

PROBLEM – 13.

ANALYTICAL AND FINITE ELEMENT SOLUTION OF A SIMPLY SUPPORTED PLATE SUBJECTED TO HYDROSTATIC LOAD

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13. ANALYTICAL AND FINITE ELEMENT SOLUTION OF A SIMPLY SUPPORTED PLATE SUBJECTED TO HYDROSTATIC LOAD

Calculate the deflection surface, the distribution of bending and twisting moments and stresses in the plate under hydrostatic load shown in Fig.13.1 by

a. analytical method utilizing the basic equations of thin plates,

b. finite element method using the code ANSYS 12,

and finally, make a comparison of the results obtained by the two different calculations! The material of the plate is linear elastic, homogeneous and isotropic.

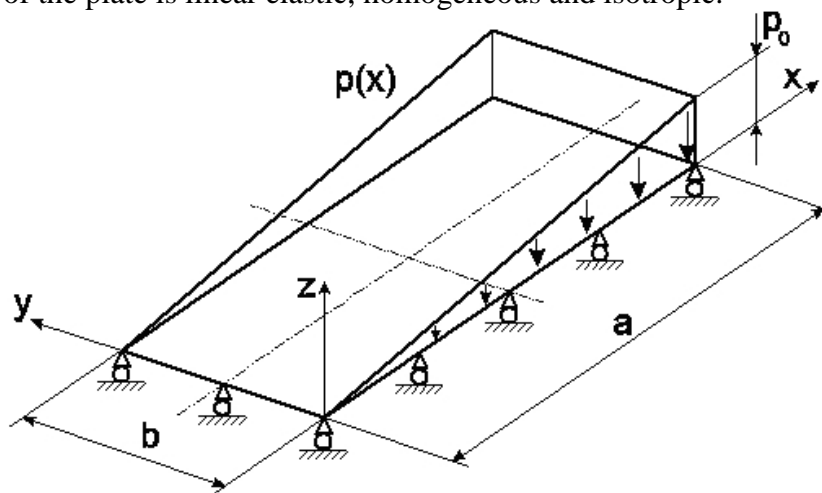


Fig.13.1. Simply supported plate subjected to hydrostatic load.

Given:

$E = 200 \text{ GPa}$, $\nu = 0,3$, $a = 500 \text{ mm}$, $b = 350 \text{ mm}$, $t = 2 \text{ mm}$, $p_0 = 10 \text{ kN/m}^2$.

13.1 Analytical solution

In section 12 we have already derived the governing partial differential equation for the deflection surface $w(x,y)$ of bent plates [1] (see Eq.(12.31)):

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y)}{I_1 E_1}, \quad (13.1)$$

where $I_1 = t^3/12$, $E_1 = (E/(1-\nu^2))$, $p(x,y)$ is the function of the load distributed on the plate surface. For simply supported plates the displacement and bending moment are zero on the corresponding boundaries, i.e. the boundary conditions are:

$$w(0,y) = 0, w(a,y) = 0, w(x,0) = 0, w(x,b) = 0, \quad (13.2)$$

$$M_x(0,y) = 0, M_x(a,y) = 0, M_y(x,0) = 0, M_y(x,b) = 0$$

Based on Eq.(12.7) (see section 12) the relationship between the bending moments and the deflection function is:

$$M_x = -I_1 E_1 (w_{,xx} + \nu \cdot w_{,yy}), M_y = -I_1 E_1 (w_{,yy} + \nu \cdot w_{,xx}), \quad (13.3)$$

$$M_{xy} = -I_1 E_1 (1-\nu) w_{,xy},$$

viz., the boundary conditions with respect to the bending moments can be reformulated using the deflection function. The boundary conditions can be satisfied if we develop the solution using a double sine series in terms of both the x and y variables. The solution function is [1,2]:

$$w(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \sin(\alpha x) \cdot \sin(\beta y), \quad (13.4)$$

where W_{mn} , α and β are the coefficients of the solution, and:

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b}. \quad (13.5)$$

The load function is provided in the form of a similar series:

$$p(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn} \sin(\alpha x) \cdot \sin(\beta y). \quad (13.6)$$

where Q_{mn} is the coefficient of the approximate function of the load. To obtain the solution we utilize the followings [1,2]:

$$\int_0^b \sin\left(\frac{n\pi y}{b}\right) \cdot \sin\left(\frac{l\pi y}{b}\right) dy = \begin{cases} 0, & \text{if } n \neq l \\ b/2, & \text{if } n = l \end{cases}. \quad (13.7)$$

We multiply $p(x,y)$ with $\sin(l\pi y/b)$ and then we integrate it from 0 to b with respect to y :

$$\int_0^b p(x, y) \cdot \sin\left(\frac{l\pi y}{b}\right) dy = \frac{b}{2} \sum_{m=1}^{\infty} Q_{ml} \cdot \sin\left(\frac{m\pi x}{a}\right) \quad (13.8)$$

Let us multiply both sides with $\sin(k\pi x/a)$ and integrate it with respect to x from 0 to a :

$$\int_0^a \int_0^b p(x, y) \cdot \sin\left(\frac{l\pi y}{b}\right) \cdot \sin\left(\frac{k\pi x}{a}\right) dy dx = \frac{ab}{4} Q_{kl}, \quad (13.9)$$

Since we can exchange the indices we obtain:

$$Q_{mn} = \frac{4}{ab} \int_0^a \int_0^b p(x, y) \cdot \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) dy dx. \quad (13.10)$$

Next, we calculate the derivatives of the solution function:

$$\begin{aligned} \frac{\partial^2 w}{\partial x^2} &= -\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \alpha^2 \sin(\alpha x) \cdot \sin(\beta y), \\ \frac{\partial^2 w}{\partial y^2} &= -\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \beta^2 \sin(\alpha x) \cdot \sin(\beta y), \\ \frac{\partial^2 w}{\partial x \partial y} &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \alpha \beta \cos(\alpha x) \cdot \cos(\beta y), \\ \frac{\partial^4 w}{\partial x^4} &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \alpha^4 \sin(\alpha x) \cdot \sin(\beta y), \\ \frac{\partial^4 w}{\partial y^4} &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \beta^4 \sin(\alpha x) \cdot \sin(\beta y), \\ \frac{\partial^4 w}{\partial x^2 \partial y^2} &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \alpha^2 \beta^2 \sin(\alpha x) \cdot \sin(\beta y). \end{aligned} \quad (13.11)$$

Taking the derivatives and the solution for $p(x,y)$ back into Eq.(13.1) we have:

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$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ -W_{mn} I_1 E_1 (\alpha^4 + 2\alpha^2 \beta^2 + \beta^4) + Q_{mn} \right\} \sin(\alpha x) \cdot \sin(\beta y) = 0, \quad (13.12)$$

and the equation above must be satisfied independently of the location, i.e. the nontrivial solution is:

$$W_{mn} (\alpha^2 + \beta^2)^2 = \frac{Q_{mn}}{I_1 E_1}, \quad (13.13)$$

and:

$$W_{mn} = \frac{Q_{mn}}{I_1 E_1 (\alpha^2 + \beta^2)^2} = \frac{Q_{mn}}{I_1 E_1 \pi^4 ((m/a)^2 + (n/b)^2)^2}. \quad (13.14)$$

Taking it back into the solution function we obtain the following:

$$w(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{Q_{mn}}{I_1 E_1 (\alpha^2 + \beta^2)^2} \sin(\alpha x) \cdot \sin(\beta y). \quad (13.15)$$

Accordingly, the solution function is obtained in terms of the coefficients of the load function. For the case of hydrostatic load the coefficients can be produced relatively easy based on Eq.(13.7) since $p(x,y) = p_0 \cdot x/a$ [1,2]:

$$Q_{mn} = \frac{4}{ab} \int_0^a \int_0^b p_0 \frac{x}{a} \cdot \sin\left(\frac{n\pi y}{b}\right) \cdot \sin\left(\frac{m\pi x}{a}\right) dy dx = -\frac{8p_0}{mn\pi^2} \cos(m\pi), \quad m, n = 1, 3, 5, \dots \quad (13.16)$$

The coefficients are summarized in Table 13.1.

| m | n | Q_{mn} |
|----------|----------|------------------------|
| 1 | 1 | $\frac{8p_0}{\pi^2}$ |
| 1 | 3 | $\frac{8p_0}{3\pi^2}$ |
| 3 | 1 | $\frac{8p_0}{3\pi^2}$ |
| 3 | 3 | $\frac{8p_0}{9\pi^2}$ |
| 3 | 5 | $\frac{8p_0}{15\pi^2}$ |
| 5 | 3 | $\frac{8p_0}{15\pi^2}$ |
| 5 | 5 | $\frac{8p_0}{25\pi^2}$ |

Table 13.1. Coefficients of the load function of a simply supported plate.

Calculate now the W_{mn} coefficients of the solution functions. The results are summarized in Table 13.2.

| m | n | W_{mn} |
|---|---|--|
| 1 | 1 | $\frac{8p_0}{I_1 E_1 \pi^6 ((1/a)^2 + (1/b)^2)^2}$ |
| 1 | 3 | $\frac{8p_0}{3I_1 E_1 \pi^6 ((1/a)^2 + (3/b)^2)^2}$ |
| 3 | 1 | $\frac{8p_0}{3I_1 E_1 \pi^6 ((3/a)^2 + (1/b)^2)^2}$ |
| 3 | 3 | $\frac{8p_0}{9I_1 E_1 \pi^6 ((3/a)^2 + (3/b)^2)^2}$ |
| 3 | 5 | $\frac{8p_0}{15I_1 E_1 \pi^6 ((3/a)^2 + (5/b)^2)^2}$ |
| 5 | 3 | $\frac{8p_0}{15I_1 E_1 \pi^6 ((5/a)^2 + (3/b)^2)^2}$ |
| 5 | 5 | $\frac{8p_0}{25I_1 E_1 \pi^6 ((5/a)^2 + (5/b)^2)^2}$ |

Table 13.2. Coefficients of the solution function of a simply supported plate.

It can be seen that we calculated only seven coefficients, with that the deflection function becomes:

$$\begin{aligned}
 w(x, y) = & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \sin(\alpha x) \cdot \sin(\beta y) = W_{11} \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{b}\right) + W_{13} \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{3\pi y}{b}\right) + \\
 & + W_{31} \sin\left(\frac{3\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{b}\right) + W_{33} \sin\left(\frac{3\pi x}{a}\right) \cdot \sin\left(\frac{3\pi y}{b}\right) + W_{35} \sin\left(\frac{3\pi x}{a}\right) \cdot \sin\left(\frac{5\pi y}{b}\right) + \\
 & + W_{53} \sin\left(\frac{5\pi x}{a}\right) \cdot \sin\left(\frac{3\pi y}{b}\right) + W_{55} \sin\left(\frac{5\pi x}{a}\right) \cdot \sin\left(\frac{5\pi y}{b}\right).
 \end{aligned}
 \tag{13.17}$$

The displacement in direction z in the midpoint of the plate was calculated by increasing the number of the considered coefficients. In the first case we considered only W_{11} , in the second case we included W_{11} and W_{13} too, etc. The results are summarized in Table 13.3 where it is shown that the displacement rapidly converges to a given value.

| number of W_{mn} coefficients [pcs] | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------------------------------|---------|---------|---------|---------|---------|--------|--------|
| $w(a/2, b/2)$ [mm] | -3.8388 | -3.8072 | -3.7102 | -3.7154 | -3.7148 | 3.7135 | 3.7138 |

Table 13.3. The convergence of the midpoint displacement of the plate increasing the number of coefficients.

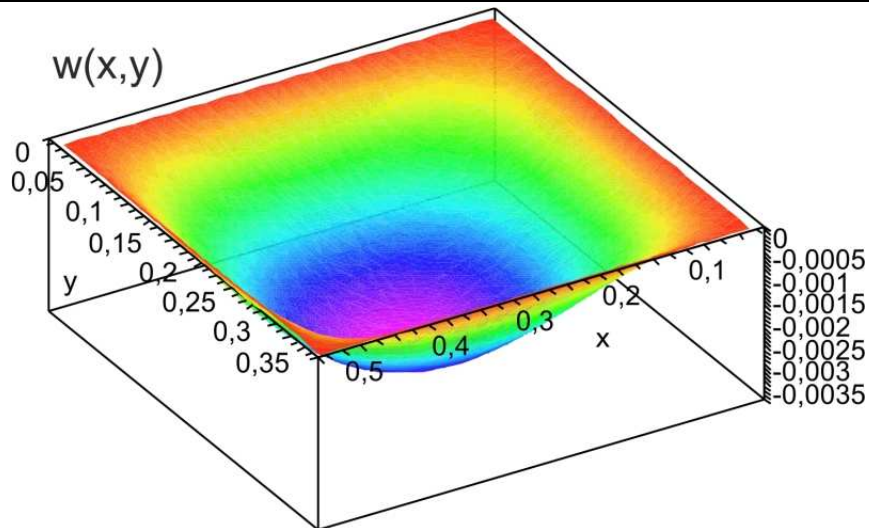


Fig.13.2. Deflection function of a simply supported plate subjected to hydrostatic load, the unit is in [m].

The deflection function is plotted in Fig.13.2, where all of the seven coefficients were taken into account. The maximum displacement takes place in the midpoint of the plate. The bending moments can be obtained using Eq.(13.3):

$$M_x = -I_1 E_1 (w_{,xx} + \nu \cdot w_{,yy}) = I_1 E_1 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} (\alpha^2 + \nu \beta^2) \sin(\alpha x) \cdot \sin(\beta y), \quad (13.18)$$

$$M_y = -I_1 E_1 (w_{,yy} + \nu \cdot w_{,xx}) = I_1 E_1 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} (\nu \alpha^2 + \beta^2) \sin(\alpha x) \cdot \sin(\beta y),$$

$$M_{xy} = -I_1 E_1 (1-\nu) w_{,xy} = I_1 E_1 (1-\nu) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn} \alpha \beta \cos(\alpha x) \cdot \cos(\beta y),$$

where M_x is the bending moment along x , M_y is the bending moment along y , M_{xy} is the twisting moment, respectively. The functions of the moments are shown by Figs.13.3-13.5.

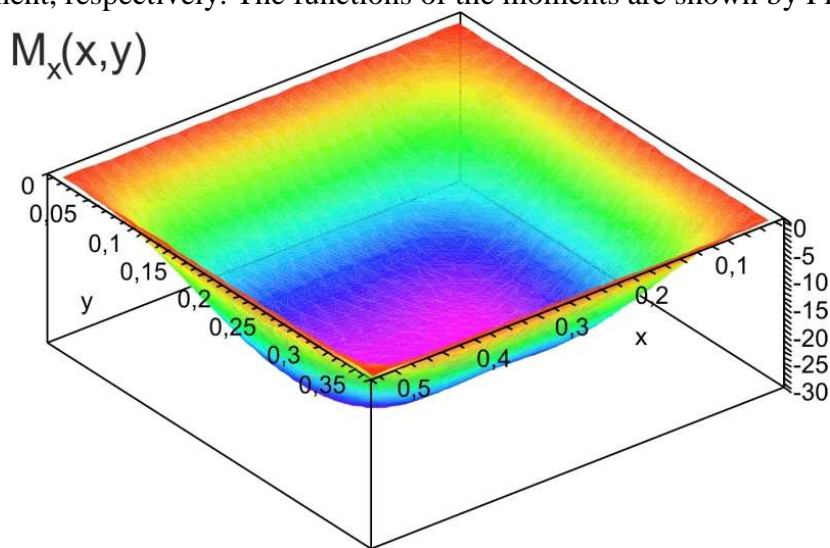


Fig.13.3. Distribution of the M_x bending moment in a simply supported plate subjected to hydrostatic load, the unit is in [Nm/m].

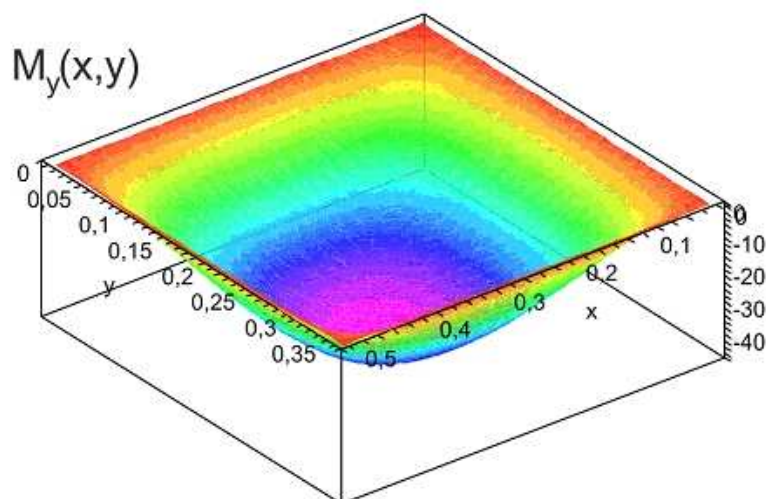


Fig.13.4. Distribution of the M_y bending moment in a simply supported plate subjected to hydrostatic load, the unit is in [Nm/m].

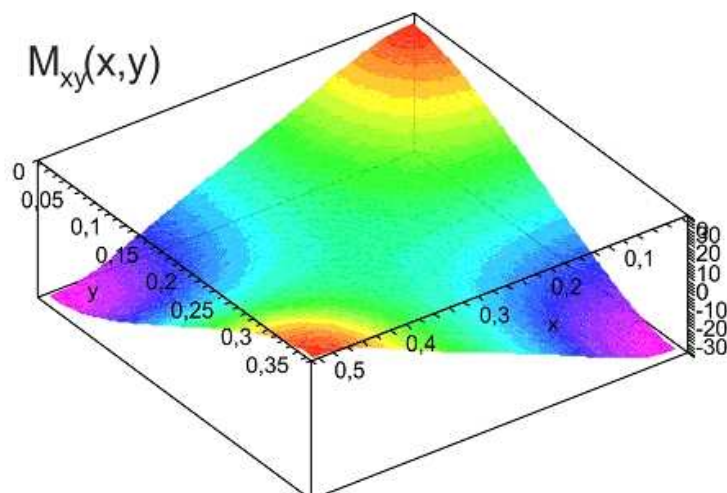


Fig.13.5. Distribution of the M_{xy} twisting moment in a simply supported plate subjected to hydrostatic load, the unit is in [Nm/m].

The shear forces can be calculated based on the equilibrium equations given by Eqs.(13.19) and (13.20) of section 14:

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = -I_1 E_1 (w_{,xx} + w_{,yy})_{,x} = -I_1 E_1 (\Delta w)_{,x}, \quad (13.19)$$

$$Q_y = \frac{\partial M_{yx}}{\partial x} + \frac{\partial M_y}{\partial y} = -I_1 E_1 (w_{,xx} + w_{,yy})_{,y} = -I_1 E_1 (\Delta w)_{,y}.$$

The function plots of the shear forces are given in Figs.13.6 and 13.7.

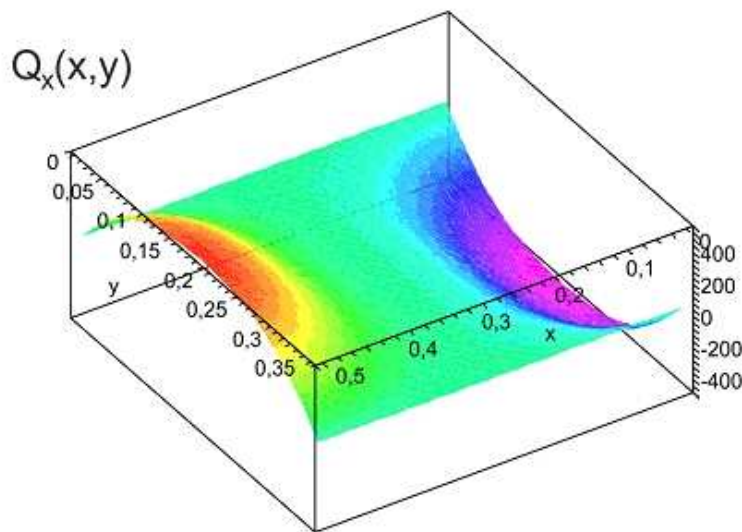


Fig.13.6. Distribution of the Q_x shear force in a simply supported plate subjected to hydrostatic load, the unit is in [N/m].

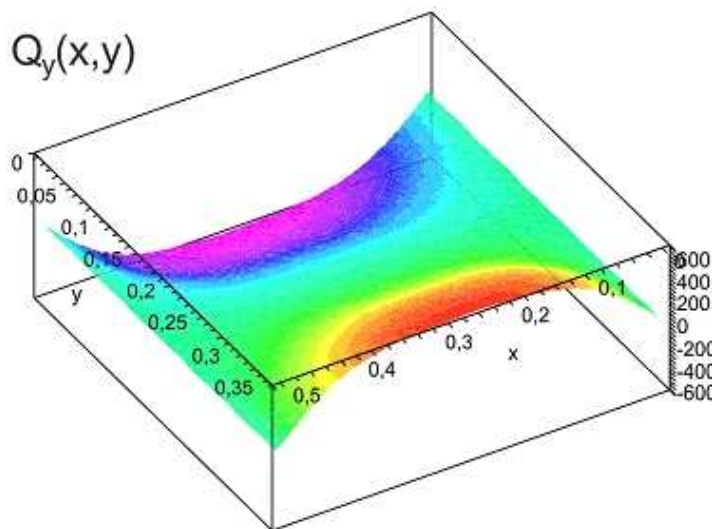


Fig.13.7. Distribution of the Q_y shear force in a simply supported plate subjected to hydrostatic load, the unit is in [N/m].

The stresses are obtained by using the following formulae [1,2]:

$$\sigma_x = \frac{M_x}{I_1} z, \quad \sigma_y = \frac{M_y}{I_1} z, \quad \tau_{xy} = \frac{M_{xy}}{I_1} z, \quad (13.20)$$

where $I_1 = t^3/12$ is the second order moment of inertia of the plate. Due to the fact that the stresses are proportional to the moments at a given z coordinate, the stress distributions are not given here. The normal stresses at the point with coordinate $z = -t/2$ in the midpoint of the plate are:

$$\sigma_x = \frac{M_x(a/2, b/2)}{I_1} \frac{t}{2} = -44,10 \text{ MPa}, \quad (13.21)$$

$$\sigma_y = \frac{M_y(a/2, b/2) t}{I_1} = -69,36 \text{ MPa} .$$

The maximum value of the shear stress takes place at the point where $x = a$ and $y = b$, i.e.:

$$\tau_y = \frac{M_{xy}(a, b) t}{I_1} = 53,38 \text{ MPa} . \quad (13.22)$$

13.2 Finite element solution

Solve the plate problem shown in Fig.13.1 by the finite element method too! Prepare the finite element model of the plate shown in Fig.13.1 and calculate the nodal displacements and stresses! Plot the distribution of the bending and twisting moments, normal and shear stresses on the surface of the plate!

The finite element solution is presented by using the code ANSYS 12. The actual commands are available in the left hand side and upper vertical menus [3]. The distances are defined in [m], the force is given in [N].

Printing the problem title on the screen

File menu / Change Title / Title: “*Simply supported plate subjected to hydrostatic load*”
- refresh the screen by the mouse roller

Analysis type definition

PREFERENCES – STRUCTURAL

Element type definition – 4 node shell element (SHELL63)

PREPROCESSOR / ELEMENT TYPES / ADD/EDIT/DELETE / ADD / SOLID / ELASTIC 4NODE 63 / OK /

PREPROCESSOR / REAL CONSTANTS / ADD/EDIT/DELETE / ADD / OK / Shell
thickness at node I TH(I) = 0.002 / OK - definition of the thickness

Material properties definition

PREPROCESSOR / MATERIAL PROPS / MATERIAL MODELS / STRUCTURAL /
LINEAR / ELASTIC / ISOTROPIC / EX = 200e9, PRXY = 0.3 / OK

Exit: Material menu / Exit

Geometry preparation

PREPROCESSOR / MODELING / CREATE / AREAS / RECTANGLES / BY 2 CORNERS / WPX = 0, WPY = 0, WIDTH = 0.5, HEIGHT = 0.35

- definition of the coordinates in the opening window

The surface of the plate is shown by Fig.13.8. Click the 9th icon on the right entitled „Fit View”, it fits the screen to the actual object size. Clicking the 1st icon on the right, entitled „Isometric View” we can view the model in 3D.

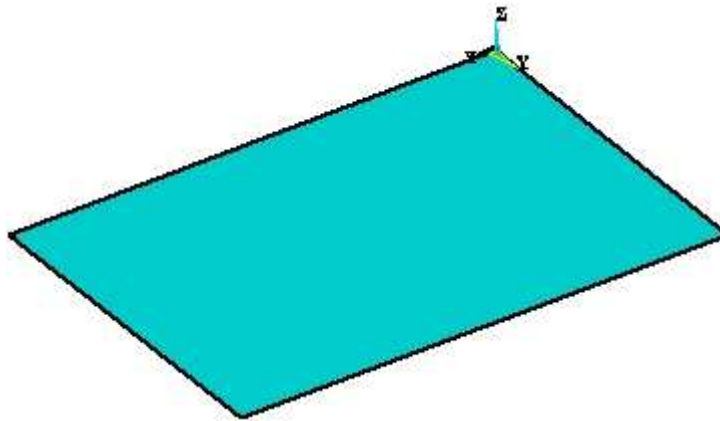


Fig.13.8. Surface of a simply supported plate.

Meshing

Element number definition along the boundary lines

PREPROCESSOR / MESHING / SIZE CNTRLS / MANUALSIZE / LINES / PICKED LINES / PICK / NO. OF ELEMENT DIVISIONS = typing in the proper number, repetition of the command

- along the edges parallel to axis x we define 50 elements
- along the edges parallel to axis y we define 35 elements

PREPROCESSOR / MESHING / MESH / AREAS / MAPPED / 3 OR 4 SIDED / PICK ALL

Plot menu / Multi-Plots - display the elements and nodes

The finite element mesh of the plate is demonstrated in Fig.13.9.

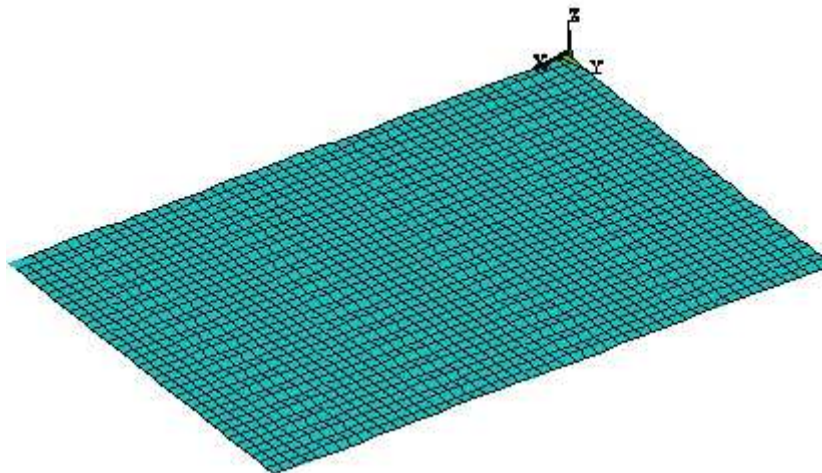


Fig.13.9. Finite element mesh of a simply supported plate.

Kinematic constraints definition

PREPROCESSOR / LOADS / DEFINE LOADS / APPLY / STRUCTURAL / DIS-PLACEMENT / ON LINES

- selection of the four boundary curves / OK / UZ / APPLY

- selection of the boundary curve at $x = 0$ / OK / UX / APPLY
- selection of the boundary curve at $y = 0$ / OK / UY / OK

The constraints are represented by small arrows acting in the proper directions, as it is shown by Fig.13.10.

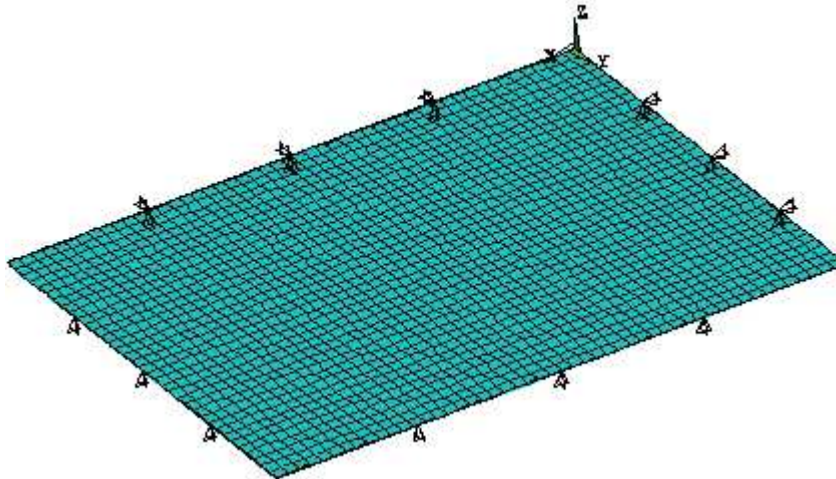


Fig.13.10. Kinematic boundary conditions of a simply supported plate.

Loading definition, linearly distributed load

The slope of the distributed load function is $p_0/a = 10000/0,5 = 20000 \text{ N/m}^3$

PREPROCESSOR / LOADS / DEFINE LOADS / SETTINGS / FOR SURFACE LD /
GRADIENT / SLOPE = - 20000 – value of the slope

SLDIR = X – the direction of the linear change

SLZER = 0 – coordinate location where slope contribution is zero

PREPROCESSOR / LOADS / DEFINE LOADS / APPLY / STRUCTURAL / PRESSURE /
ON AREAS / VALUE Load PRES Value = 0 (explanation: the slope and the “SLZER”
were defined before)

Display the arrows of the distributed load

PlotCtrls menu / Symbols / Surface Load Symbols: “Pressures”

Show pres and convect as: „Arrows”

Solution

SOLUTION / SOLVE / CURRENT LS

„SOLUTION IS DONE!”

Plotting and listing of the results

GENERAL POSTPROC / PLOT RESULTS / DEFORMED SHAPE / select Def + undef
edge / OK

PlotCtrls menu / Animate / Deformed Shape - animation

Nodal displacements, stresses and strains with color scale

GENERAL POSTPROC / PLOT RESULTS / CONTOUR PLOT / NODAL SOLU /

| | |
|----------------------------|--|
| NODAL SOLUTION: | nodal displacements |
| DOF SOLUTION: UX, UY, USUM | displacements with color scale |
| STRESS: | normal and shear stresses, principal stresses, equivalent stresses |
| ELASTIC STRAIN: | strains and shear strains, principal strains, equivalent strains |

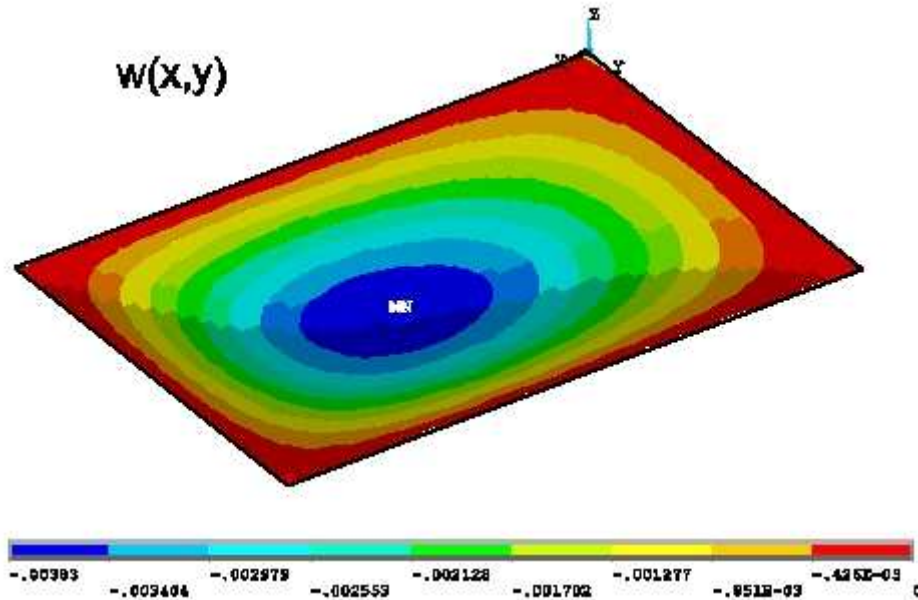


Fig.13.11. Transverse displacement (z) of a plate under hydrostatic load in [m].

Transverse displacement of the points of the plate is shown in Fig.13.11, where it is shown that the maximum displacement takes place in the midpoint of the plate, its value is: 3,83 mm.

GENERAL POSTPROC / PLOT RESULTS / CONTOUR PLOT / ELEMENT SOLU /

| | |
|-------------------|--|
| ELEMENT SOLUTION: | element (averaged) solutions |
| STRESS: | normal and shear, principal stresses, equivalent stresses, |
| ELASTIC STRAIN: | strains and shear strains, principal strains, equivalent strains |

Bending and twisting moments over the surface of the plate

Definition of element tables to display the moment distributions

GENERAL POSTPROC / ELEMENT TABLE / DEFINE TABLE / ADD

Lab – User label for item: MX

Item, Comp Result data item: By sequence number / SMISC, 4 / APPLY

Lab – User label for item: MY

Item, Comp Result data item: By sequence number / SMISC, 5 / APPLY

Lab – User label for item: MXY

Item, Comp Result data item: By sequence number / SMISC, 6 / OK

CLOSE

Display the content of element tables

GENERAL POSTPROC / ELEMENT TABLE / PLOT ELEM TABLE /

Itlab Item to be plotted

MX, v. MY, v. MXY

Avglab Average at common nodes?

Yes - average

The distribution of bending and twisting moments are shown in Figs.13.12-13.14.

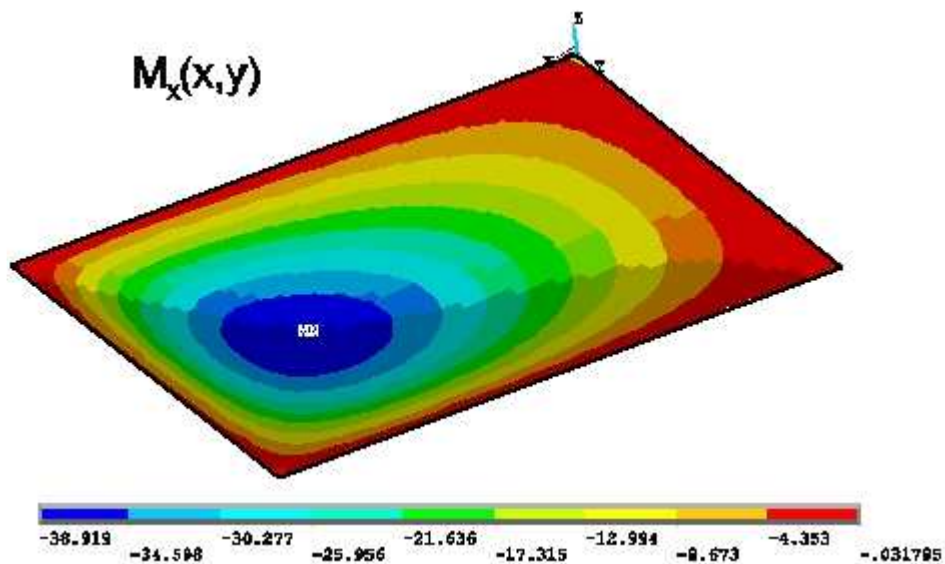


Fig.13.12. Distribution of the M_x bending moment in [Nm/m] in a simply supported plate under hydrostatic load.

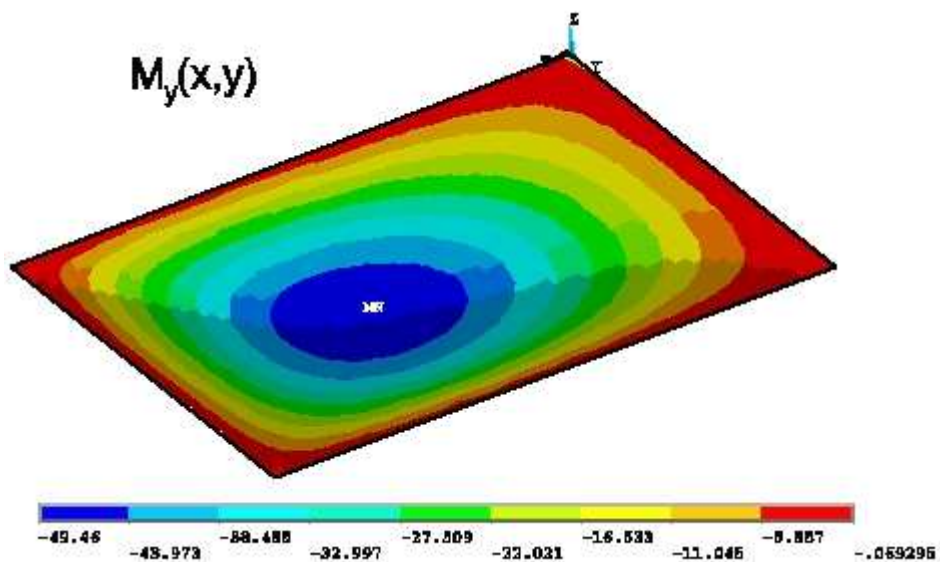


Fig.13.13. Distribution of the M_y bending moment in [Nm/m] in a simply supported plate under hydrostatic load.

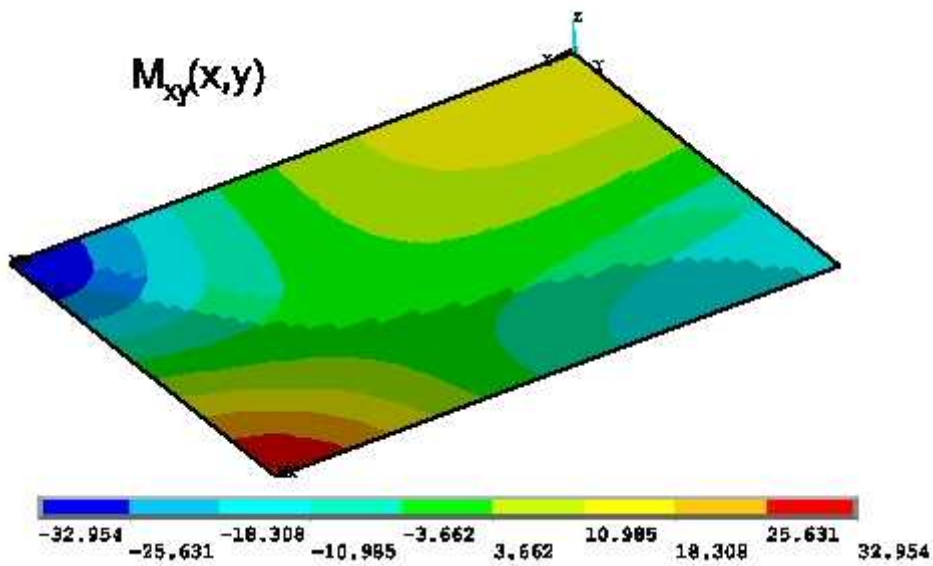


Fig.13.14. Distribution of the M_{xy} twisting moment in [Nm/m] in a simply supported plate under hydrostatic load.

The stress distributions based on the averaged nodal stresses are shown in Figs.13.15-13.17.

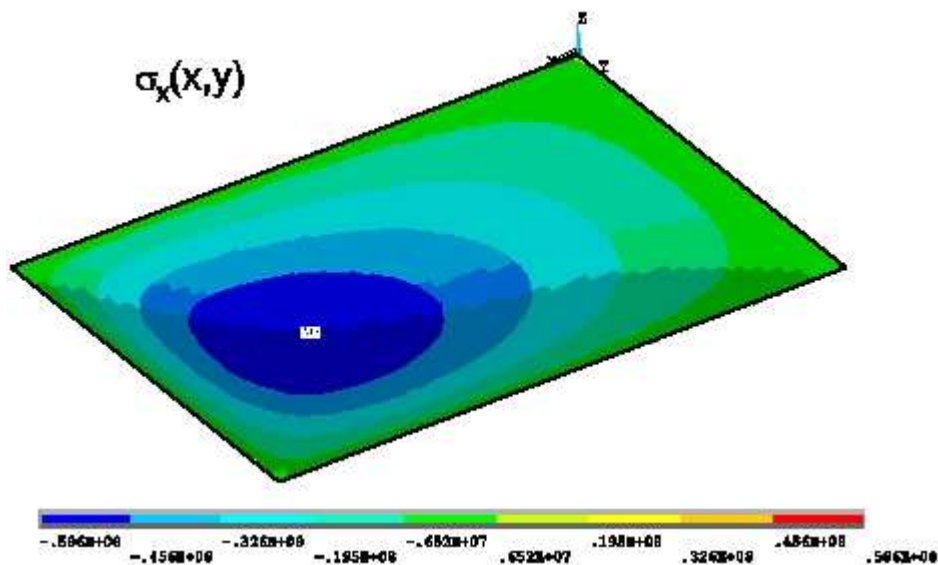


Fig.13.15. Distribution of the σ_x normal stress in [Pa] in a simply supported plate on the $z = t/2$ top surface under hydrostatic load.

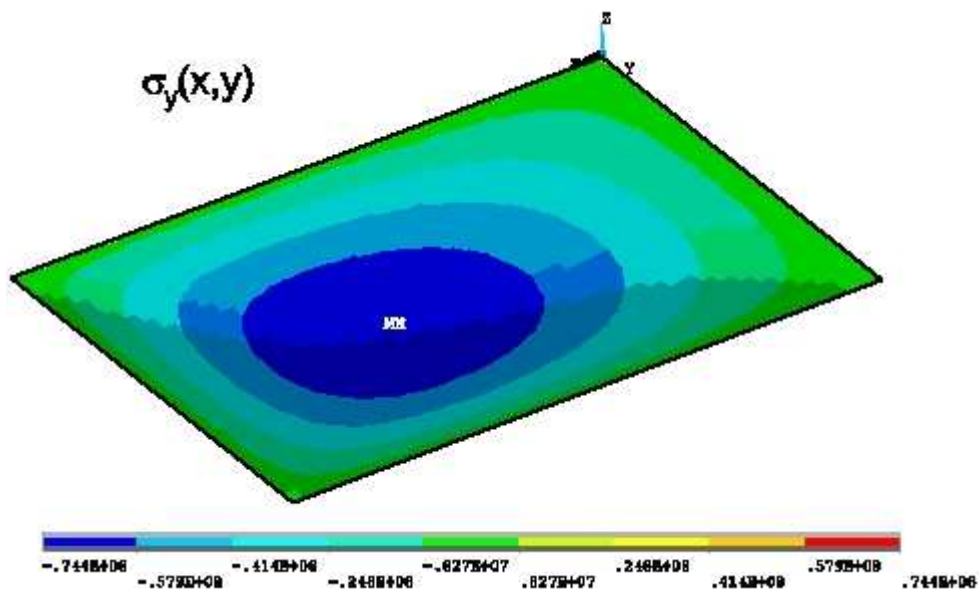


Fig.13.16. Distribution of the σ_y normal stress in [Pa] in a simply supported plate on the $z = t/2$ top surface under hydrostatic load.

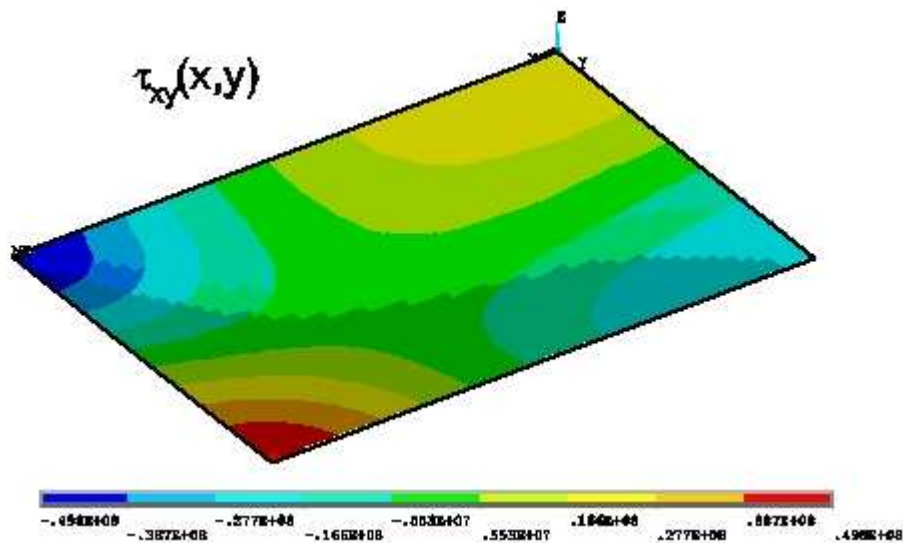


Fig.13.17. Distribution of the τ_{xy} shear stress in [Pa] in a simply supported plate on the $z = t/2$ top surface under hydrostatic load.

Deformation of the plate and the development of the normal stresses in x -direction can be seen in the attached animation files (*pt_anim_13-01.avi*, *pt_anim_13-02.avi*).

Listing the results

List menu / Results / Nodal solution / Nodal Solution
 / DOF solution / selection of the component
 / Stress / selection of the stress component
 / Elastic strain / selection of the strain component

- / Element solution – element solutions
- / Reaction solution – listing the reactions
- / Element Table Data – plot the content of the element tables

Identification of the results by the mouse

GENERAL POSTPROC / QUERY RESULTS / SUBGRID SOLU – selection of the component

In a separated window

GENERAL POSTPROC / RESULTS VIEWER – selection of the component

13.3 Comparison of the results by analytical and finite element solutions

The results obtained from the two different solutions are listed in Table 13.4. The displacement, bending moment and the normal stresses were calculated in the midpoint of the plate, the twisting moment and shear stress were determined at the corner point, respectively. Based on the table it is seen that there is a very good agreement between the analytical and finite element solutions. In each case the result by finite element solution is higher than that obtained from analysis, except for the case of the twisting moment and the shear stress.

| | Analytical solution | Finite element solution (SHELL63) | Difference [%] |
|--|---------------------|-----------------------------------|----------------|
| Displacement $w(a/2,b/2)$ [mm] | -3,714 | -3,83 | -3,03 |
| Bending moment $M_x(a/2,b/2)$ [Nm/m] | -29,40 | -30,61 | -3,95 |
| Bending moment $M_y(a/2,b/2)$ [Nm/m] | -46,24 | -47,15 | -1,93 |
| Twisting moment [Nm/m] $M_{xy}(a,b)$ | 35,59 | 32,95 | 8,01 |
| Normal stress $\sigma_x(a/2,b/2,t/2)$ [MPa] | -44,10 | -46,07 | -4,28 |
| Normal stress $\sigma_y(a/2,b/2,t/2)$ [MPa] | -69,37 | -70,93 | -2,20 |
| Shear stress $\tau_{xy}(a,b,t/2)$ [MPa] | 53,38 | 49,78 | 7,23 |

Table 13.4. Comparison of the solutions by analytical and finite element calculations.

13.4 Bibliography

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