ECONOMIC STATISTICS

Author: Anikó Bíró

Supervised by Anikó Bíró

June 2010

Week 11

AR models

AR(p) model

- Up to now: AR(1) model
  - Slope – stationarity
- AR(p) model: autoregression of order p

$$Y_t = \alpha + \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + e_t$$

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + ... + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$

$$\rho = \phi_1 + ... + \phi_p - 1$$

$\rho = 0$ – unit root $-2 < \rho < 0$ - stationary

AR(p) model – modified form

$$Y_t = \alpha + \phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p} + e_t$$

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \ldots + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$

$$\gamma_{p-1} = -\phi_p$$

$$\gamma_{p-2} = -\phi_p - \phi_{p-1}$$

$$\vdots$$

$$\gamma_1 = -\phi_p - \phi_{p-1} - \ldots - \phi_2$$

$$\rho = \phi_1 + \ldots + \phi_p - 1$$

Unit root

- Y has a unit root – cannot be included in the regression!
  Exemption: cointegration

- Differenced value (ΔY) has to be used!
- ΔY stacionary – Y difference stationary
- Y: has stochastic trend

Deterministic trend

- Example:

$$Y_t = \alpha + \phi Y_{t-1} + \delta t + e_t, \quad |\phi| < 1$$

- Y stationary – trend stationary
- Graph: similar to stochastic trend – not enough to make a decision on unit root

Example – AR(4) model

- AR(4) model with deterministic trend:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \gamma_3 \Delta Y_{t-3} + \delta t + e_t$$

- Generate differenced variables
- Differenced variables: 3 lags
- Trend: @trend
- Coefficient of $Y_{-1} = 0$?

Seasonality

- Pattern recurring at regular intervals
- Example: consumption, agricultural production, export
- Treatment: variables indicating seasonality
  - Quarterly: 3 dummies!
  - Monthly: 11 dummies!

  Or: seasonal adjustment
- KSH: seasonally adjusted time series

Specification choice

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + ... + \gamma_{p-1} \Delta Y_{t-p+1} + \delta t + e_t$$

- Maximal lag length (p_{max})
- Estimate AR(p_{max}) model with or without deterministic trend (according to the dependent variable, based on assumption!)
- Test $\Gamma_{pmax-1} = 0$ (t-test) – if satisfied: decrease lag length by one

Unit root test

Testing ρ=0: usual t-test cannot be used!

Dickey–Fuller-test: use t-statistic, but critical values are corrected

Problem: "weak" test – can find unit root even if it is not present

- Example: trend stationary time series, structural break

Dickey–Fuller-test

- Question: include trend?

- Null hypothesis: unit root
- Large p-value: has unit root, not stationary

Unit root test – example

Montly export data

- Seasonally adjusted
- Trend

Null Hypothesis: EXPORT_SA has a unit root

Exogenous: Constant, Linear Trend

Lag Length: 3 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.
Augmented Dickey–Fuller-test stat.		-2,1186	0,5310
Test critical values:	1% level	-4,0180	
	5% level	-3,4389	
	10% level	-3,1438	

Summary

- AR(p) model, modified form
- Unit root in AR(p) models
- Trend stationarity
- Seasonality
- Dickey–Fuller-test

AR models

Seminar 11

AR(p) model

- AR(p) model: autoregression of order p

$$Y_t = \alpha + \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + e_t$$
$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + ... + \gamma_{p-1} \Delta Y_{t-p+1} + e_t$$
$$\rho = \phi_1 + ... + \phi_p - 1$$

ρ=0 – unit root   -2<ρ<0 - stationary

Unit root

- Y has a unit root – cannot be included in the regression!

    Exemption: cointegration

- Differenced value (ΔY) has to be used!

- ΔY stationary – Y difference stationary
- Y: has stochastic trend

Example – monthly export

MNB data (m EUR)

- Estimation of AR(4) model with deterministic trend:

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \gamma_3 \Delta Y_{t-3} + \delta t + e_t$$

- Generate differenced variables
- Differenced variables: 3 lags
- Trend: @trend
- Coefficient of $Y_{-1} = 0$?

Seasonality

- Pattern recurring at regular intervals
- Treatment: variables indicating seasonality
  - Quarterly: 3 dummies
  - Monthly: 11 dummies

Seasonality – example

- Monthly export data – 11 seasonal dummies
- @seas(1) @seas(2) …
- 12 seasonal dummies: multicollinearity – EViews error message
- EViews: Procs/Seasonal adjustment

Specification choice

$$\Delta Y_t = \alpha + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + ... + \gamma_{p-1} \Delta Y_{t-p+1} + \delta t + e_t$$

- Maximal lag length (p_{max})
- Estimate AR(p_{max}) mode with or without deterministic trend
- Test $\Gamma_{pmax-1}=0$ (t-test) – if satisfied: decrease lag length by one
- Test the significance of trend after lag length selection
- Example: AR(p) model for first differenced log export time series (use seasonally adjusted data!)

Dickey–Fuller-test

- Test unit root
- View/Unit root test
- Option: automatic lag length selection
- Question: include trend?
- Null hypothesis: unit root
- Large p-value: has unit root, not stationary

Unit root test

- Monthly export data (MNB)
    - Seasonally adjusted
    - Trend?
    - Interpret output
    - Is the differenced variable stationary?
- Quarterly public debt data (MNB)
    - Trend?