ECONOMIC STATISTICS

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HUNGARY'S RENEWAL

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Week 4

Simple regression – fit, nonlinearity, confidence interval

Simple regression – reminder

- Regression model:

$$Y_i = \alpha + \beta X_i + e_i$$
$$Y_i = \hat{\alpha} + \hat{\beta} X_i + u_i$$

- Estimation: OLS

Example

70 tropical countries, relationship between X: population density (capita/1000 ha), and Y: deforestation rate (%)

	Coefficients
Intercept	0,60
X variable	0,001

Interpretation?

Measure of fit

- OLS: finding the best fitting line
- How good is the fit?
  - Measure: R^2
  - Simple (univariate) regression:

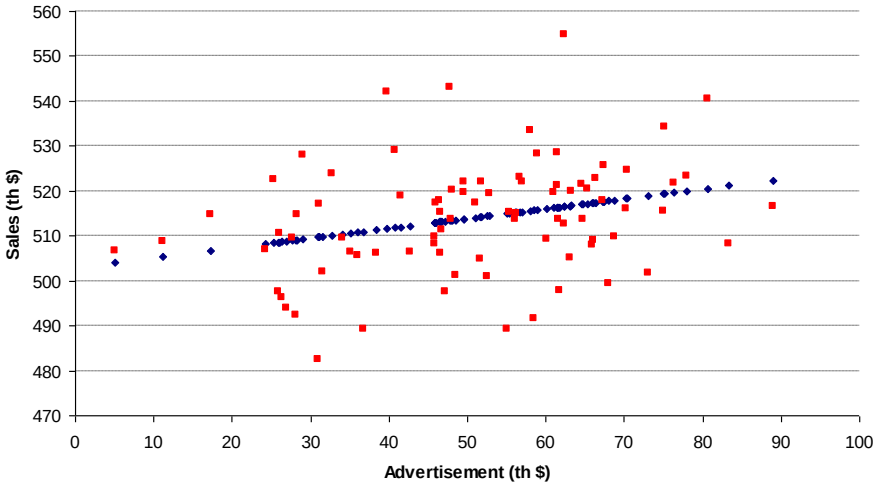
    square of correlation= R^2

Estimated value

- Regression line: $Y = \alpha + \beta X + e$
- Estimated/fitted/forecasted value: $\hat{Y} = \hat{\alpha} + \hat{\beta} X$

Comparison of the two – how good is the fit

Advertisement example:



Residual: $u = Y - \hat{Y}$

Residual vs. error term!

R2

To Total sum of squares : $TSS = \sum (Y_i - \bar{Y})^2$

Variance : TSS/(N -1)

Regression sum of squares : $RSS = \sum (\hat{Y}_i - \bar{Y})^2$

Sum of squared residuals : $SSR = \sum (Y_i - \hat{Y}_i)^2 = \sum u_i^2$

$$TSS = RSS + SSR$$

$$R^2 = 1 - \frac{SSR}{TSS} = \frac{RSS}{TSS}$$

Interpret R2

- What percentage of the variance of Y is explained by X

$$0 \leq R^2 \leq 1$$

- R^2=1 – perfect fit

Deforestation example

Regression statistics	
r-squared	0,434

ANALYSIS OF VARIANCIE

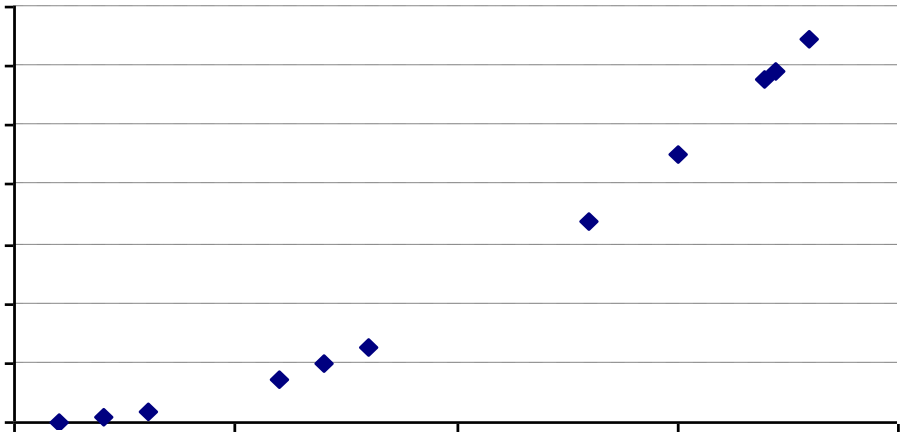
	df	*SS*
Regression	1	25,828
Residual	68	33,618
Total	69	59,446

Nonlinearity

Nonlinear relationship between X and Y

Common examples:

- Quadratic:



- Logarithmic

Logarithmic form

- Can ensure linear relationship
- Easy to interpret – elasticity:

$$\ln Y = \alpha + \beta \ln X$$

$$\beta = \frac{d \ln Y}{d \ln X}$$

If X increases by one %, Y increases by beta % on average

- Unit of measurement does not matter
- Approximation of % change: $100 \cdot d \ln Y$

Logarithmic form, cont.

How to interpret the slope coefficients?

$$Y_i = \alpha + \beta \ln X_i + e_i$$

$$\ln Y_i = \alpha + \beta X_i + e_i$$

Uncertainty

- Real values of the coefficients are unknown
- Estimated based on a sample
  - Estimated value is not exactly equal to the true value
- Point estimation: does not reveal the uncertainty

Factors influencing the precision of the OLS estimation

See textbook graphs

- More observations – more precise estimation
- Smaller error terms – more precise estimation
- Larger variance of X – more precise estimation
  - Example: effect of education level on income

Confidence interval

$$\left(\hat{\beta} - t_b s_b, \hat{\beta} + t_b s_b\right)$$

$$s_b = \sqrt{\frac{SSR}{(N-2)\sum (X_i - \overline{X})^2}}$$

s_b : standard deviation of $\hat{\beta}$

t_b : Student's t - distribution

Larger confidence level $\rightarrow$ larger t_b

More observations $\rightarrow$ smaller t_b

Interpretation

- Most common:
  95% confidence interval
  ”There is 95% chance that the true value of the coefficient lies in the given interval”
- Large N, 95%: t=1,96
- Table of t-distribution

- Excel: confidence level can be chosen

Deforestation example

	Coefficients	*Standard dev.*	*Bottom 95%*	*Top 95%*
Intercept	0,6000	0,1123	0,3758	0,8241
X variable	0,0008	0,0001	0,0006	0,0011

Summary

- Interpretation of estimated coefficients
- R-squared
- Nonlinearity, logarithmic form
- Uncertainty, confidence interval

Simple regression – fit, nonlinearity, confidence interval

Seminar 4

R2

$$\text{Total sum of squares}: TSS = \sum (Y_i - \bar{Y})^2$$

$$\text{Variance}: \text{TSS/(N -1)}$$

$$\text{Regression sum of squares}: RSS = \sum (\hat{Y}_i - \bar{Y})^2$$

$$\text{Sum of squared residuals}: SSR = \sum (Y_i - \hat{Y}_i)^2 = \sum u_i^2$$

$$TSS = RSS + SSR$$

$$R^2 = 1 - \frac{SSR}{TSS} = \frac{RSS}{TSS}$$

Interpret R2

- What percentage of the variance of Y is explained by X

$$0 \leq R^2 \leq 1$$

- R^2=1 – perfect fit
- Examples: advertisement regression, KSH unemployment rate regression

Examples for nonlinearity

Textbook: 4.5, 4.6

Uncertainty

- Real values of the coefficients are unknown
- Estimated based on a sample
  - Estimated value is not exactly equal to the true value
- Point estimation: does not reveal the uncertainty
- Confidence interval:

$$\left(\hat{\beta} - t_b s_b, \hat{\beta} + t_b s_b\right)$$

$$s_b = \sqrt{\frac{SSR}{(N-2)\sum (X_i - \overline{X})^2}}$$

Examples

- Advertisement – sales example: confidence interval of the estimated slope parameter (various confidence levels)

- Real estate prices – lot size (hprice.xls)

Homework 3 (groups)

- Analyze the relationship between two variables from a cross sectional sample (KSH, Eurostat, OECD, Penn World tables)
  - Descriptive statistics of both variables
  - Correlation
  - Regression
    - Functional form?
    - Fit?
    - Interpret the estimation results (confidence interval, as well)