CHAPTER 1

ZERO RESISTANCE

THE ELECTRICAL resistivity of all metals and alloys decreases when they are cooled. To understand why this should be, we must consider what causes a conductor to have resistance. The current in a conductor is carried by "conduction electrons" which are free to move through the material. Electrons have, of course, a wave-like nature, and an electron travelling through a metal can be represented by a plane wave progressing in the same direction. A metal has a crystalline structure with the atoms lying on a regular repetitive lattice, and it is a property of a plane wave that it can pass through a perfectly periodic structure without being scattered into other directions. Hence an electron is able to pass through a perfect crystal without any loss of momentum in its original direction. In other words, if in a perfect crystal we start a current flowing (which is equivalent to giving the conduction electrons a net momentum in the direction of the current) the current will experience no resistance. However, any fault in the periodicity of the crystal will scatter the electron wave and introduce some resistance. There are two effects which can spoil the perfect periodicity of a crystal lattice and so introduce resistance. At temperatures above absolute zero the atoms are vibrating and will be displaced by various amounts from their equilibrium positions; furthermore, foreign atoms or other defects randomly distributed can interrupt the perfect periodicity. Both the thermal vibrations and any impurities or imperfections scatter the moving conduction electrons and give rise to electrical resistance.

We can now see why the electrical resistivity decreases when a metal or alloy is cooled. When the temperature is lowered, the thermal vibrations of the atoms decrease and the conduction electrons are less frequently scattered. The decrease of resistance is linear down to a temperature equal to about one-third of the characteristic Debye temperature of the material, but below this the resistance decreases progressively less rapidly as the temperature falls (Fig. 1.1). For a perfectly pure metal, where the electron motion is impeded only by the thermal vibrations of the lattice, the resistivity should approach zero as the temperature is reduced towards 0°K. This zero resistance which a hypothetical "perfect" specimen would acquire if it could be cooled to absolute zero, is *not*, however, the phenomenon of superconductivity. Any real specimen of metal cannot be perfectly pure and will contain some impurities. Therefore the electrons, in addition to being scattered by thermal vibrations of the lattice atoms, are scattered by the impurities, and this impurity scattering is more or less independent of temperature. As a result, there is a certain "residual resistivity" (ρ_0, Fig. 1.1) which remains even at the lowest temperatures. The more impure the metal, the larger will be its residual resistivity.

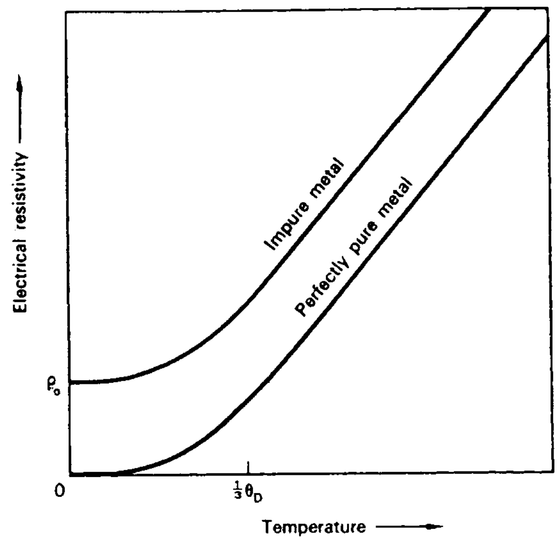

FIG. 1.1. Variation of resistance of metals with temperature.

Certain metals, however, show a very remarkable behaviour; when they are cooled their electrical resistance decreases in the usual way, but on reaching a temperature a few degrees above absolute zero they suddenly lose all trace of electrical resistance (Fig. 1.2). They are then said to have passed into the *superconducting* state.† The transformation

† In this book we use the term *superconductor* for a material which shows superconductivity if cooled. We use the adjective *superconducting* to describe it when it is exhibiting superconductivity, and *normal* when it is not exhibiting superconductivity (e.g. when above its transition temperature).

to the superconducting state may occur even if the metal is so impure that it would otherwise have had a large residual resistivity.

1.1. Superconducting Transition Temperature

The temperature at which a superconductor loses resistance is called its superconducting *transition temperature* or *critical temperature*; this temperature, written T_c, is different for each metal. Table 1.1 shows the transition temperatures for metallic elements. In general the transition temperature is not very sensitive to small amounts of impurity, though

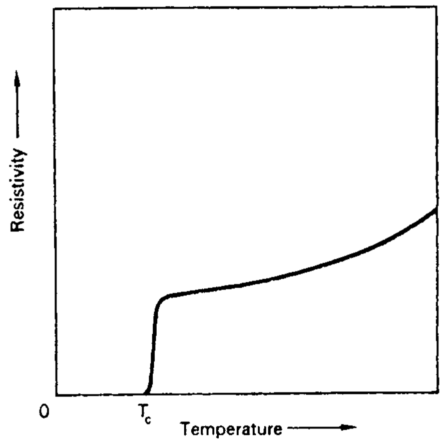

FIG. 1.2 Loss of resistance of a superconductor at low temperatures.

magnetic impurities tend to lower the transition temperature. (We shall see in Chapter 9 that ferromagnetism, in which the spins of electrons are aligned parallel to each other, is incompatible with superconductivity.) The superconductivity of a few metals, such as iridium and molybdenum, which in the pure state have very low transition temperatures, may be destroyed by the presence of minute quantities of magnetic impurities. Such elements, therefore, only exhibit superconductivity if they are extremely pure, and specimens of these metals of normal commercial purity are not superconductors. Not all pure metals have been found to be superconductors; for example, copper, iron and sodium have not shown superconductivity down to the lowest temperature to which they have so far been cooled. Of course, experiments at even lower temperatures may reveal new superconductors, but there is no fundamental reason why all metals should show superconductivity, even at absolute zero. Nevertheless, it should be realized that superconductivity is not a rare phenomenon; about half the metallic elements are known to be superconductors and, in addition, a large number of alloys are superconductors. It is possible for an alloy to be a superconductor, even if it is composed of two metals which are not themselves superconductors (e.g. Bi–Pd). Superconductivity can be shown by conductors which are not metals in the ordinary sense; for example, the semiconducting mixed oxide of barium, lead and bismuth is a superconductor, and the conducting polymer, polysulphur nitride $(SN)_x$, has been found to become superconducting at about 0·3°K.

TABLE 1.1. THE SUPERCONDUCTING ELEMENTS
T_c is the superconducting transition temperature
H_0 is the critical magnetic field at 0°K (see Chapter 4)

		H_0	
	T_c(°K)	(Amp m^{-1})	(gauss)
Aluminium	1·2	0·79 × 10^4	99
Americium fcc	1·1		
Americium dhcp	0·79		
Cadmium	0·52	0·22 × 10^4	30
Gallium	1·1	0·41 × 10^4	51
Indium	3·4	2·2 × 10^4	276
Iridium	0·11	0·13 × 10^4	16
Lanthanum α	4·8		
Lanthanum β	4·9		
Lead	7·2	6·4 × 10^4	803
Lutecium	0·1	2·8 × 10^4	350
Mercury α	4·2	3·3 × 10^4	413
Mercury β	4·0	2·7 × 10^4	340
Molybdenum	0·9		
Niobium	9·3	Type-II (see Chap. 12)	
Osmium	0·7	~0·5 × 10^4	~63
Protactinium	0·4	Type-II (see Chap. 12)	
Rhenium	1·7	1·6 × 10^4	201
Rhodium	3·3 × 10^{-4}	4	0·05
Ruthenium	0·5	0·53 × 10^4	66
Tantalum	4·5	6·6 × 10^4	830
Technetium	7·9	Type-II (see Chap. 12)	
Thalium	2·4	1·4 × 10^4	171
Thorium	1·4	1·3 × 10^4	162
Tin	3·7	2·4 × 10^4	306
Titanium	0·4		
Tungsten	0·016	0·0096 × 10^4	1·2
Uranium α	0·6		
Uranium β	1·8		
Vanadium	5·4	Type-II (see Chap. 12)	
Zinc	0·9	0·42 × 10^4	53
Zirconium	0·8	0·37 × 10^4	47

Niobium is the metallic element with the highest transition temperature (9·3°K), but some alloys and metallic compounds remain superconducting up to even higher temperatures (Table 1.2). For example, Nb_3Ge has a transition temperature of about 23°K. These alloys with relatively high transition temperatures are of great importance in the engineering applications of superconductivity. Recently some ceramic metal oxides have been developed which are superconducting up to temperatures as high as about 100°K. These are of great interest because such temperatures, unlike those within a few tens of degrees of Absolute Zero, can be obtained easily and cheaply (by, for example, the use of liquid nitrogen).

TABLE 1.2. SUPERCONDUCTING TRANSITION TEMPERATURES OF SOME ALLOYS AND METALLIC COMPOUNDS COMPARED WITH THEIR CONSTITUENT ELEMENTS

	Ta–Nb	Pb–Bi	3Nb–Zr	Nb_3Sn	Nb_3Ge
T_c (°K)	6·3	8	11	18	23

	Nb	Pb	Ta	Sn	Zr	Bi	Ge
T_c (°K)	9·3	7·2	4·5	3·7	0·8	not s/c	not s/c

On cooling, the transition to the superconducting state may be extremely sharp if the specimen is pure and physically perfect. For example, in a good gallium specimen, the transition has been observed to occur within a temperature range of 10^{-5} degrees. If, however, the specimen is impure or has a disturbed crystal structure, the transition may be considerably broadened. Figure 1.3 shows the transition in pure and impure tin specimens.

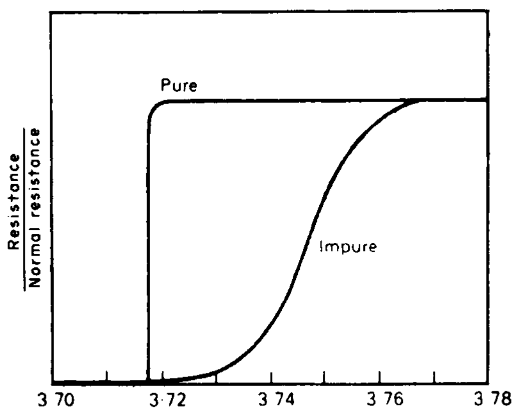

FIG. 1.3. Superconducting transition in tin.

1.2 Zero Resistance

Even when the transition is spread over a considerable temperature range the resistance still seems to disappear completely below a certain temperature. We naturally ask whether in the superconducting state the resistance has indeed become zero or whether it has merely fallen to a very small value. Of course, it can never be proved by experiment that the resistance is in fact zero; the resistance of any specimen may always be just less than the sensitivity of our apparatus allows us to detect. However, no experiment has been able to detect any resistance in the superconducting state. We may look for resistance quite simply by passing a current through a wire of superconductor and seeing if any voltage is recorded by a sensitive voltmeter connected across the ends of the wire. A more sensitive test, however, is to start a current flowing round a closed superconducting ring and then see whether there is any decay in the current after a long period of time. Suppose the self-inductance of the ring is L; then, if at time $t = 0$ we start a current $i(0)$ flowing round the ring (for ways of doing this see § 1.3), at a later time t the current will have decayed to

$$i(t) = i(0)e^{-(R/L)t}, \tag{1.1}$$

where R is the resistance of the ring. We can measure the magnetic field that the circulating current produces and see if this decays with time. The measurement of the magnetic field does not draw energy from the circuit, and we should be able to observe whether the current circulates indefinitely. Gallop has been able to show from the lack of decay of a current circulating round a closed loop of superconducting wire that the resistivity of the superconducting metal was less than 10^{-26} ohm-metres (i.e. less than 10^{-18} the resistivity of copper at room temperature). It seems, therefore, that we are justified in treating the resistance of a superconducting metal as zero.

1.3. The Resistanceless Circuit

A closed circuit, such as a ring, formed of superconducting metal has an important and useful property resulting from its zero resistance. *The total magnetic flux threading a closed resistanceless circuit cannot change so long as the circuit remains resistanceless.* Suppose as in Fig. 1.4a, a ring of metal is cooled below its transition temperature in an applied field of uniform flux density B_a. If the area enclosed by the ring is $\mathscr{A}$, an amount of flux $\Phi = \mathscr{A} B_a$ will thread the ring. Suppose the applied field is now changed to a new value. By Lenz's law, when the field is changing currents are induced and circulate round the ring in such a direction as to create a flux inside the ring which tends to cancel the flux change due to the alteration in the applied field. While the field is changing there is an e.m.f., $-\mathscr{A} dB_a/dt$, and an induced current i given by

$$-\mathscr{A}\frac{dB_a}{dt} = Ri + L\frac{di}{dt},$$

where R and L are the total resistance and inductance of the circuit. In a normal resistive circuit the induced currents quickly die away and the flux threading the ring acquires the new value. In a superconducting circuit, however, $R = 0$ and

$$-\mathscr{A}\frac{dB_a}{dt} = L\frac{di}{dt},$$

so that

$$Li + \mathscr{A} B_a = \text{constant}. \tag{1.2}$$

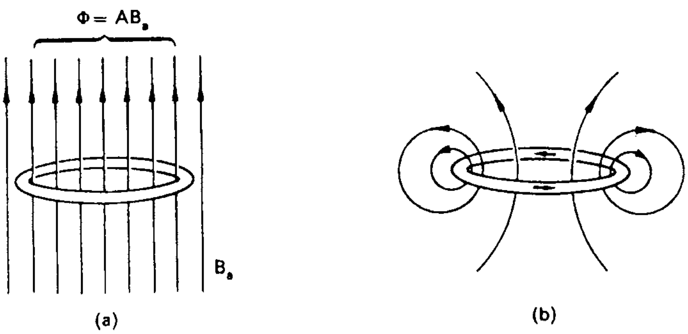

FIG. 1.4. Resistanceless circuit.

But $Li + \mathscr{A} B_a$ is the total magnetic flux threading the circuit; we have therefore demonstrated that the total flux threading a resistanceless circuit cannot change. If the applied magnetic field strength is changed an induced current is set up of such a magnitude that it creates a flux which exactly compensates the change in the flux from the applied magnetic field. Since the circuit is resistanceless the induced current flows for ever and the original amount of flux is maintained indefinitely. Even if the external field is reduced to zero, as in Fig. 1.4b, the internal flux will be maintained by the induced circulating current.

This property can be made use of when solenoids wound from superconducting wire are employed to generate magnetic fields. In Fig. 1.5 current to the refrigerated superconducting solenoid S is derived from the d.c. power supply P. Once the current has been adjusted by the rheostat R to a value which gives the desired magnetic field strength, the superconducting switch XY can be closed. XY and S now form a closed resistanceless circuit in which the flux must remain constant. Hence the field strength generated by S does not vary in time and we can, if we wish, disconnect the power supply and the field will be maintained by the current i flowing round the resistanceless circuit XYS. A superconducting solenoid operating in this fashion is said to be in the *persistent mode.*

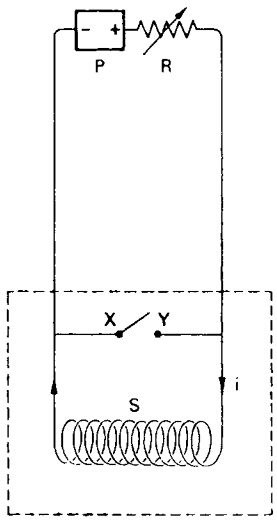

FIG. 1.5. Superconducting solenoid.

Note that, although the *total* amount of flux enclosed in a resistanceless circuit remains constant, there can be a change in the flux density **B** at any point due to a redistribution of the flux within the circuit. Thus in Fig. 1.4b the flux density has become stronger near the wire and weaker in the centre of the enclosed space compared to the uniform distribution in Fig. 1.4a. In both cases, however, the total flux ($= \iint_{\mathscr{A}} \mathbf{B} \cdot d\mathscr{A}$) is the same.

We have seen that if a closed circuit of superconductor is cooled below its transition temperature while in an applied magnetic field, the flux it encloses remains constant in spite of any changes in the applied field. On the other hand, if the circuit is cooled in the absence of an applied magnetic field, so that there is initially no flux inside, and an external magnetic field is subsequently applied, the net internal flux remains zero in spite of the presence of the external field. This property enables us to use hollow superconducting cylinders to shield enclosures from external magnetic fields. The shielding is only perfect for the case of a long hollow cylinder, in which case the induced currents generate a uniform compensating flux density throughout its interior. For other configurations, such as a short ring, it is only the *total* flux which is maintained at zero and the local magnetic flux density generated by the induced current will not be uniform within the ring. Hence the flux density due to the persistent currents will in some places be stronger and in other places weaker than that of an applied field, and there will not everywhere be exact cancellation. In other words, though $\iint_{\mathscr{A}} \mathbf{B} \cdot d\mathscr{A} = 0$, $\mathbf{B}$ itself is not necessarily

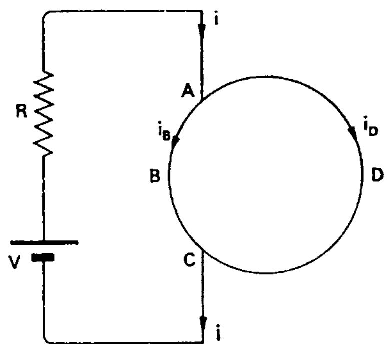

FIG. 1.6. Division of current between two parallel paths.

everywhere equal to zero. In practice, however, superconducting shielding can be used to give very good screening against magnetic fields.

We now consider what determines the distribution of currents in a network of resistanceless conductors. Consider, for example, the simple circuit of Fig. 1.6. If the ring *ABCD* is resistanceless, how will the current i divide between branches B and D? Clearly Kirchhoff's laws are of no help, because both paths have zero resistance and the second law will be obeyed for all possible divisions of the current i. However, though the two branches have no resistance, they do contribute inductance to the circuit. We shall now show that the division of current is determined by these inductances. The voltage difference between A and C equals

$$L_B \frac{di_B}{dt} + M_{BD} \frac{di_D}{dt} = L_D \frac{di_D}{dt} + M_{BD} \frac{di_B}{dt},$$

where L_B and L_D are the inductances of the branches B and D, and M_{BD} is the mutual inductance between them. Rearrangement gives

$$(L_B - M_{BD}) \frac{di_B}{dt} = (L_D - M_{BD}) \frac{di_D}{dt},$$

and integrating this we obtain

$$(L_B - M_{BD})\, i_B = (L_D - M_{BD})\, i_D + \text{constant}.$$

If $i_B = 0 = i_D$ at $t = 0$, then the constant $= 0$, and we have

$$\frac{i_B}{i_D} = \frac{L_D - M_{BD}}{L_B - M_{BD}},$$

and it can be seen that the division of the current is controlled by the inductances of the paths. It often happens that the magnetic coupling between two parallel paths is small so that their mutual inductance can be neglected. Under this circumstance we can deduce the rule that in resistanceless networks ***the currents carried in parallel paths are inversely proportional to the self-inductances of those paths.***

1.4. A.C. Resistivity

The fact that a superconducting metal has no resistance means, of course, that there is no voltage drop along the metal when a current is passed through it, and no power is generated by the passage of the current. This, however, is only strictly true for a direct current of constant value. If the current is changing an electric field is developed and some power is dissipated. To understand the reason for this we must first discuss briefly some aspects of the behaviour of conduction electrons in superconductors.

Many of the properties of superconductors can be explained if it is supposed that below the transition temperature the conduction electrons divide into two classes, some behaving as "superelectrons" which can pass through the metal without resistance (i.e. suffering no collisions), the remainder behaving as "normal" electrons which can be scattered and so experience resistance just like conduction electrons in a normal metal. The fraction of superelectrons appears to decrease as the temperature is raised towards the transition temperature. At 0°K all conduction electrons behave like superelectrons, but, if the temperature is raised, a few begin to behave as normal electrons, and on further heating the proportion of normal electrons increases. Eventually, at the transition temperature, all the electrons have become normal electrons and the metal loses its superconductive properties. Hence a superconductor below its transition temperature appears to be permeated by two electron fluids, one of normal electrons and one of superelectrons. The relative electron density in the two fluids depends on the temperature. This "two-fluid model" is suggested by thermodynamic arguments based on the results of specific heat and similar measurements on superconductors, which will be discussed in Chapter 5.

In a superconducting metal the current can in general be carried by both the normal and superelectrons. However, in the special case of a *constant* direct current all the current is carried by the superelectrons. We can see that this will be so by noting that, if the current is to remain constant, there must be no electric field in the metal, otherwise the superelectrons would be accelerated continuously in this field and the current would increase indefinitely. If there is no field there is nothing to drive the normal electrons and so there is no normal current. We see, therefore, that for a constant value of total current all the current is carried by the superelectrons. A superconducting metal is like two conductors in parallel, one having a normal resistance and the other zero resistance. We can say that the superelectrons "short circuit" the normal electrons. To put this another way, if we suddenly apply a voltage source, such as a battery, across a superconductor, the current tends to rise to infinity but is in fact limited by the internal resistance of the source. While the current is changing, an electric field must be present to accelerate the electrons. Electrons do, however, have a small inertial mass and so the supercurrent does not rise instantaneously but only at the rate at which the electrons accelerate in the electric field. If we apply an *alternating* field, the supercurrent will therefore lag behind the field because of the inertia of the superelectrons. Hence the superelectrons present an inductive impedance† and, because there now is an electric field present, some of the current will be carried by the normal electrons. The current is not, therefore, carried entirely by the superelectrons as in the d.c. case. Of course, the normal electrons also have an inertial mass but their resulting inductive reactance is completely swamped by the

† This inherent inductive impedance is, of course, quite distinct from, and is additional to, the ordinary inductance of the conductor due to its geometry.

resistance resulting from their being scattered in the metal. We can, in fact, represent the bulk properties of a superconducting metal by a perfect inductance in parallel with a resistance.

The fraction of the current diverted through the normal electrons dissipates power in the usual way. The mass of electrons is, however, extremely small, so the inductance due to their inertia is also extremely small. The inductance in henrys of a typical superconductor due to the inertia of its superelectrons is only about 10^{-12} of its normal resistance in ohms, so at 1000 Hz, for example, only about 10^{-8} of the total current is carried by the normal electrons and there is only a minute dissipation of power. Nevertheless, this contrasts with the absolutely zero resistance in the d.c. case.

If the frequency of an applied field is sufficiently high, however, a superconducting metal responds in the same way as a normal metal. This is because, as we shall see in Chapter 9, superelectrons are in a lower energy state than normal electrons, but, if the frequency of the applied field is high enough, the photons of the electromagnetic field have enough energy to excite superelectrons into the higher state where they behave as normal electrons. This happens for frequencies greater than about 10^{11} Hz (i.e. greater than the frequency of very long wave infrared). The behaviour of a superconductor at optical frequencies is therefore no different from that of a normal metal and there is, for example, no change in the visual appearance of a superconductor as it is cooled below its transition temperature.

It is tempting to suppose that the superelectrons in a superconductor behave like electrons in a vacuum. The electrons in the beam of a cathode ray tube, for example, are resistanceless in the sense that they flow without undergoing any collisions. There is, however, a significant difference between the two cases. It is possible to maintain a potential drop along an electron beam while the current remains at a constant value. This is because, though the current must be the same all along the path of the beam, the electron density need not remain constant. Hence the electrons accelerate from the cathode towards the anode and the electron density is relatively high near the cathode and decreases as the anode is approached. However, the product of the electron density and electron velocity, i.e. the current, remains constant along the beam. The fact that the electrons are able to accelerate allows us to maintain the electric field. In a superconductor, however, conditions are different. The metal must remain everywhere electrically neutral and, since the positive metal ions are fixed in the crystal, the electron density cannot vary through the material. Hence for a constant current to be maintained through the metal, the velocity of all the electrons along the current path must be the same. The electrons, therefore, do not accelerate and an electric field cannot exist in the metal.