

POLITICAL ECONOMY

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Alternatives to the majority rule

Simple alternatives defined

- **Majority rule:** Choose the candidate who is ranked first by more than half of the voters.
- **Majority rule, runoff election:** If one of the m candidates receives a majority of first-place votes, this candidate is the winner. If not, a second election is held between the two candidates receiving the most first-place votes on the first ballot. The candidate receiving the most votes on the second ballot is the winner.
- **Plurality rule:** Choose the candidate who is ranked first by the largest number of voters.
- **Condorcet criterion:** Choose the candidate who defeats all others in pairwise elections using majority rule.
- **The Hare system:** Each voter indicates the candidate he ranks highest of the m candidates. Remove from the list of candidates the one ranked highest by the

fewest voters. Repeat the procedure for the remaining $m-1$ candidates. Continue until only one candidate remains. Declare this candidate the winner.

- **The Coombs system:** Each voter indicates the candidate he ranks lowest of the m candidates. Remove from the list of candidates the one ranked lowest by the most voters. Repeat the procedure for the remaining $m-1$ candidates. Continue until only one candidate remains. Declare this candidate the winner.
- **Approval voting:** Each voter votes for the k candidates ($1 \leq k \leq m$) he ranks highest of the m candidates, where k can vary from voter to voter. The candidate with the most votes is the winner.
- **The Borda count:** Give each of the m candidates a score of 1 to m based on the candidate's ranking in a voter's preference ordering; that is, the candidate ranked first receives m points, the second one $m-1$, the lowest ranked candidate one point. The candidate with the highest number of points is declared the winner.

A game

- Let's vote on which country should be the next European Champion in football (from this set):
 - England
 - Germany
 - Italy
 - Spain

Let us use different voting mechanisms.

The results:

	Majority	Maj. runoff	Plurality	Condorcet	Hare	Coombs	Approval	Borda
Eng.	0		x					x
Ger.	0	x		x	x	x	x	x
Ita.	0							
Sp.	0							

Which one is the good one?

We have a plethora of voting methods, so we will need some criteria to judge them by.

For two alternatives, $m=2$, they will produce the same result.

For $m>2$, majority and Condorcet might not deliver a solution at all.

Let us use the Condorcet criterion as a ... criterion.

Y is the Condorcet winner. But what about the other voting methods?

Plurality

V ₁	V ₂	V ₃	V ₄	V ₅
X	X	Y	Z	W
Y	Y	Z	Y	Y
Z	Z	W	W	Z
W	W	X	X	X

Majority, (runoff)

V ₁	V ₂	V ₃	V ₄	V ₅
X	X	Y	Z	z
Y	Y	Z	Y	Y
Z	Z	W	W	w
W	W	X	X	X

Does Hare deliver the Condorcet winner?

Hare

V ₁	V ₂	V ₃	V ₄	V ₅
Y	W	X	Y	W
X	Z	Z	Z	X
Z	X	W	X	Z
W	Y	Y	W	Y

Borda

V ₁	V ₂	V ₃	V ₄	V ₅
X	X	X	Y	Y
Y	Y	Y	Z	Z
Z	Z	Z	X	X

Simulations

Table 7.4. Condorcet efficiency for a random society (25 voters)

Voting system	Number of candidates				
	3	4	5	7	10
Runoff	96.2	90.1	83.6	73.5	61.3
Plurality	79.1	69.4	62.1	52.0	42.6
Hare	96.2	92.7	89.1	84.8	77.9
Coombs	96.3	93.4	90.2	86.1	81.1
Approval	76.0	69.8	67.1	63.7	61.3
Borda	90.8	87.3	86.2	85.3	84.3
Social utility maximizer	84.6	80.2	77.9	77.2	77.8

Source: Merrill (1984, p. 28).

Utilitarian efficiency

- „Tyranny of the majority”
 - Although Y would be picked by other methods, X is forced upon two voters w/ majority rule (Borda or approval would pick Y)
- Criterion: Utilitarian efficiency

V ₁	V ₂	V ₃	V ₄	V ₅
X	X	X	Y	Y
Y	Y	Y	Z	Z
Z	Z	Z	X	X

$$W = \sum_i U_i,$$

A closer look at the Borda count

It seems that Borda count fares well with Condorcet as well as with the utilitarian criterion.

You can have May-theorem like axiomatic approach that only Borda satisfies. Young's key assumptions (that only Borda satisfies):

Neutrality – issues/candidates do not matter.

Cancellation (anonymity) – voter order does not matter.

Faithfulness – if one voter votes the decision is her best element.

Consistency: ...

Borda count

- *Consistency : Let N_1 and N_2 be two groups of voters who are to select an alternative from the set S . Let C_1 and C_2 be the respective sets of alternatives that the two groups select using voting procedure B . Then if C_1 and C_2 have any elements in common (i.e., $C_1 \cap C_2$ is not empty), then the winning issue under procedure B when these two subgroups are brought together ($NT = N_1 \cup N_2$) is contained in this common set of elements ($CT = C_1 \cap C_2$).*

Is the majority rule consistent?

Table 7.6.

N_1			N_2			
V_1	V_2	V_3	V_4	V_5	V_6	V_7
z	x	y	z	z	x	x
x	y	z	x	x	y	z
y	z	x	y	y	z	y

Choice set (x,y,z) and (x,z) but N_1 and N_2 combined results in z.

Borda count and the tyranny of the majority

V_1	V_2	V_3	V_4	V_5
X	X	X	Z	Z
Y	Y	Y	X	X
Z	Z	Z	W	W
W	W	W	Y	Y

Simple majority and plurality always pick V_1 - V_2 - V_3 coalition choices.

Borda does not, but it is also open to strategic manipulation! (Table 7.2) – as is every other method.

Borda vs. approval

- Both score high on Condorcet and the utilitarian criterion as well.
- But whilst Borda is relatively hard to implement (e.g. many candidates, or unknown issues – think about the upcoming lg elections and all the candidates you would have to rank) approval voting is simple.

Further criteria and comparison

- http://en.wikipedia.org/wiki/Voting_system

Bottom line:

every voting system can be considered to be a criterion as well,

there is no one perfect mechanism,

so you have to tailor the voting rule to the exigencies of the situation.

Complicated alternatives to the majority rule

Is that all we have?

- In a way, voting systems are still simple,
- and, as we have seen, they leave a lot to be desired.
- They are also limited in terms of the information they let aggregate.
- So a lot of further mechanisms have been proposed,
- although their use in practice remains limited.

The valuation – Revelation challenge

- Challenge: to make people (who have to pay for it, too) to truthfully reveal their valuations of a public good and thus arrive at a socially optimal result.
- Requirements:
 - Enough tax be raised,
 - the alternative with the highest (aggregate, social) valuation be chosen,
 - so that everyone follows their interests (cf. strategic voting).

Isn't this like game theory?

- Yes it is. We are designing games here. So let us use game theory, or, rather, implementation theory to analyze what is going on here.
- (Separate handout on implementation theory, the revelation theorem and the Groves-Clarke mechanism, based on chapter 10 of Martin J. Osborne and Ariel Rubinstein: A Course in Game Theory (Cambridge, MIT Press, 1994))

Implementation theory

Consider a simple, normal form, two player game like this:

	Player 2	Action b1	Action b2
Player 1			
Action a1		2	3
		2	0
Action a2		0	1
		3	1

Formal representation

- Players: Player 1, Player 2
- Actions available $A_1=\{a_1,a_2\}$, $A_2=\{b_1,b_2\}$,
- Consequences $C=\{C_1, C_2, C_3, C_4\}$
- Outcome function $g(a_1,b_1) =\{C_1\}$, $g(a_1,b_2) =\{C_2\}$, $g(a_2,b_1) =\{C_3\}$, $g(a_2,b_2) =\{C_4\}$,
- Preference orderings

$C_3 \text{ Pref}_1 C_1 \text{ Pref}_1 C_4 \text{ Pref}_1 C_2$

$C_2 \text{ Pref}_2 C_1 \text{ Pref}_2 C_4 \text{ Pref}_2 C_3$

- Solution concept: DSE (dominant strategy equilibrium) or Nash of this game is:
 $(a_2,b_2)\dots$

The Groves–Clarke mechanism

- For a choice between the status quo and a costly alternative.
- Everyone is required to announce their valuations for the alternative.
- Those are aggregated and the decision to chose made based on them in a specific way:
- The decision if to go for the alternative is determined by whether the sum of valuations exceeds the cost of the alternative.
- Amount to contribute only depends on the decision for or against the alternative and others' valuations (e.g. the shortfall between the cost and the sum of the others' valuations plus a constant term).