CHAPTER 2

PERFECT DIAMAGNETISM

2.1. Magnetic Properties of a Perfect Conductor

WE HAVE seen in the previous chapter that a superconductor below its transition temperature appears to have no resistance. Let us now try to deduce the magnetic properties of such a resistanceless conductor.

Suppose that we cool a specimen which, below its transition temperature, becomes perfectly conducting. The resistance around an imaginary closed path within the metal is zero; and therefore, as shown in the previous chapter, the amount of magnetic flux enclosed within this path cannot change. This is true for *any* such imaginary circuit and this can only be so if the flux density at every point within the metal does not vary with time, i.e.

$$\dot{\mathbf{B}} = 0.$$

Consequently the flux distribution in the metal must remain as it was when the metal became resistanceless.

Consider now the behaviour of a perfect conductor under various circumstances. Suppose that a specimen loses its resistance in the absence of any magnetic field and that a magnetic field is then subsequently applied. Because the flux density in the metal cannot change, it must remain zero even after the application of the magnetic field. In fact the application of the magnetic field induces resistanceless currents which circulate on the surface of the specimen in such a manner as to create a magnetic flux density which everywhere inside the metal is exactly equal and opposite to the flux density of the applied magnetic field.† Because

† We must define here exactly what we mean by an "applied magnetic field". An "applied" field is a field generated by some agency (solenoid, permanent magnet, etc.) external to a specimen. Its strength H_a and flux density B_a are those which would be measured if the specimen were not there. In the case of a uniform applied field the strength and flux density are the same as those measured far away from the specimen, i.e. where any perturbing effects due to the magnetic properties of the specimen are negligible.

these currents do not die away, the net flux density inside the material remains at zero. This is illustrated in Fig. 2.1a: the surface currents i generate a flux density B_i that exactly cancels the flux density B_a of the applied magnetic field everywhere inside the metal. These surface currents are often referred to as ***screening currents.***

The flux density created by the persistent surface currents does not, of course, disappear at the boundary of the specimen, but the flux lines form continuous closed curves which return through the space outside (Fig. 2.1a). Though the density of this flux everywhere *inside* the specimen is equal and opposite to the flux from the applied field, this is not so *outside* the specimen. The net distribution of flux resulting from the superposition of the flux from the specimen and that from the applied field is shown in Fig. 2.1b. The pattern is as though the sample

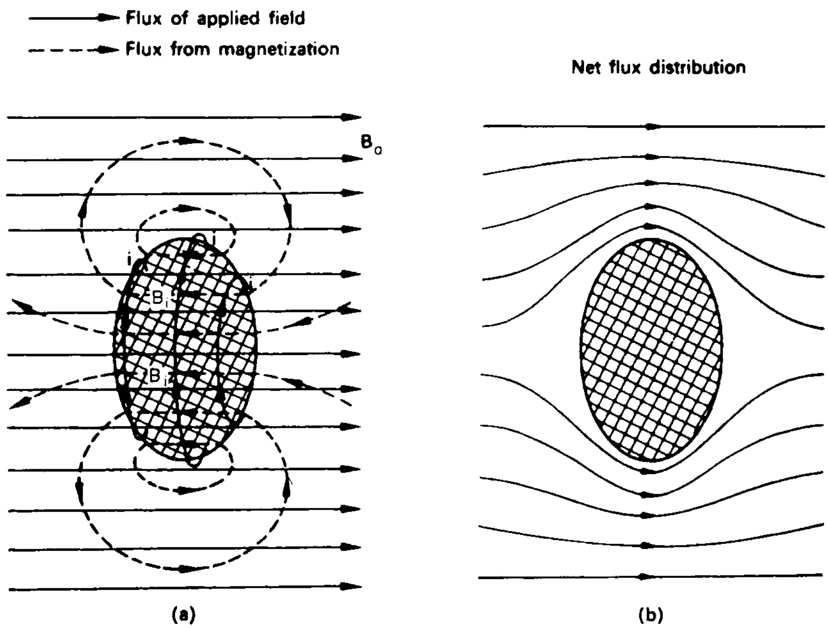

FIG. 2.1. Distribution of magnetic flux about a perfectly diamagnetic body.

had prevented entry into it of the flux of the applied field. A sample in which there is no net flux density when a magnetic field is applied is said to exhibit ***perfect diamagnetism.*** If we now reduce the applied magnetic field to zero the specimen is left in its original unmagnetized condition. The above sequence of events is illustrated in Fig. 2.2 a–d.

Let us now consider a different sequence of events. Suppose that the magnetic field B_a is applied to the specimen while it is above its transi-

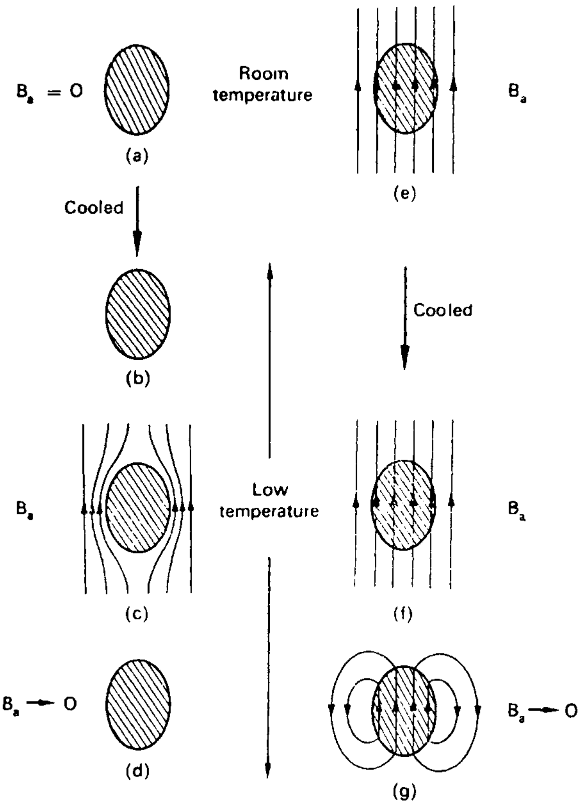

FIG. 2.2. Magnetic behaviour of a "perfect" conductor. (a)–(b) Specimen becomes resistanceless in absence of field. (c) Magnetic field applied to resistanceless specimen. (d) Magnetic field removed.

(e)–(f) Specimen becomes resistanceless in applied magnetic field. (g) Applied magnetic field removed.

tion temperature (Fig. 2.2e). Most metals (other than the special ferromagnetics, iron, cobalt and nickel) have values of relative magnetic permeability very close to unity, and so the flux density inside is virtually the same as that of the applied field. The specimen is now cooled to a low temperature so that it loses its electrical resistance. This disappearance of resistance has no effect on the magnetization, and the flux distribution remains unaltered (Fig. 2.2f). We next reduce the applied field to zero. The flux density inside the perfectly conducting metal cannot change, and persistent currents are induced on the specimen, maintaining the flux inside, with the result that the specimen is left permanently magnetized (Fig. 2.2g).

It is important to notice that in (c) and (f) of Fig. 2.2 the sample is under the same conditions of temperature and applied magnetic field, and yet its state of magnetization is quite different in the two cases. Similarly (d) and (g) show different states of magnetization under identical external conditions. We see that the state of magnetization of a perfect conductor is not uniquely determined by the external conditions but depends on the sequence by which these conditions were arrived at.

2.2. Special Magnetic Behaviour of a Superconductor

2.2.1. Meissner effect

In the previous section we have deduced the magnetic behaviour of a resistanceless conductor by applying simple and well-known fundamental principles of electromagnetism, and for 22 years after the discovery of superconductivity it was assumed that the effect of a magnetic field on a superconductor would be as shown in Fig. 2.2. However, in 1933 Meissner and Ochsenfeld measured the flux distribution outside tin and lead specimens which had been cooled below their transition temperatures while in a magnetic field.† They found that the expected situation of Fig. 2.2f did not in fact occur, but that at their transition temperatures the specimens spontaneously became perfectly diamagnetic, cancelling all flux inside, as in Fig. 2.2c, even though they had been cooled in a magnetic field. This experiment was the first to demonstrate that superconductors are something more than materials which are perfectly conducting; they have an additional property that a merely resistanceless metal would not possess: ***a metal in the superconducting state never allows a magnetic flux density to exist in its interior.*** That is to say, inside a superconducting metal we always have

$$\mathbf{B} = 0,$$

whereas inside a merely resistanceless metal there may or may not be a flux density, depending on circumstance (Fig. 2.2). When a superconductor is cooled in a weak magnetic field, at the transition temperature persistent currents arise on the surface and circulate so as to cancel the flux density inside, in just the same way as when a magnetic field is applied

† The magnetic field used in experiments of this kind must not be too strong because, as we shall see in Chapter 4, a metal loses its superconducting properties if the applied magnetic field exceeds a certain strength.

after the metal has been cooled (Fig. 2.3). This effect, whereby a superconductor never has a flux density inside even when in an applied magnetic field, is called (with injustice to Ochsenfeld) ***the Meissner effect.***

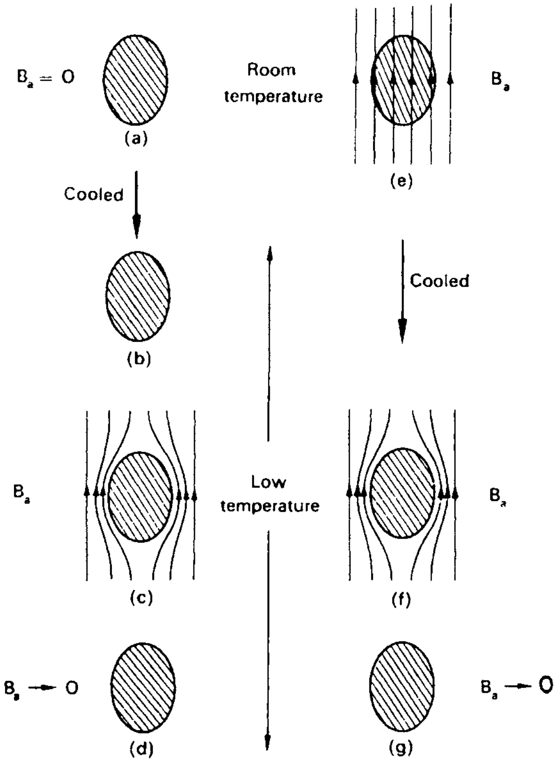

FIG. 2.3. Magnetic behaviour of a superconductor. (a)–(b) Specimen becomes resistanceless in absence of magnetic field. (c) Magnetic field applied to superconducting specimen. (d) Magnetic field removed.

(e)–(f) Specimen becomes superconducting in applied magnetic field. (g) Applied magnetic field removed.

For convenience we call a hypothetical metal which simply has no resistance and would behave as shown in Fig. 2.2 a "perfect conductor" in contrast to *super*conductors which in the superconducting state never permit a magnetic flux density to exist inside them (Fig. 2.3). The state of magnetization of a "perfect conductor" would depend on the order in which the final conditions of applied magnetic field and temperature were obtained, but the magnetization of a superconductor depends only on the actual values of the applied field and temperature and not on the way they were arrived at.

2.2.2. Permeability and susceptibility of a superconductor

Suppose a magnetic field of flux density B_a is applied to a superconductor.† In order that we may neglect demagnetizing effects (see Chapter 6) we consider a long superconducting rod with the field applied parallel to its length. An applied magnetic field of flux density B_a produces in the material a flux density equal to $\mu_r B_a$, where μ_r is the relative permeability of the material. Metals, other than ferromagnetics, have a relative permeability which is very close to unity, i.e. $\mu_r = 1$, so the flux density within them due to the applied magnetic field is equal to B_a. However, as we have seen, the total flux density inside a superconducting body is zero. This perfect diamagnetism arises because surface screening currents circulate so as to produce a flux density B_i which everywhere inside the metal exactly cancels the flux density due to the applied field; $B_i = -B_a$. A rod-shaped superconducting specimen therefore behaves like a long solenoid with circulating current that creates a flux density exactly equal in magnitude, but opposite in direction, to the flux density due to the applied magnetic field. To create a flux density of $-B_a$, the magnitude of the circulating surface current per unit length must, from the ordinary solenoid formula, be $|j| = B_a/\mu_0$. In other words $|j| = H_a$, where H_a is the applied field strength.

We can, however, describe the perfect diamagnetism in another way. Because we cannot actually observe the surface screening currents which arise when a magnetic field is applied, we could suppose that the perfect diamagnetism arises from some special bulk magnetic property of the superconducting metal, and we can describe the perfect diamagnetism simply by saying that for a superconducting metal $\mu_r = 0$, so that the flux density inside, $B = \mu_r B_a$, is zero. Here we do not consider the mechanism by which the diamagnetism arises; the effect of the screening currents is included in the statement $\mu_r = 0$. The strength H_a of the applied magnetic field is given by

$$H_a = \frac{B_a}{\mu_0},$$

and the flux density in a magnetic material is related to the strength of the applied field by

$$B = \mu_0 (H_a + I),$$

† As explained in Appendix A, we take the view that a magnetic field is best described by its flux density.

where I is the magnetization (often called the "intensity of magnetization") of the material. The magnetization of a superconductor, in which $B = 0$, must therefore be

$$I = -H_a,$$

and the magnetic susceptibility, i.e. the ratio of the magnetization to the field strength, must be

$$\chi = -1.$$

The two descriptions are entirely equivalent because, as shown in Appendix A, the magnitude of I is equal to the equivalent surface current density j.

We now summarize the two alternative ways of regarding the perfect diamagnetism.

(i) Screening-current diamagnetism

The material of the superconductor, like other metals, is non-magnetic, and an applied magnetic field produces a flux density B_a in the metal. However, screening currents generate an internal flux density which everywhere is exactly equal and opposite to this flux density and consequently the net flux density is zero.

(ii) Bulk diamagnetism

The material can be considered to have a relative permeability $\mu_r = 0$, so the flux density produced in it by an applied magnetic field is always zero. The material behaves as though in a magnetic field it acquires a negative bulk magnetization $I = -H_a$.

Appendix A shows that these two ways of regarding the perfect diamagnetism are entirely equivalent. We shall make use of both descriptions, the choice in each case depending on which is more convenient for the particular situation under discussion.

2.3. Surface Currents

The fact that a superconducting metal does not allow magnetic flux to exist in its interior has an important effect on any electric currents that flow along it; ***currents cannot pass through the body of a superconducting metal, but flow only on the surface.*** To see why this should be, let us take the first viewpoint of a superconductor expressed at (i) above and treat the material as having the same relative magnetic permeability as any ordinary metal, i.e. $\mu_r = 1$. At any point in a material of unit relative permeability the relation between the magnetic flux and current density is, from Maxwell's equation,

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{J}. \tag{2.1}$$

If the metal is superconducting **B** is zero inside, and so curl **B** must also be zero inside.† It follows, therefore, from Maxwell's equation that, as a consequence of **B** being zero, the current density **J** must also be zero within the superconductor. There is, of course, no reason why **B** must be

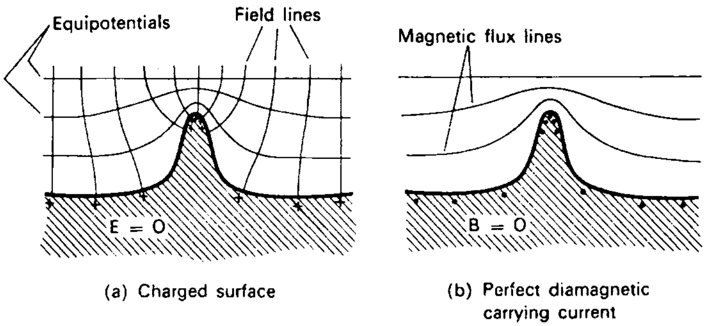

FIG. 2.4. Analogy between distribution of electrostatic charge and surface current.
+ + + electric charges
. . . current perpendicular to plane of page.

zero *outside* the superconductor so, if there is a current, it flows not through the metal but on the surface. This is true both of currents passed along the superconductor from some external source such as a battery (we call these "transport" currents), and of diamagnetic screening currents. Any transport current will flow all over the surface, creating a magnetic flux outside but not inside the conductor. If a magnetic field is applied, the diamagnetic screening currents which flow so as to cancel the flux density inside also circulate on the surface.

There is an interesting and useful analogy between the distribution of current on the surface of a superconducting metal and the distribution of electrostatic charge on a conducting body. Consider part of the surface of a charged conductor as shown in Fig. 2.4a. In the equilibrium state $E = 0$ inside the conductor but, if the body carries a surface charge, this charge will produce an electric field outside the conductor. The compo-

† As explained in Appendix A, we take the view that **J** affects **B** but not **H**. So the vanishing of **B** does not necessarily imply that **H** is zero.

nent of the electric field parallel to the surface, $E_{\parallel}$, is continuous across the surface and therefore, since $E = 0$ in the conductor, $E_{\parallel}$ must be zero just outside the surface. The electric field lines must consequently meet the conductor at right angles. The surface itself is an equipotential and the electric field lines are orthogonal to the equipotentials. It can be seen that the field lines are crowded together where the boundary has a high convex curvature, so the electric charge, being proportional to the normal component of the field, will be concentrated into these regions. Figure 2.4b shows a section of part of a superconducting metal carrying a current in a direction normal to the plane of the paper. Inside the perfectly diamagnetic material we have $B = 0$, but if current is flowing on the surface there will be a magnetic flux density outside. The component of B normal to the surface is continuous across the boundary, so just outside the surface the flux lines must be parallel to the surface. This flux density is proportional to the surface current density. In fact the external magnetic field due to the surface current has the same form as the equipotentials due to surface charges in Fig. 2.4a. The magnetic field lines are crowded together close to the region where the surface has high convex curvature, so the surface current density must be greatest at these regions. We might expect, therefore, that the distribution of surface current on a perfectly diamagnetic body has the same form as the distribution of electric charge on a charged conductor of the same shape, and a proper mathematical analysis shows that this is indeed so.

2.3.1. Hole through a superconductor

In later chapters we shall on several occasions need to consider the properties of a superconducting body with a hole right through it. Though the body of a superconductor is perfectly diamagnetic, flux can exist inside a hole.

Consider for example a long hollow body as shown in Fig. 2.5. This forms a closed circuit. We have already discussed the properties of closed resistanceless circuits in § 1.3, but, when the body is a superconductor, we must also take into account the perfect diamagnetism of the material itself.

First suppose that the body shown in Fig. 2.5 is cooled below its superconducting transition temperature in the absence of any applied magnetic field and that, after it has become superconducting, a magnetic field of flux density B_a is applied (Fig. 2.5a). The superconducting material is perfectly diamagnetic so there will be no flux density in the body of the material. The perfect diamagnetism of the superconducting material is maintained by currents i_d which circulate on the outer surface so as to cancel the flux in the metal. However, the flux density generated by these diamagnetic screening currents also cancels the flux density due to the applied field in the hole, so in this case there is no flux density in the hole. In this situation the superconducting body is behaving no differently from a merely resistanceless body. In both cases the current induced on the outer surface by the application of the magnetic field cancels the flux inside.

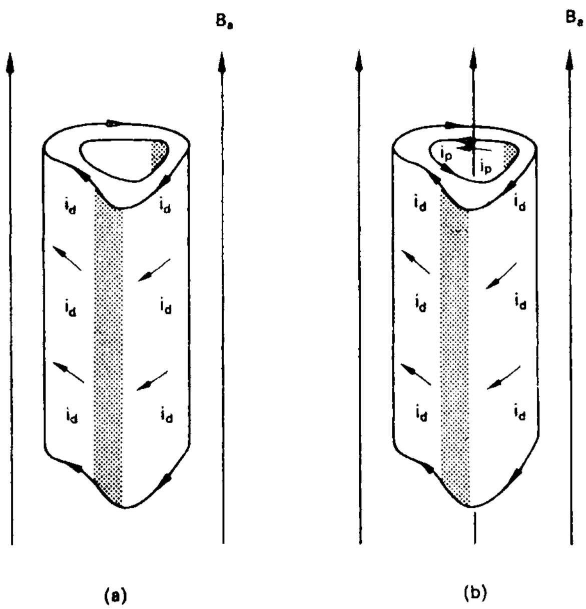

FIG. 2.5. Hollow superconductor. (a) Magnetic field applied when material is in the superconducting state. (b) Material becomes superconducting in an applied magnetic field.

Now consider a situation in which a superconductor behaves differently from a merely resistanceless body, or "perfect" conductor. Suppose the magnetic field is applied *before* the body is cooled below its transition temperature. Above the transition temperature the magnetic flux passes both through the body and the hole. In the case of the perfect conductor this flux distribution will not change when the body loses its resistance, and no currents will appear on the surface. A superconductor, however, behaves differently; below the transition temperature the material becomes perfectly diamagnetic but though. there is no flux through the material, flux remains in the hole (Fig. 2.5b). Currents must circulate to maintain these differences in flux density. As we have just seen, the diamagnetic surface currents i_d which cancel the flux in the superconducting material would also cancel the flux in the hole, so if there is magnetic flux threading the hole, this must be generated by currents i_p circulating in the opposite ("paramagnetic") direction around its periphery. We have, therefore, the result that flux threading a hole or normal region through a superconductor is always associated with a current circulating around the boundary between this region and the superconductor.

Note that the net circulating current $i_p - i_d$ is just that which generates a flux density equal to the difference between the flux density in the hole and the flux density outside the superconducting body.

As we saw in Chapter 1, the flux threading any resistanceless circuit cannot change. Consequently once flux has been established through a hole in a superconducting body, this flux, and the circulating current i_p associated with it, will persist even if the applied magnetic field strength is changed or reduced to zero.

2.4. Penetration Depth

In § 2.3 we have seen that the perfect diamagnetism of a superconductor prevents electric currents flowing through the body of the material. On the other hand, currents cannot be confined entirely to the surface because, if this were so, the current sheet would have no thickness and the current density would be infinite, which is a physical impossibility. In fact the currents flow within a very thin surface layer whose thickness is of the order of 10^{-5} cm, although the exact value varies for different metals. We shall see that, though this thickness is so small, it plays a very important part in determining the properties of superconductors.

When a superconducting sample is in an applied magnetic field, the screening currents which circulate to cancel the flux inside must flow within this surface layer. Consequently, the flux density does not fall abruptly to zero at the boundary of the metal but dies away within the region where the screening currents are flowing. For this reason the depth within which the currents flow is called the *penetration depth*, because it is the depth to which the flux of the applied magnetic field appears to penetrate. Thus, although we speak of a superconductor as being perfectly diamagnetic, there is in fact a very slight penetration of magnetic flux, the flux density dying away at the surface as shown in

Fig. 2.6. (This is somewhat like the "skin depth" to which high frequency alternating fields penetrate in a normal conductor.) Consider the boundary of a semi-infinite slab, as shown in Fig. 2.6. If at a distance x into the metal the flux density falls to a value $B(x)$, we can define the penetration depth λ by

$$\int_0^\infty B(x)\,dx = \lambda B(0), \tag{2.2}$$

where $B(0)$ is the flux density at the surface of the metal. In other words, there would be the same amount of flux inside the superconductor if the flux density of the external field remained constant to a distance λ into the metal.

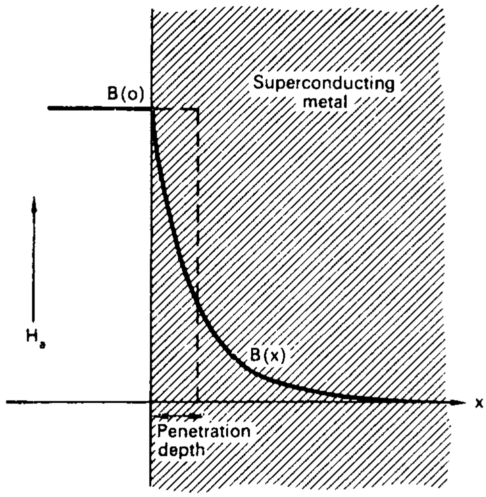

FIG. 2.6. Penetration of magnetic flux into surface of superconductor.

The London theory of superconductivity, which we shall discuss in the next chapter, predicts that in a specimen which is much thicker than the penetration depth the magnetic flux density decays exponentially as it penetrates into the metal, i.e.

$$B(x) = B(0)e^{-x/\lambda}.$$

However, in simple calculations it is often sufficient to use the approximation that the flux density $B(0)$ of the applied field remains constant to a distance λ into the metal and then abruptly falls to zero.

Because the penetration depth is so small, we do not notice the flux penetration in magnetic measurements on ordinary sized specimens,† and these appear to be perfectly diamagnetic with $\mathbf{B} = 0$. We shall, for convenience, continue to refer to reasonably large superconducting bodies as being "perfectly" diamagnetic, the very small magnetic flux at the surface being included in this term. The penetration of the magnetic flux becomes noticeable, however, if we make measurements on small samples, such as powders or thin films, whose dimensions are not much greater than the penetration depth. In these cases there is an appreciable magnetic flux density right through the metal; there is no longer perfect diamagnetism, and consequently the properties are rather different from those of the bulk superconducting metal. We shall deal fully with the special features of thin specimens in Chapter 8.

2.4.1. Variation with temperature

The penetration depth does not have a fixed value but varies with temperature, as shown in Fig. 2.7. At low temperatures it is nearly in-

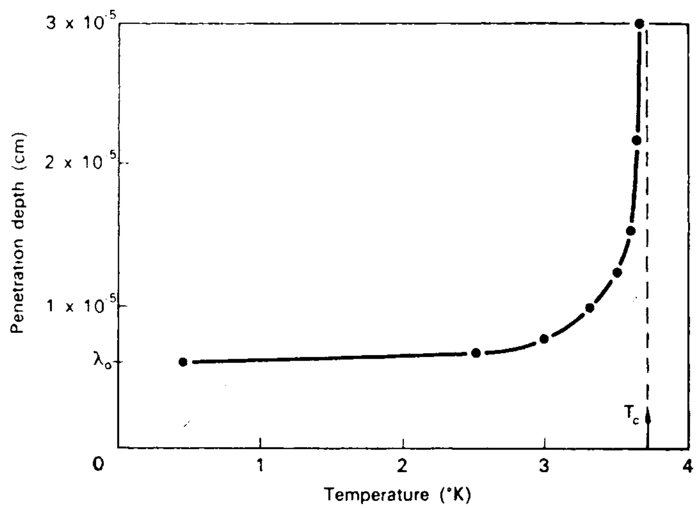

FIG. 2.7. Variation with temperature of penetration depth in tin (after Schawlow and Devlin).

dependent of temperature and has a value λ_0 characteristic of the particular metal (Table 2.1). Above about 0·8 of the transition temperature, however, the penetration depth increases rapidly and approaches infinity as the temperature approaches the transition temperature.

† The methods by which such measurements can be made are discussed in § 4.4.

The variation of penetration depth with temperature is found to fit very closely the relation

$$\lambda = \frac{\lambda_0}{(1-t^4)}, \tag{2.3}$$

where t is the temperature relative to the transition temperature, $t = T/T_c$.

Perfect diamagnetism does not, therefore, occur in specimens which are very close to their transition temperature. The decrease in penetration depth is, however, so rapid as the temperature falls below T_c that

TABLE 2.1. SOME VALUES OF THE PENETRATION DEPTH AT 0°K

	In	Al	Pb
λ_0 (cm)	$6{\cdot}4 \times 10^{-6}$	$5{\cdot}0 \times 10^{-6}$	$3{\cdot}9 \times 10^{-6}$

any *large* departure from perfect diamagnetism would be extremely difficult to detect in bulk specimens because of the difficulty of holding the temperature sufficiently constant during a measurement. For example, to observe a penetration depth of about 1 mm a specimen would, according to (2.3), have to be held at a temperature only 10^{-7} per cent below the transition temperature! Furthermore, we would require a specimen so pure and uniform that the transition to the superconducting state would take place within a temperature range which was less than 10^{-7} per cent of the transition temperature. The experimental relation (2.3) has been obtained by observing relatively small increases in the penetration depth as a superconducting metal is warmed towards its transition temperature. In one experimental arrangement† a rod of pure superconductor is surrounded by a closely fitting solenoid. When the temperature of the metal is lowered so that it becomes superconducting there can be no magnetic flux in the metal, except just below the surface within the penetration depth. Because the solenoid fits the rod very closely, its self-inductance is largely dependent on the magnitude of this penetration depth. A capacitor is connected across the solenoid, the combination forming the tank circuit of an oscillator of about 100 kHz frequency. When the inductance alters, because of a change in penetration depth, the oscillator frequency shifts. The frequency can be measured with great precision on an accurate frequency meter. In the

† A. L. Schawlow and G. E. Devlin, *Phys. Rev.* **113**, 120 (1959).

experiment of Schawlow and Devlin a frequency shift of 0·1 Hz was equivalent to a change of 4×10^{-8} cm in the penetration depth in the metal core. Such an experiment needs to be done with great care because spurious effects, not due to the change in penetration depth, may alter the resonant frequency of the solenoid and capacitor. By this method the temperature variation of the penetration depth in tin was observed, and the results are those shown in Fig. 2.7. The very rapid increase in penetration depth as the transition temperature is approached is in good accord with (2.3).

Unless we specify otherwise, when in this book we speak of the penetration depth, we mean the value λ_0 to which λ tends when the metal is at temperatures which are appreciably below its transition temperature.

The penetration depth in a superconducting metal also depends on the purity, the penetration depth increasing as the metal becomes more impure. For example, in tin containing 3% indium the penetration depth is twice that of pure tin.