Exercise in tensile testing

Exercise:

After tensile testing a round test specimen with d_0=10 mm initial diameter (L_0=10d_0) the next data were measured:

- offset yield point at 0,2% strain: $F_{p0,2}$=6,3 kN;
- rate of necking: Z=75%;
- ultimate force: F_m=11,5 kN.

Marks on the specimen with d_0 lenght were elongated as follows: 11, 11, 11, 12, 13, 16, 13, 12, 11, 11 mm.

Let us determine the value of offset yield strenght ($R_{p0,2}$), the true stress belonging to the ultimate force (σ_m), the engineering elongation (ε_m), and the maximum elongation in the true (φ_u) and engineering (A) system!

Solution:

The offset yield strenght in the engineering system:

$$R_{p0,2} = \frac{F_{p0,2}}{S_0} = \frac{F_{p0,2}}{\frac{d_0^2\pi}{4}} = \frac{6300\text{N}}{\frac{(10\text{mm})^2\pi}{4}} = 80{,}21\text{MPa} .$$

For the true ultimate tensile strenght we should calculate the cross sectional area belonging to the ultimate force (S_m). Calculating it we should use the volumetric constancy:

$$S_0 l_0 = S_m l_m .$$

In the equation above S_0 is the initial cross sectional area, S_m is the cross sectional area at the maximum force, l_0 is the initial lenght of marks on the specimen (equal to 10 mm), l_m is the lenght of marks after rupture, but not on the necking part (in this case l_m=11 mm).

So:

$$S_m = S_0 \frac{l_0}{l_m} = \frac{d_0^2\pi}{4}\frac{l_0}{l_m} = \frac{(10\text{mm})^2\pi}{4}\frac{10\text{mm}}{11\text{mm}} = 71{,}40\text{mm}^2 ,$$

and:

$$\sigma_m = \frac{F_m}{S_m} = \frac{11500\text{N}}{71{,}40\text{mm}^2} = 161{,}06\text{MPa} .$$

The engineering elongation in this part:

$$\varepsilon_m = \frac{l_m - l_0}{l_0}\cdot 100 = \frac{11\text{mm} - 10\text{mm}}{10\text{mm}}\cdot 100 = 10\% .$$

For calculating of the true elongation belonging to the last point we should calculate the cross sectional area of the specimen before rupture. For it we use the equation of necking rate:

$$Z = \frac{S_0 - S_u}{S_0} \Rightarrow S_u = S_0(1-Z) = \frac{d_0^2\pi}{4}(1-Z) = \frac{(10\text{mm})^2\pi}{4}(1-0{,}75) = 19{,}6349\text{mm}^2 .$$

and

$$\varphi_u = \ln\frac{S_0}{S_u} = \ln\frac{\left(\frac{d_0^2\pi}{4}\right)}{S_u} = \ln\frac{\left(\frac{(10\text{mm})^2\pi}{4}\right)}{19{,}6349\text{mm}^2} = 1{,}3863 .$$

The value of maximum elongation in the engineering system:

$$A = \frac{L_u - L_0}{L_0}\cdot 100 = \frac{121\text{mm} - 100\text{mm}}{100\text{mm}}\cdot 100 = 21\% .$$

Where L_u is the sum of the length of marks on the specimen after rupture (in this case: 121 mm – implicitly: L_0=100mm).