

KOVÁCS ENDRE, PARIPÁS BÉLA,

PHYSICS II.

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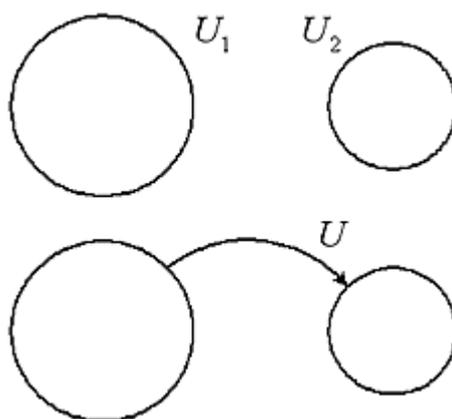
A Műszaki Földtudományi Alapszak tananyagainak kifejlesztése a
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II. ELECTRIC CURRENT

1. BASIC CONCEPTS

The free electrons in an isolated metallic conductor are in random motion; their motion has no net direction. If we consider a hypothetical surface in the conductor, the rate at which electrons pass through it from right to left is the same as from left to right, so the net rate is zero.

Consider now two charged conductors. Suppose that their potentials are different: let $U_1 > U_2$.



If we connect the two conductors with a conductor wire, then charge flows through the wire until the potential of the united conductor is equalized. The charge transfer is directed from the higher potential to the lower one. The motion of charge carriers during this rearrangement is temporary or transient. Due to the potential difference an electric field will be set up at every point within the wire, and this field will act on the charge carriers and give them a resultant motion. We can say that an ordered motion is superposed onto the irregular motion of the carriers.

The electric current is defined as the ordered motion of electric charges. Due to agreement the direction of the current is the real or imaginary direction of motion of the positive charge carriers.

In a conductor, a charge collides randomly with the atoms, but due to the field it accelerates until it collides with another particle. The charged particle thus gives up some of its kinetic energy, accelerates again until it collides, and so on. This is a random motion with a gradual drift in the direction of the field. The inelastic collisions with the stationary particles transfer energy to them: this increases their vibration energy and hence the temperature of the conductor.

Current density and Current

First we introduce the **current density vector** \vec{j} . The magnitude of the current density vector numerically equals the net charge flowing through a unit perpendicular (to the flow) area in unit time. The direction of \vec{j} is taken as the velocity of the ordered motion of the positive charge carriers (or opposite to the velocity of the ordered motion of the negative charge carriers).

Unit of the current density:

$$[J] = 1 \frac{C}{m^2 s} = 1 \frac{A}{m^2}.$$

The current density vector consists of two parts:

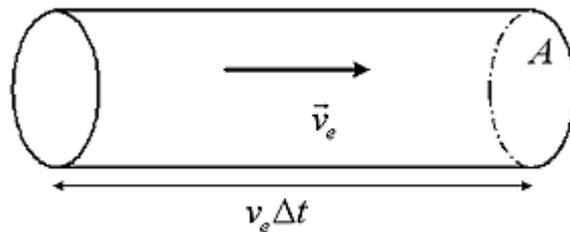
$$\vec{J} = \rho \vec{v} + \vec{j}$$

- $\rho \vec{v}$: convection current density is associated with beams of charged particles or motion of charged insulator, ρ is the volume charge density and \vec{v} is its velocity
- \vec{j} : conduction current density

Consider now a crystallized or metallic conductor at rest. As the volume charge density $\rho = 0$, there is no convection current density. It is easy to see that the conduction current density:

$$\vec{j} = -en_e\vec{v}_e,$$

where $e = 1.6 \cdot 10^{-19} \text{ C}$ is the elementary charge (the magnitude of the charge of an electron), n_e is the number density of the free conducting electrons, \vec{v}_e is the average drift speed of the electrons.



After a time interval Δt all the free electrons within the volume $A v_e \Delta t$ will have passed through the end face.

The charge will have passed:

$$\Delta Q = en_e v_e \Delta t A$$

The current density:

$$|\vec{j}| = \frac{\Delta Q}{A \Delta t} = \frac{en_e v_e \Delta t A}{A \Delta t} = en_e v_e$$

$$\vec{j} = -en_e \vec{v}_e$$

The direction of the current density is opposite that of the electron average drift speed. If we know the current density vector at each point along a surface A , we can calculate the **electric current** passing through this surface:

$$I = \int_A \vec{j} \cdot d\vec{A}$$

Current is a scalar algebraic quantity. The sign of the current is determined by the choice of the surface normal vector $d\vec{A}$. The current through an area is the net charge flowing through the area in unit time. This is the rate of flow of charge.

$$I = \frac{dQ}{dt}$$

The charge Q that passes a given cross-section in a given time interval is given by:

$$Q = \int_{t_1}^{t_2} I dt$$

Units:

$$[I] = 1 \frac{\text{C}}{\text{s}} = 1 \text{ A}$$

$$[J] = 1 \frac{A}{m^2}$$

Electromotive Force (EMF)

If only an electric field acts on the charge carriers, the charge flows through the wire from the higher potential to the lower potential (positive carriers) and this would lead to the equalization of potentials and stopping of the current.

To maintain a current for sufficiently long time it is necessary to have the aid of forces of non-electrostatic origin, called extraneous forces. The extraneous forces move the positive charge carriers from the lower potential back to the higher potential. These extraneous forces may be of chemical nature in a battery or cell, or magnetic nature in an alternating-current generator. Extraneous force appears as the magnetic field varies with time. (This will be discussed later.) The device in which extraneous forces act to maintain the potential difference is called a seat of electromotive force. It has two terminals, the higher potential is called the positive terminal and the lower potential is called the negative terminal.

The symbol of the extraneous force is: \vec{F}^* .

The strength of the extraneous field is:

$$\vec{E}^* = \frac{\vec{F}^*}{q}$$

The work done by the extraneous forces within the seat of emf between the two terminals:

$$W_{-+}^* = \int_{-}^{+} \vec{F}^* \cdot d\vec{r} = \int_{-}^{+} q \vec{E}^* \cdot d\vec{r}$$

The **electromotive force** emf is defined as the work done by the extraneous forces on a unit positive charge as it moves between the terminals:

$$\mathcal{E}_{-+} = \frac{W_{-+}^*}{q} = \int_{-}^{+} \vec{E}^* \cdot d\vec{r}.$$

Unit:

$$[\mathcal{E}] = 1 \frac{J}{C} = 1V = 1\text{volt}.$$

We suppose that the emf is independent of the path taken within the seat of emf. A section of a circuit on which no extraneous force $\vec{E}^* = 0$ acts is called homogeneous (metallic conductor). The section in which the charge carriers experience extraneous force is called inhomogeneous (seat of emf or current source), $\vec{E}^* \neq 0$. In the metallic conductor the current flows from the higher potential to the lower, while in the seat of emf the current flows from the lower potential to the higher.

The integral form of Ohm's Law

Ohm's Law was discovered experimentally: the current passing through a homogeneous conductor is proportional to the potential difference across its terminals. Their ratio is called the electric resistance of the conductor R .

$$R = \frac{U}{I}$$

Unit of resistance:

$$[R] = 1 \frac{V}{A} = 1\Omega = 1\text{ohm}.$$

Multiloop Circuits, Kirchoff's Rules

The calculation of multiloop circuits or network is considerably simplified if we use two rules formulated by Kirchoff.

First we define some concepts:

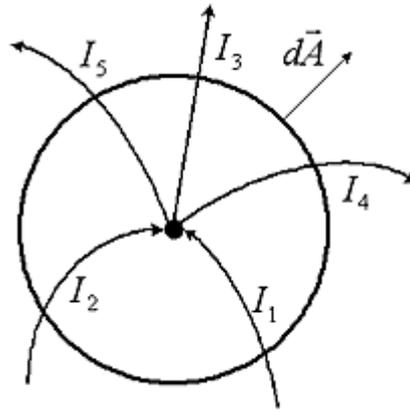
- The branch point or junction in a network is a point where three or more conductors are joined.
- The branch is a part of the network whose end points are junctions but there is no inner junction on it. In a branch the current is the same at all points. The circuit elements in a branch are connected series.
- A loop is any closed conducting path.
- The electrical consumers are connected in parallel if their terminals are at the same potentials.

Kirchoff's First Rule

In the case of a stationary or steady-state current the conservation of charge is described by the simplified continuity equation:

$$\oint_A \vec{J} \cdot d\vec{A} = 0$$

Consider the next closed mathematical surface:



$$I > 0 \text{ if } \vec{J} \cdot d\vec{A} > 0$$

$$I < 0 \text{ if } \vec{J} \cdot d\vec{A} < 0$$

In the example: $I_3 + I_4 + I_5 - I_1 - I_2 = 0$.

In general:

$$\sum_{i=1}^n I_i = 0$$

The algebraic sum of the currents at a junction is zero.

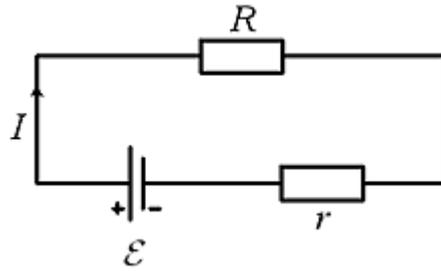
Kirchoff's Second Rule

The algebraic sum of the potential differences in any loop must equal zero.

$$\sum_{i=1}^n U_i = 0$$

Integral form of Ohm's Law for a closed circuit

Consider now a closed circuit consist of a seat of emf and a consumer. Denote the emf by \mathcal{E} and the resistance of the seat of emf by r , called internal resistance. The figure shows the internal resistance r and the emf separately, although they occupy the same region of space. The continuous resistance of the external circuit is denoted by R and symbolized by a rectangle, the straight lines are conductors having negligible resistance.



Use \mathcal{E} for the electromotive force:

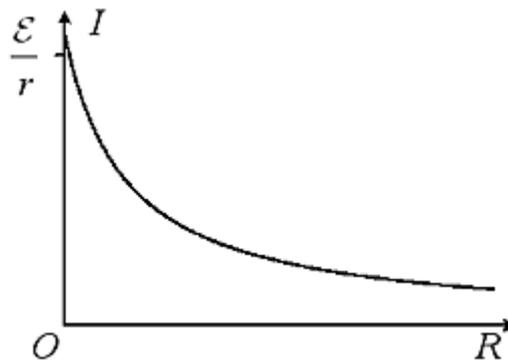
$$\mathcal{E} = I(R+r) \rightarrow I = \frac{\mathcal{E}}{R+r}$$

This is Ohm's Law for a complete circuit. The current equals the emf of the source divided by the total circuit resistance, external plus internal.

The terminal potential difference under closed circuit conditions:

$$U_x = IR = \mathcal{E} - Ir$$

The terminal potential difference is less than the emf \mathcal{E} if $I \neq 0$. The variation of the current I as a function of the external resistance R is shown in the next figure:

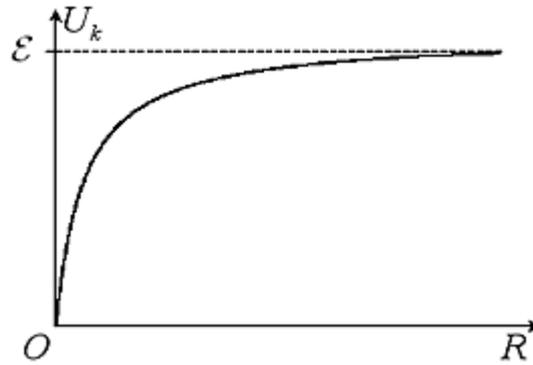


If the terminals of a source are connected by a conductor of zero (or negligible) resistance, the source is said to be short circuited. The short-circuit current is:

$$I_s = \frac{\mathcal{E}}{r}$$

The terminal voltage:

$$U_x = IR = \mathcal{E} \frac{R}{R+r}$$

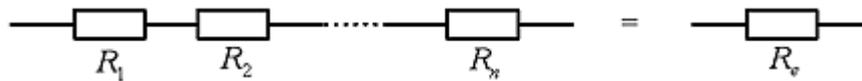


- if $R = 0$ or $R \ll r$, then it is a short circuit, and $U_k = 0$,
- if $R = \infty$ or $R \gg r$, then it is an open circuit, and $U_k = \mathcal{E}$.

The terminal potential difference of a battery is less than \mathcal{E} unless the battery has no internal resistance ($r = 0$), or if it is an open circuit ($R = \infty$), then the terminal potential difference is equal to \mathcal{E} .

Applications of Kirchoff's Rules

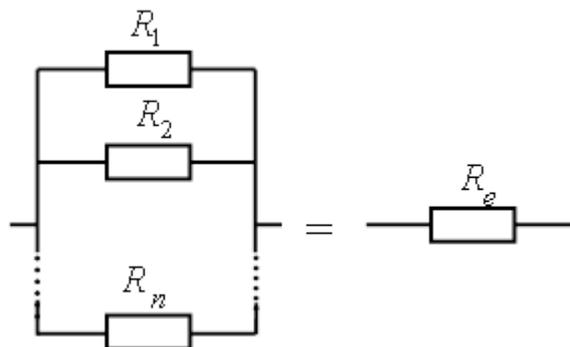
Series combination of resistors



$$R_e = \sum_{i=1}^n R_i$$

The equivalent resistance of any number of resistors in series equals the sum of their individual resistances.

Parallel combination of resistors



$$\frac{1}{R_e} = \sum_{i=1}^n \frac{1}{R_i}$$

For any number of resistors in parallel, the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances.

2. THE DIFFERENTIAL OHM'S LAW

Resistance as the function of geometrical sizes

As was shown previously, Ohm's Law was discovered experimentally: the current passing through a homogeneous conductor is proportional to the potential difference across its terminals. Their ratio is the electric resistance of the conductor R .

$$R = \frac{U}{I}$$

Consider a homogeneous cylindrical conductor of length l , cross-sectional area A . The increase of the length of the conductor by a factor of two is the same as the connection of another conductor in series to the original one, so the resistance is also increased by a factor of two. Therefore the resistance is proportional to the length of the wire. The increase of the cross-section of the conductor by a factor of two is the same as the connection of another conductor in parallel, so in this case the resistance is decreased by half. Therefore the resistance is inversely proportional to the cross-section of the conductor.

$$R = \rho \frac{l}{A}$$

The proportionality factor ρ is termed the resistivity of the substance, its unit is Ωm or $\frac{\Omega mm^2}{m}$.

The resistivity depends on the:

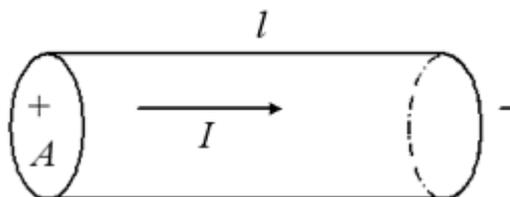
- substance,
- presence of residual mechanical stresses,
- temperature.

The differential form of Ohm's Law

The electric resistance of the conductor R can be expressed by the electric field and current density vectors inside the conductor.

$$R = \frac{U}{I} = \frac{\int_{-} \vec{E} \cdot d\vec{r}}{\int_{+} \vec{J} \cdot d\vec{A}}$$

In a homogeneous cylindrical conductor of length l , cross-sectional area A , and resistivity ρ it is very simple:



$$U = \int_{+} \vec{E} \cdot d\vec{r} = \int_{+} E dr = E \int_{+} dr = El$$

$$I = \int_{A} \vec{J} \cdot d\vec{A} = \int J dA = JA, \text{ and}$$

$$R = \frac{U}{I} = \frac{El}{JA}$$

On the other hand the resistance is proportional to the length and inversely proportional to the cross-section of the conductor:

$$R = \rho \frac{l}{A}$$

From these equations we can find that $\vec{E} = \rho \vec{J}$. The electric field and current density vectors remain parallel in the general case, too:

$$\vec{E} = \rho \vec{J}$$

This equation is the differential form of Ohm's Law. Introducing the electrical conductivity γ as the reciprocal value of the resistivity of the substance:

$$\rho = \frac{1}{\gamma},$$

we obtain another form of the differential Ohm's Law: $\vec{J} = \gamma \vec{E}$

In the case of metals, with increasing temperature the conductivity decreases, while for semiconductors it increases.

The differential Ohm's Law is not a strict proportionality between \vec{j} and \vec{E} , because in some cases γ and \vec{j} are not independent.

The resistance of a large group of metals and alloys vanishes in jump at a temperature close to the absolute zero. This is called superconductivity, and was discovered by **Kammerling** and **Onnes** (1911).

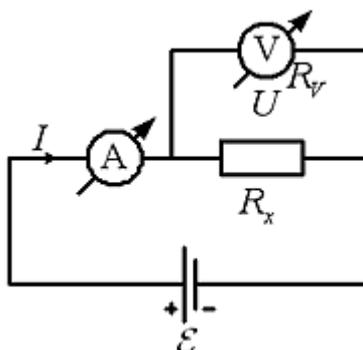
For example, in the case of lead $T_{\sigma} = 7K$ and $\frac{1}{\gamma} = 0$.

3. PRACTICAL DEVICES AND METHODS

The measurement of resistance by ammeter – voltmeter method

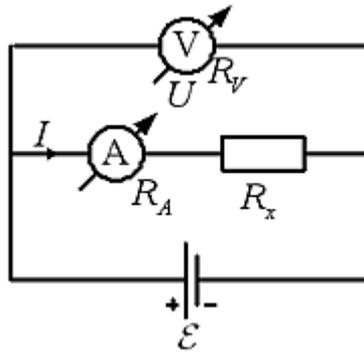
Using the defining equation $R_x = \frac{U}{I}$, measure I using an ammeter in series and U with a voltmeter in parallel. The two possibilities:

a) if $R_x \ll R_V$ (internal resistance of the voltmeter)



$$R_x \approx \frac{U}{I}$$

b) if $R_x \gg R_A$ (internal resistance of the ammeter)

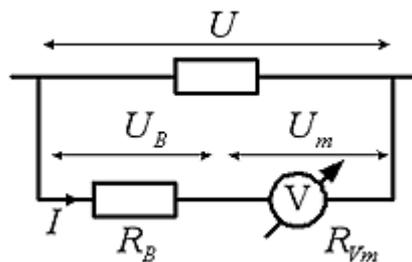


$$R_x \approx \frac{U}{I}$$

In case of digital multimeters an operational amplifier is used to increase the internal resistance of the voltmeter to ~ 10 MW. This resistance is so large that it does not disturb the circuit. The current measurement in the reality is a potential difference measurement on a high accuracy resistor.

Broadening the range of the potential difference can be measured by a voltmeter

A voltmeter always measures the potential difference or voltage between two points, and its terminals must be connected to these points, that is parallel with the resistance or other circuit components on which the voltage is to be measured. Voltmeters should therefore have a high resistance. The range of a voltmeter may be extended by connecting a resistor R_B in series with the meter, as in the figure, so that only some fraction of the total potential difference appears across the meter itself, and the remainder across R_B . The series resistor is called a bobbin. Denote the full-scale reading of the voltmeter by U_m .



Suppose that we need a voltmeter with a range of U , based on a meter U_m , and internal resistance R_{Vm} . Calculate the bobbin resistance R_B .

From Kirchoff's 2nd Rule:

$$U_m + U_B - U = 0.$$

Due to the serial connection of R_B , and R_{Vm} :

$$I = \frac{U_m}{R_{Vm}} = \frac{U_B}{R_B}$$

$$U_B = U_m \frac{R_B}{R_{Vm}}$$

$$U = U_m + U_m \frac{R_B}{R_{Vm}} = U_m \left(1 + \frac{R_B}{R_{Vm}} \right)$$

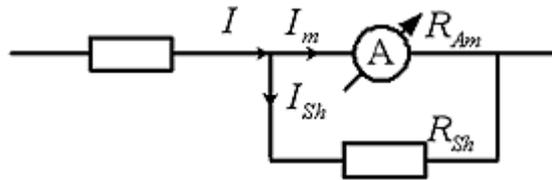
$$n = \frac{U}{U_m} = 1 + \frac{R_B}{R_{Vm}}$$

Thus the bobbin resistance R_B :

$$R_B = (n - 1) R_{Vm}$$

Broadening the range of the current can be measured by an ammeter

An ammeter is always placed in series with the resistance or other circuit components through which the current is to be measured. Ammeters should therefore have a low resistance compared with that of the rest of the circuit. An ammeter can be adapted to measure currents larger than its full-scale reading by connecting a resistor parallel with it, as shown in the figure, so that some of the current bypasses the meter. The parallel resistor is called a shunt.



Suppose that we need an ammeter with a range of I , based on a meter I_m , and internal resistance R_{Am} . Calculate the shunt resistance R_{Sh} .

Due to Kirchoff's 1st Rule:

$$I = I_m + I_{Sh}$$

Due to Kirchoff's 2nd Rule:

$$I_m R_{Am} = I_{Sh} R_{Sh}$$

$$I_{Sh} = I_m \frac{R_{Am}}{R_{Sh}}$$

$$I = I_m + I_m \frac{R_{Am}}{R_{Sh}} = I_m \left(1 + \frac{R_{Am}}{R_{Sh}} \right)$$

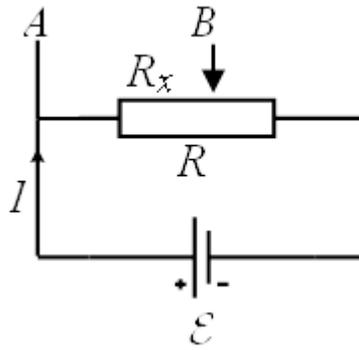
$$n = \frac{I}{I_m} = 1 + \frac{R_{Am}}{R_{Sh}}$$

The shunt resistance R_{Sh} :

$$R_{Sh} = \frac{R_{Am}}{n - 1}$$

The Potential Divider

If the potential divider is open:

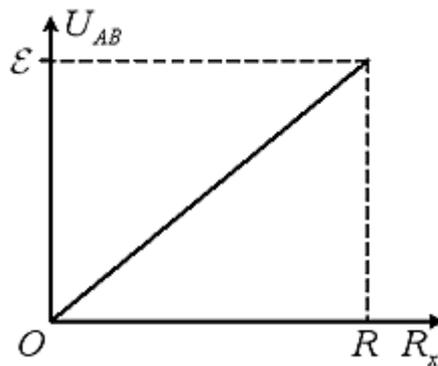


$$I = \frac{\mathcal{E}}{R}$$

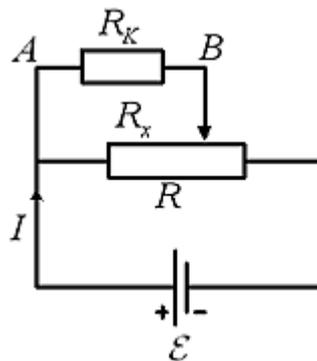
$$U_{AB} = IR_x = \mathcal{E} \frac{R_x}{R}$$

$$0 \leq U_{AB} \leq \mathcal{E}.$$

The value of U_{AB} can be varied by moving the sliding contact of the resistor. This voltage is a linear function of R_x .



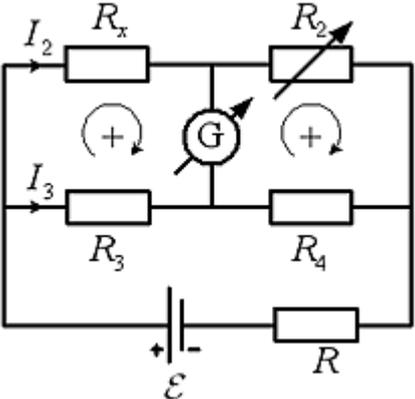
The potential divider is loaded if the variable potential difference is applied on a consumer R_x .



In the case of loaded divider the potential difference is not a linear function of the R_x resistance.



The measurement of resistance by the Wheatstone Bridge



This arrangement of resistors is called a Wheatstone bridge. R_x is the unknown resistance.

One resistor is varied until the current in the galvanometer G is zero. The bridge is then said to be balanced. $I_G = 0$

From the loop equations:

$$I_2 R_x - I_3 R_3 = 0 \quad I_2 R_x = I_3 R_3$$

$$I_2 R_2 - I_3 R_4 = 0 \quad I_2 R_2 = I_3 R_4$$

we obtain the value of R_x :

$$R_x = R_2 \cdot \frac{R_3}{R_4}$$

This method is suitable for $1\Omega - 10^6\Omega$.

4. WORK AND POWER IN STATIONARY CURRENT CIRCUIT

Consider a consumer in a circuit, having a current I and potential difference U_{12} between the two terminals. As charge passes through this circuit element the electric field does work on the charge. In time t the amount of charge passes through the consumer, in case of stationary current is $Q = It$, so the work is:

$$W = QU_{12} = U_{12}It$$

By means of this work the electric field transfers energy into this portion of the circuit. The rate of the energy transfer is the power:

$$P = \frac{W}{t} = U_{12}I$$

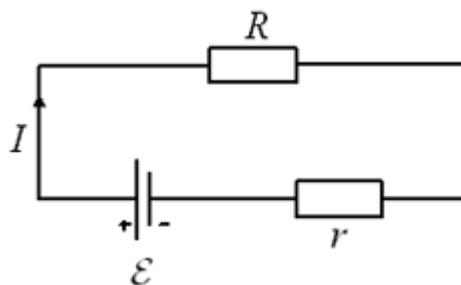
In case of pure resistance:

$$U_{12} = IR$$

$$P = U_{12}I = I^2R = \frac{U_{12}^2}{R}$$

This is the power input to the resistor. The circulating charges give up energy to the atoms of the resistor when they collide with them and the temperature of the resistor increases. We say that energy is dissipated in the resistor at a rate IR^2 .

Consider the next circuit: The source has an emf \mathcal{E} and internal resistance r . The external resistance is R .



The terminal potential difference across the resistance R is:

$$U_{12} = \mathcal{E} - Ir \quad | \cdot I$$

$$U_{12}I = \mathcal{E}I - I^2r$$

- $U_{12}I$ is the input power to the resistor R ,
- $\mathcal{E}I$ is the rate of conversion of non-electrical to electrical energy (power output of the source),

- $I^2 r$ is the rate of energy dissipation in the source.