Engineering optimization example

The title of example:	Optimization of the three bar-truss problem
Number of example:	OPT-BME-2
Level of example:	basic
Short description of the example:	In connection with the first chapter of the Engineering Optimization teaching material this example illustrates the definition of basic elements for solving an optimization problem. The definition of the design variables, optimization constraints and goal function is to be outlined. Application of only two design variables makes the graphical appearance possible.

1. Definition of the problem:

To illustrate the formulation of design variables, consider the symmetric three-bar truss example shown in the figure. This example was first presented by Schmit. The structural material is steel, and the structure is subjected to two distinct loadings, P_I, and P_{II}, respectively. The design parameters represent the material properties (modulus of elasticity, density, yield stress, etc.), the topology of the structure (three members between the nodes A-B, A-C, and A-D), and the coordinates of the nodes which are configurational parameters. The design variables represent the members' cross-sectional areas X_1, and X_2.

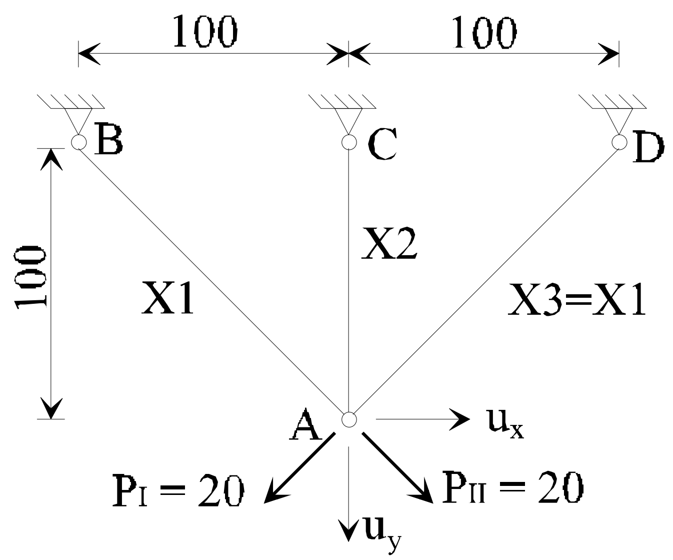
Applied loads: P_I, = 20 P_{II} = 20

Allowable stresses, tension (upper bound) and compression (lower bound) are the following:

$$\sigma^F = 20,\ \sigma^A = -15$$

Lower bounds on cross-sectional areas are: $X_1 \geq 0$ and $X_2 . \geq 0$.

The minimum weighted structure should be achieved by determining the optimal X_1, and X_2 values.



2. The steps of the solution:

1st step: Determining the design variables and design parameters

Due to symmetry of loading and geometry, the number of design variables is reduced to two (X_1, and X_2) and only one loading condition is considered. This made the graphical representation of the problem possible. From a mathematical point of view the design variables are categorized in the continuous type and from the physical point of view they belong to the cross sectional design variables. The all other properties of the structure are unchangeable (material properties and topology of the structure), so they are called design parameters or preassigned values.

2nd step: Definition of the optimization constraints

For this problem inequality type design constraints should be applied.

The applied two side constraints defines the lower bound of the design variables in inequality form (they are denoted with g):

$$g_1:\ -X_1 \leq 0$$

$$g_2:\ -X_2 \leq 0$$

There is no special condition for the upper bounds of the design variables, only the production possibilities delimit them. From the defined minimum weight problem, it is obvious that it is not necessary to prescribe upper bounds, because these constraints would never be active.

Behaviour constraints guarantee the functionality of the structure. In this example we have stress type behaviour constraints. All the three bars, the maximum allowable tension and compression we have stresses should be considered. This results all together 6 inequality behaviour constraints, which are denoted from $g_{3,}$ to g_8.

So the stress type behaviour constraints are the following:

$$g_3:\ \sigma_1 - 20 \leq 0$$

$$g_4:\ -\sigma_1 - 15 \leq 0$$

$$g_5:\ \sigma_2 - 20 \leq 0$$

$$g_6:\ -\sigma_2 - 15 \leq 0$$

$$g_7:\ \sigma_3 - 20 \leq 0$$

$$g_8:\ -\sigma_3 - 15 \leq 0$$

Stresses can be obtained in terms of the design variables in closed form without applying numerical simulation technique.

Only constraints which may affect the design must be considered. Since σ_1 and σ_2 will always be positive, and σ_3 negative, we should consider only the 3., 5. and 8. stress constraints.

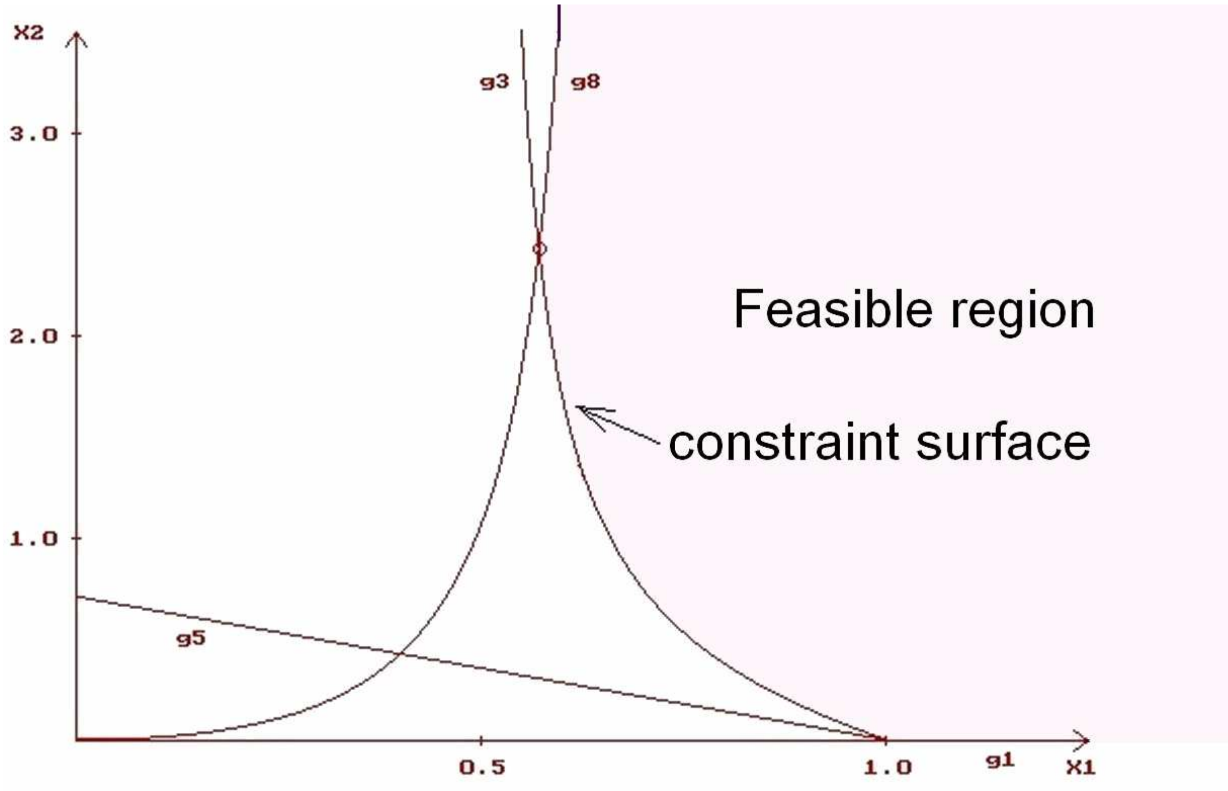
Since the structural analysis in this case can be performed analytically, the three mentioned stress constraints are the following:

$$g_3 : \sigma_1 - 20 = 20\frac{X_2 + \sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1^2} - 20 \le 0$$

$$g_5 : \sigma_2 - 20 = 20\frac{\sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1^2} - 20 \le 0$$

$$g_8 : -\sigma_3 - 15 = 20\frac{X_2}{2X_1X_2 + \sqrt{2}X_1^2} - 15 \le 0$$

The design space and the constraint surfaces for the three-bar truss example are shown in the following figure:



The set of all feasible designs form the feasible region. The portions of the respective constraint surfaces that bound the feasible region form the composite constraint surface. Points within the feasible region are called free points, or unconstrained designs. All structures, which are represented with unconstrained designs points, satisfy all the design requirements and generally there are infinite numbers of such structures. Points on the surface (i.e., feasible designs for which at least one $g_i(X) = 0$) are called bound points or constrained designs. It is possible that the feasible region is composed of two or more disjoint subregions, but this is rare in real engineering design problems. The subspace where two or more constraints $g_j(X)=0$ is called an intersection. In a two dimensional space, two constraints intersect in a point. If the constraint is violated, then the corresponding design is unfeasible.

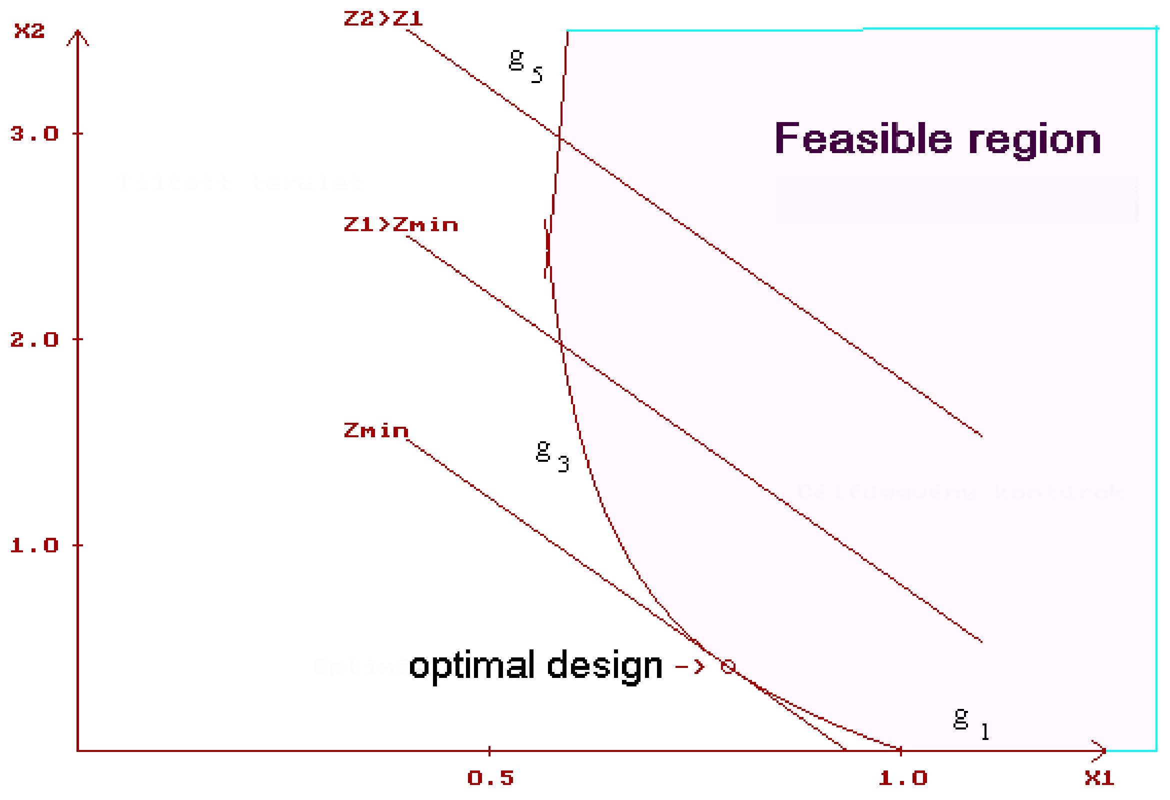
3th step: Definig the goal funtion

There usually exist infinite number of feasible designs. In order to find the best one, it is necessary to form a function of the variables to use for comparison of feasible design alternatives. In this example volume are chosen as goal function since the density of the structure is homogeneous.

The volume of the middle bar is $100 \cdot X_2$ and the volume of each side bar is $100 \cdot \sqrt{2} X_1$, so we can obtain the following goal function:

$$Z = 2 \cdot 100 \cdot \sqrt{2} X_1 + 100 \cdot X_2 = 282.8 X_1 + 100 X_2$$

All points satisfying F(X) =constant forms a surface. For each constant there corresponds a different member of a family of surfaces. Every design on a particular contour has the same volume. It can be observed that the minimum value of F(X) in the feasible region occurs at point, which represents the optimal design (shown in the figure below).



4th step: Determining the optimal structure

The graphical solution of this problem in the two-dimensional design variables space is shown in the last figure. The optimal design is feasible and has minimal value of the objective function in the marked point. The corresponding design variables are: $X_1 = 0.788$ $X_2 = 0.410$ and the minimum goal function value is: $Z = 263.9$. This type of graphical solution can be applied advantageously only for two (or maximum three) design variables. It is worth to notice, that in the optimum only one constraint is active.