maximum at propagation direction)
- Application to ultrasound transducers: pressure field result of sum of (directional) point sources across transducer surface

Reflection and refraction

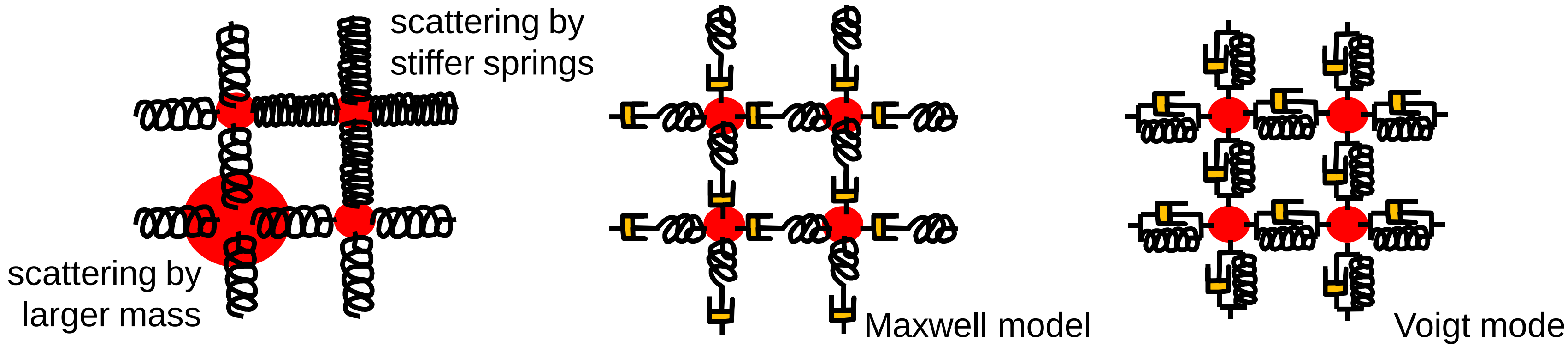
- Reflection and refraction governed by change in characteristic acoustic impedance $Z=\rho c$ across boundary.
- Ratio of pressure reflected: $(Z_2\text{-}Z_1)/(Z_1+Z_2)$
- Z has units of Rayls
- For planar waves, $p/|\mathbf{v}| = Z$, where **v** is velocity field

[Kaye&Laby]	Air	Water	Blood	Bone
Z (MRayl)	4e-4	1.5	1.1	3.5–4.6

- Over 99.9% of pressure is reflected at air-water boundary!
- Refraction governed by Snell's law: $\sin\theta_1/\sin\theta_2 = c_1/c_2$

Attenuation in simple conceptual terms

- Ordered vibrations of a wave gradually
  - re-transmitted in other directions (scattering)
  - turned into unordered, random mechanical (i.e. thermal) fluctuations (absorption)
- Simple model of wave propagation: particles held together by springs
- Wave propagation due to reaction force of springs and inertia of particles
- Scattering caused by variations in particle mass and spring stiffness
- Absorption: addition (series or parallel) of dashpots to springs [Gao *et al.* 1996]



Attenuation in tissue [Sehgal and Greenleaf 1984]

- Scattering from density and compressibility changes (*cf*. mass-spring model)
- Classical thermoviscous model: absorption arises from phase difference between p, ρ [Lighthill 2001 pp. 78-79]

$$p = c^2\rho + \delta\partial\rho/\partial t\text{; leading to } \partial\rho/\partial t - c^{-2}\,\partial\rho/\partial t + \delta c^{-2}\partial^3\rho/(\partial z^2\partial t)=0$$

- Such phase difference may arise from [Cobbold 2007, pp. 84-86]
  - heat conduction
  - viscosity
  - molecular (thermal and structural) relaxation
- Scattering: diffuse to diffractive single particules ($ka\leq 1$) $\alpha_s \sim f^{\,2\text{-}4}$ predicted
- Absorption: thermoviscous model predicts $\alpha_a \sim f^{\,2}$ (sim. to Kelvin-Voigt model)

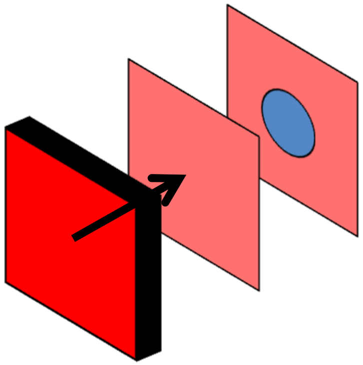
In contrast, α_s, α_a both ~ $f^{\,1.1\text{-}1.2}$ in tissue! Modify models:

- α_s : spatial auto-correlation for $\Delta\rho$,$\Delta\kappa$ [Sehgal and Greenleaf 1984]
- α_a: [Szabo 2004, pp. 77-83]; large mass-spring-dashpot arrangements [Gao *et al*. 1996]

Attenuation by single objects [Cobbold 2007, pp. 270-271]

- Consider intensity I plane wave impinging on object with cross-section (c.s.) A
- If object removes all incident intensity ("full attenuator") , $P_{removed} = IA$
- Object with c.s. A removes e.g. half of I acts like full attenuator of c.s. $A/2$
- Define acoustic c.s. as equivalent c.s. of full attenuator
- Total acoustic c.s. (area) sum of attenuation c.s. and scattering c.s.

$$\sigma = \sigma_a + \sigma_s = P_{removed}/ I$$



- Differential scattering c.s.(area/solid angle)

  $\sigma_{ds}(\theta) = P_s(\theta)/I$ (unlike attenuation, scattering θ-dependent)
- Differential backscattering c.s. (area /solid angle)

  $\sigma_{dbs} = \sigma_{ds}(\theta=[\pi\ 0])$ (arises in pulse-echo ultrasonics)
- Backscattering coefficient (area/solid angle/volume) [Cobbold 2007, p. 308]

  $\sigma_{BSC} = \sigma_{ds}(\theta=[\pi\ 0])/V$ (gives "density" of scattering)

References

[Au *et al.* 2007] Modeling the detection range of fish by echolating bottlenoise dolphins and porpoises

[Brunner 2002] Ultrasound system considerations and their impact on front-end components

[Cobbold 2007] Foundations of biomedical ultrasound

[Coussios 2005] Biomedical ultrasonics lecture notes

[Gao *et al.* 1996] Imaging of the elastic properties of tissue – a review

[Gyöngy 2010] Passive cavitation mapping for monitoring ultrasound therapy

[Hill *et al.* 2004] Physical principles of medical ultrasonics

[Kaye and Laby] Tables of physical and chemical constants. http://www.kayelaby.npl.co.uk/

[Lighthill 2001] Waves in fluids

[McLaughlin and Renzi 2006] Shear wave speed recovery in transient elastography and supersonic imaging using propagating fronts

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References

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[Neppiras 1980] Acoustic cavitation

[Olympus 2006] Ultrasonic transducers technical notes. http://www.olympus-ims.com/data/File/panametrics/UT-technotes.en.pdf

[Sehgal and Greenleaf] Scattering of ultrasound by tissues

[Sekuler and Blake 1994] Észlelés

[Ye and Farmer 1996] Acoustic scattering by fish in the forward direction

[Wells 1999] Ultrasonic imaging of the human body