CHAPTER 4

THE CRITICAL MAGNETIC FIELD

WE SHALL see in Chapter 9 that, if a metal is to remain superconducting, the net momentum of the superelectrons must not exceed a certain value. For this reason there is a limit to the density of resistanceless current† that can be carried by any region in the metal. Let us call this the *critical current density* J_c of the metal. This critical current density applies both to a current passed along the specimen from an external source and to screening currents which shield the specimen from an applied magnetic field. We now show that, as a result of this critical current density, a superconducting metal will be driven normal if a sufficiently strong magnetic field is applied to it.

As we have seen, the perfect diamagnetism of a superconductor arises because, in an applied magnetic field, resistanceless surface currents circulate so as to cancel the flux density inside. If the strength of the applied magnetic field is increased, the shielding currents must also increase in order to maintain perfect diamagnetism. If the applied magnetic field is increased sufficiently, the critical current density will be reached by the shielding currents and the metal will lose its superconductivity. The shielding currents then cease and the flux due to the applied magnetic field is no longer cancelled within the metal.‡ There is therefore a limit to the strength of magnetic field which can be applied to a superconductor if it is to remain superconducting. This destruction of superconductivity by a sufficiently strong magnetic field is one of the most important properties of a superconductor.

At any point in a superconductor there is a definite relationship

† Strictly speaking it is not the current but the metal which is resistanceless. However, the expression "resistanceless current" is often used and we shall make use of this convenient phrase to avoid circumlocution.

‡ The transition from the perfectly diamagnetic superconducting state to the non-magnetic normal state is a *reversible* transition. The screening currents do not die away with dissipation of energy and do not generate heat in the material. In fact, as we shall see from thermodynamic arguments in § 5.2.2, if a thermally isolated specimen is driven normal by a magnetic field its temperature *falls*.

between the supercurrent density J_s and the magnetic flux density, though the values of these are only appreciable within the penetration depth. This relationship can be obtained from the London equations (Chapter 3). The critical current density will therefore be reached at the surface when the flux density of the applied magnetic field (i.e. the flux density at the surface) is increased to a certain value. We may call this the "critical flux density" B_c. However, the flux density *outside* the metal always equals $\mu_0 H$, where H is the magnetic field strength, so we may equally well refer to a ***critical magnetic field strength***, $H_c = B_c/\mu_0$. Notwithstanding our view that B rather than H is the basic magnetic quantity (see Appendix A), it is usual in the literature to refer to the critical magnetic field strength H_c rather than the critical flux density B_c. In this book we shall follow this convention, and refer to the critical magnetic field strength H_c remembering that this is always related to the critical flux density by $H_c = B_c/\mu_0$.

4.1. Free Energy of a Superconductor

We saw in § 2.2.1 that the state of magnetization of a superconductor depends only on the *values* of the applied magnetic field and temperature and not on the way these external conditions were arrived at. This implies that, whether or not there is an applied magnetic field, the transition from the superconducting to the normal state is reversible, in the thermodynamic sense. We may therefore apply thermodynamic arguments to a superconductor, using the temperature and magnetic field strength as thermodynamic variables.

It is possible to deduce something about the critical magnetic field by considering what effect the application of a magnetic field has on the free energy of a superconducting specimen. We are interested in the free energy because in any system the stable state is that with the lowest free energy. In considering the critical magnetic field of a superconductor we are interested in the *Gibbs* free energy, because we want to compare the difference in the magnetic contribution to the free energy of two phases, superconducting and normal, when they are in the same applied magnetic field (i.e. with the *intensive* thermodynamic variable constant).

Consider a specimen of superconductor in the form of a long rod. (At this stage we consider a long rod so as to reduce to a negligible degree special demagnetizing effects which are due to the ends of the specimen. These effects are discussed in § 6.2.) When the specimen is cooled below its transition temperature it becomes superconducting. Therefore, below the transition temperature the free energy of the superconducting state must be less than that of the normal state, otherwise the metal would remain normal. Suppose that at a temperature T, and in the absence of an applied magnetic field ($H_a = 0$), the Gibbs free energy per unit volume of the superconducting state is $g_s(T, 0)$ and that of the normal state is $g_n(T, 0)$ (Fig. 4.1). Now let a magnetic field of strength H_a be applied parallel

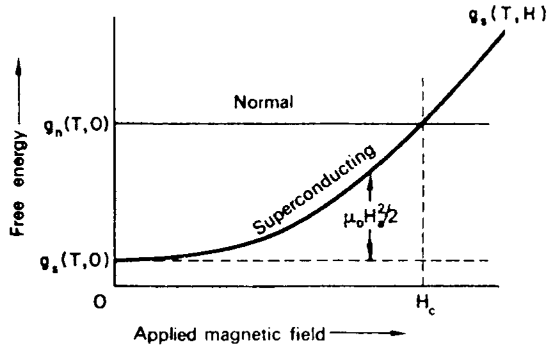

FIG. 4.1. Effect of applied magnetic field on Gibbs free energy of normal and superconducting states.

to the length of the rod. Any substance, which in an applied field H_a acquires a magnetization I, changes its free energy per unit volume by an amount†

$$\Delta g(H_a) = -\mu_0 \int_0^{H_a} I dH_a. \tag{4.1}$$

So in the case of the field producing a positive magnetization, i.e. the magnetization in the same direction as the magnetic field, the free energy is lowered. [Note that (4.1) implies that, when a substance is magnetized by an applied magnetic field, its free energy changes by an amount proportional to the area under its magnetization curve (I v. H_a). This is a useful general result, which we shall employ several times.] In the case of a superconducting specimen the application of a magnetic field produces a *negative* magnetization which, if penetration of the field is neglected, exactly cancels the flux due to the applied field, so that $I = -H$. The free energy per unit volume is therefore *increased* to

$$g_s(T, H) = g_s(T, 0) + \mu_0 \int_0^{H_a} |I|\, dH_a.$$

† See Appendix B.

In fact $|I| = H$, so the magnetic field raises the free energy density to

$$g_s(T, H) = g_s(T, 0) + \mu_0 \frac{H_a^2}{2}. \tag{4.2}$$

So, when we apply a magnetic field to a superconductor, its free energy increases to this value due to the magnetization (Fig. 4.1). The *normal* state, however, is virtually non-magnetic and acquires negligible magnetization in an applied magnetic field. Consequently the application of a magnetic field does not change the free energy of the normal state though it raises that of the superconducting state. If the field strength is increased enough, the free energy of the superconducting state will be raised above that of the normal state, and, in this case, the metal will not remain superconducting but will become normal. This occurs when $g_s(T, H) > g_n(T, 0)$, which with (4.2) gives

$$\mu_0 \frac{H_a^2}{2} > [g_n(T, 0) - g_s(T, 0)].$$

There is therefore a maximum magnetic field strength that can be applied to a superconductor if it is to remain in the superconducting state. This critical magnetic field strength is given by

$$H_c(T) = \left\{ \frac{2}{\mu_0} [g_n(T, 0) - g_s(T, 0)] \right\}^{\frac{1}{2}}. \tag{4.3}$$

This critical magnetic field which we have derived by a thermodynamic argument is the same as the critical field which we discussed on p. 40 in terms of a critical current density.

The critical magnetic field strength can be measured quite simply by applying a magnetic field parallel to a wire of superconductor and observing the strength at which resistance appears.

4.2. Variation of Critical Field with Temperature

If the critical magnetic field of a superconductor is measured, its value is found to depend on the temperature (Fig. 4.2), falling from some value H_0, at very low temperatures, to zero at the superconducting transition temperature T_c. A diagram such as Fig. 4.2 can be called the ***phase diagram*** of a superconductor. The metal will be superconducting for any combination of temperature and applied field which gives a point, such as P, lying within the shaded region. As the arrows indicate, the metal can be driven into the normal state by increasing either the temperature or the applied field, or both.

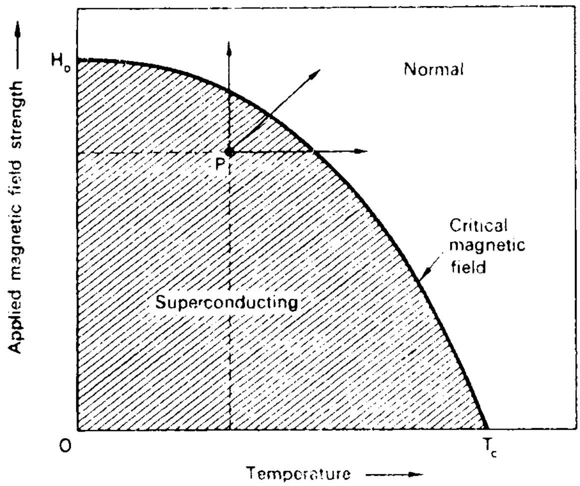

FIG. 4.2. Phase diagram of a superconductor, showing variation with temperature of the critical magnetic field.

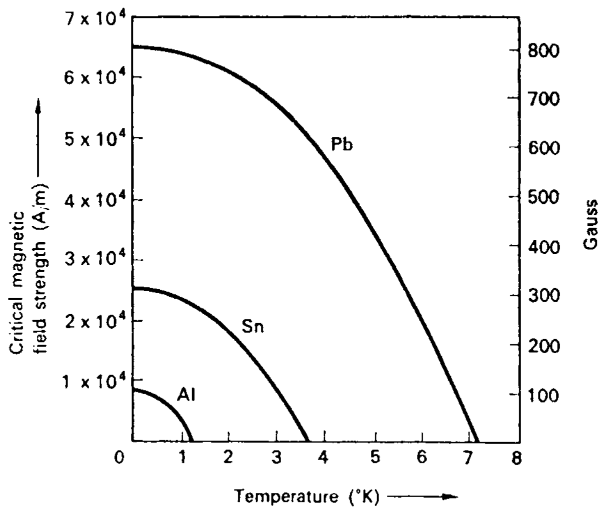

FIG. 4.3. Critical fields of some superconductors.

The value of H_0 is different for each superconducting metal; metals with low transition temperatures have low critical fields at absolute zero. So each superconductor has a different phase diagram (Fig. 4.3). From experiment it has been found that the critical fields fall off almost as the square of the temperature, so the critical field curves are closely approximated by parabolas of the form

$$H_c = H_0\,[1 - (T/T_c)^2], \tag{4.4}$$

where H_0 is the critical field at absolute zero and T_c is the transition temperature. Each superconductor can be characterized by its particular values of H_0 and T_c, and, knowing these, we can use (4.4) to find the critical field at any temperature. Table 1.1 lists values of H_0 and T_c for the superconducting elements.

We may remark here that there is no fundamental significance in the relation between critical field and temperature being a parabola. It has merely been found experimentally that the variation can be conveniently described to within a few per cent by a parabolic curve. The experimental curves are not in fact exactly parabolas, and to describe them accurately one would need a polynomial expression. For most calculations, however, it is sufficient to use the parabola given by (4.4).

The Cryotron

The existence of a critical magnetic field has been made use of in a controlled switch called a *cryotron* (Fig. 4.4). The current $\mathscr{I}$ which is to be controlled flows along a straight wire of tantalum called the "gate".

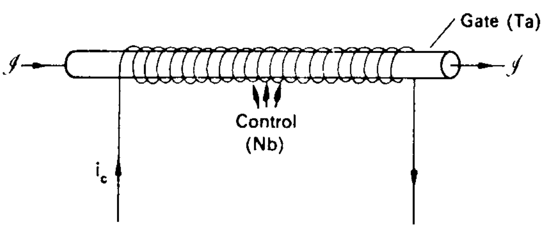

FIG. 4.4. Cryotron of tantalum and niobium.

Around this, but insulated from it, is a niobium wire wound in a long single-layer coil called the "control". When cooled to 4·2°K by immersion in liquid helium both the tantalum gate and the niobium control are superconducting, and the gate offers no resistance to the passage of the current $\mathscr{I}$. If, however, a current i_c is passed through the control coil, it generates a magnetic field along the gate, and, if the control current is large enough, the gate is driven normal by the magnetic field, and the appearance of resistance reduces the current $\mathscr{I}$. The control coil, however, remains resistanceless because the critical field of niobium is considerably higher than that of tantalum. Hence the current $\mathscr{I}$ through the gate can be controlled by a smaller current in the control, and the device is analogous to a relay. Small cryotrons were first developed as fast acting switches for possible use in digital computers. Large cryotrons can be used to control the currents in superconducting magnet circuits (see § 1.3).

4.3. Magnetization of Superconductors

We now discuss how the magnetization of a superconducting specimen varies as a magnetic field of increasing strength is applied. Let us again consider a rod of superconductor and imagine a magnetic field H_a to be applied parallel to its length. Figure 4.5a shows how the flux density B inside the specimen varies as we increase the strength of the

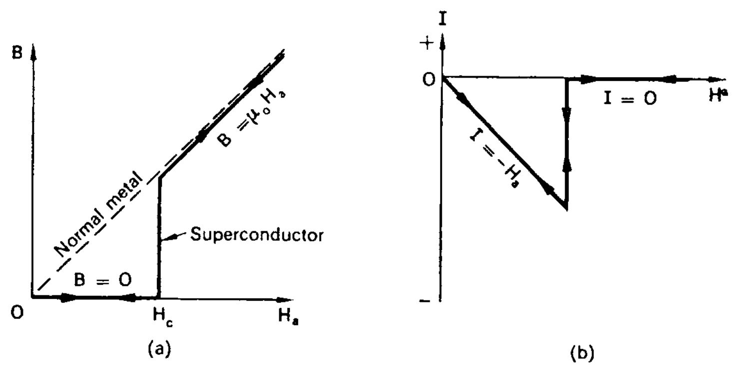

FIG. 4.5. Magnetic behaviour of a superconductor.

applied field. Normal metals (excluding the special ferromagnetic metals, such as iron) are virtually non-magnetic and so the flux density B inside them is proportional to the strength of the applied field, $B = \mu_0 H_a$, as shown by the dotted line in Fig. 4.5a. A superconductor, however, is perfectly diamagnetic if we neglect the penetration depth, and as the applied magnetic field strength is increased, the flux density within the specimen remains at zero. But when the applied field strength reaches the critical value H_c, the superconductor is driven into the normal state, and the flux due to the applied field is no longer cancelled inside. At all higher applied field strengths the superconductor behaves just like a normal metal. For a pure specimen this behaviour is reversible; if the applied magnetic field is decreased from a high value, the specimen goes back into the superconducting state at the value H_c and below this there is no net flux inside.

We can describe the magnetic behaviour of a superconductor in another way. We have seen that, when a metal is in the superconducting state, there is no magnetic flux inside, because surface currents circulate to give the specimen a magnetization I exactly equal and opposite to the applied field, so that $I = -H_a$. Figure 4.5b shows how the magnetization of a superconductor varies with the strength of the applied magnetic field. When the applied field strength reaches H_c the superconductor becomes normal and the negative magnetization disappears. At higher applied fields the superconductor has, like any normal metal, virtually no magnetization. Figure 4.5 is, of course, just two equivalent ways of presenting the same information. We need to be familiar with the form of both curves, because sometimes it is convenient to consider the internal flux density, whereas on other occasions it is more convenient to consider magnetization.

4.3.1. "Non-ideal" specimens

The magnetic properties we have been considering so far in this chapter are those that would be shown by "ideal" specimens, i.e. those containing no impurities or crystalline faults. Any real specimen is, however, not perfect, and its behaviour will depart to some extent from the behaviour we have just described. Nevertheless, it is possible with great care to produce specimens so nearly perfect that they have properties very closely approximating to the ideal. However, the greater the degree of imperfection the greater will be the departure from ideal behaviour.

An ideal specimen has a sharply defined critical field strength and its magnetization curve is completely reversible. Figure 4.6 illustrates the

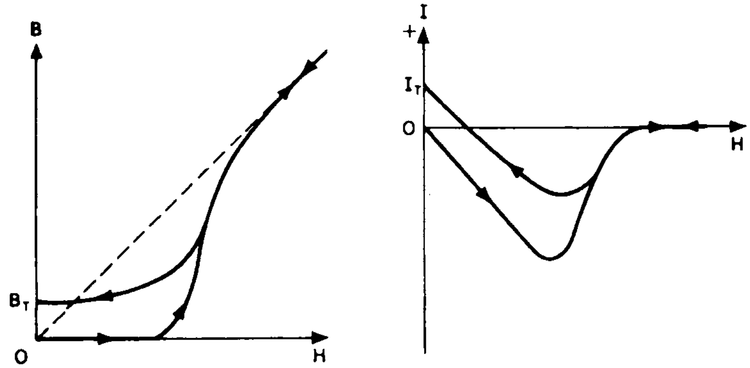

FIG. 4.6. Magnetic behaviour of a non-ideal superconductor.

magnetic behaviour of an imperfect sample. It can be seen that there is no longer a sharply defined critical magnetic field, the transition from the superconducting to normal state being "smeared out" over a range of applied field strengths. Furthermore, the magnetization is not reversible; in decreasing fields the curves follow different paths from those traced in the original increasing field. We call this *hysteresis*. Finally, when the applied field has been reduced to zero, there may remain some positive magnetization of the sample, giving rise to a residual flux density B_T and magnetization I_T. We say the sample has *trapped flux*. In this condition the superconductor is like a permanent magnet.

We see, therefore, that a non-ideal specimen may show:

1. Ill-defined critical magnetic field.
2. Magnetic hysteresis.
3. Trapped flux.

These three departures from ideal behaviour do not necessarily all occur together. For example, a specimen may not have a sharp critical field and may show hysteresis but still not trap any flux. Defects involving large numbers of atoms, such as particles of another substance or the chains of displaced atoms known as dislocations, tend to give rise to hysteresis and trapped flux, whereas impurity atoms and unevenness of composition reduce the sharpness of the critical field. The reasons why different kinds of impurity and imperfection produce the various departures from ideal behaviour are complicated and not yet fully understood, and so we shall not discuss them in detail here. However, these effects are of considerable practical importance and we shall return to them again in Chapter 12.

4.4. Measurement of Magnetic Properties

The techniques which can be used to measure the magnetic characteristics of superconductors do not differ in principle from those used in measurements on ordinary magnetic materials, such as a ferromagnetics; but they must, of course, be suitable for use at very low temperatures. The methods fall into two classes: those which measure the flux density B in the sample and those which measure the sample's magnetization I (Fig. 4.5). Though either of these gives the full information on the magnetic properties of the sample, it may be convenient to choose one or other method of measurement, depending on circumstance. Many different types of apparatus have been used, of various degrees of complexity depending on the sensitivity, degree of automation, etc., required. However, these are all based on the simple methods we now describe.

4.4.1. Measurement of flux density

This is a very simple measurement which does not require any moving parts in the low temperature region. The principle is to measure the magnetic flux which appears in a specimen when a magnetic field is applied. On the specimen X (Fig. 4.7) is wound a "pick-up" coil C consisting of a few hundred turns of fine wire. The ends of this coil are connected to a ballistic galvanometer G outside the low temperature apparatus. A magnetic field H can be applied parallel to the axis of the specimen by means of the solenoid S, and when the magnetic field is suddenly applied by closing the switch, the ballistic galvanometer will be deflected by an amount proportional to the flux threading the pick-up coil C, i.e. the deflection is proportional to the flux density B. Hence by successively applying stronger magnetic fields we can follow the varia-

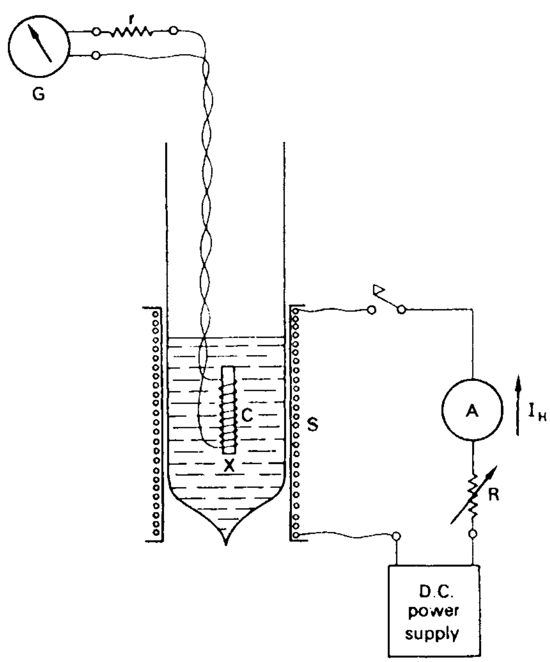

FIG. 4.7. Measurement of magnetic flux density in a superconductor.

tion of B with field strength H. Figure 4.8 shows a B versus H characteristic obtained in this way. The critical field H_c at which the specimen ceases to be perfectly diamagnetic can clearly be seen.

In the case of a non-ideal specimen some flux will be trapped and so, when the field is switched off, there will be a smaller reverse deflection of

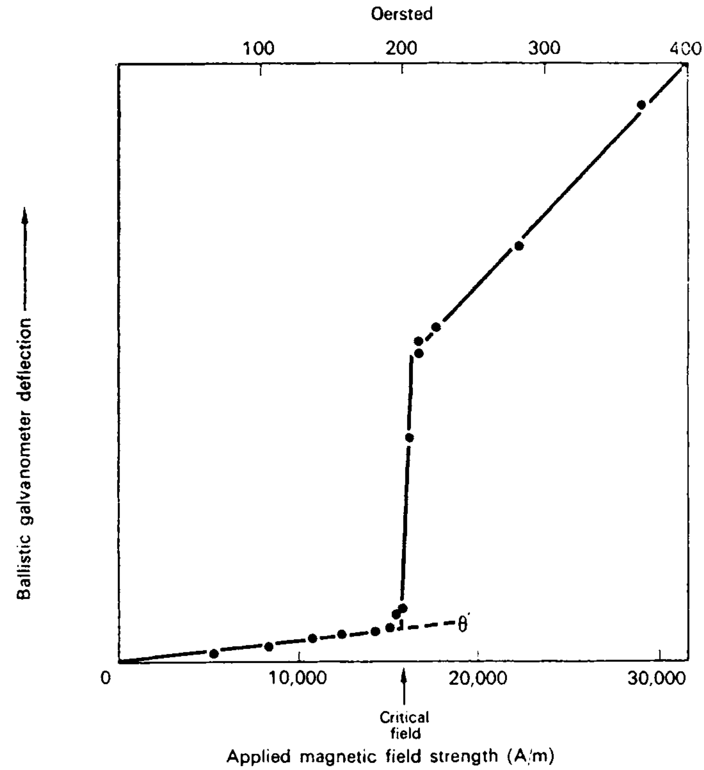

FIG. 4.8. Experimental results of ballistic flux measurement on a superconductor (tantalum at 3·7°K). The small "background" deflection θ' is due to the flux density produced by the applied field in the actual wire of the pick-up coil (after Rose-Innes).

the galvanometer. It is therefore necessary to note the galvanometer deflection both when switching on and when switching off the applied magnetic field in order to observe how much flux is trapped.

4.4.2. Measurement of magnetization

In this method the specimen is moved into and out of a pick-up coil which is connected to a ballistic galvanometer. The throw of the galvanometer will be proportional to the flux carried by the sample and this will be proportional to its magnetization. The method is illustrated in Fig. 4.9. The specimen X is mounted on the end of a sliding rod R so that it can be moved from the upper pick-up coil A to the lower one B. A and B are identical coils, except that they are wound in opposite direc-

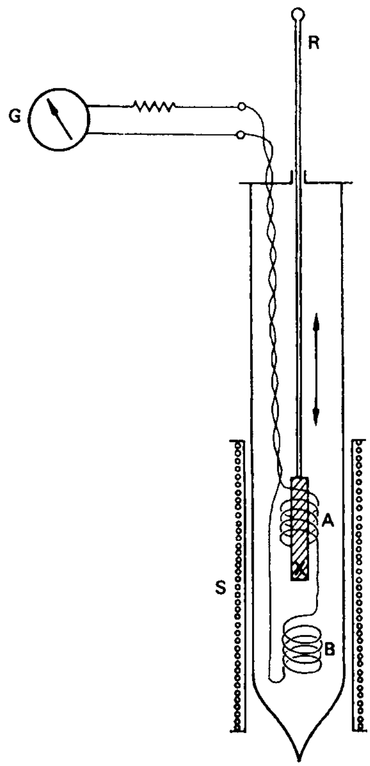

FIG. 4.9. Apparatus to measure magnetization.

tions. To measure the magnetization of the sample a steady magnetic field of the required magnitude is applied by means of the solenoid S and the specimen is then moved quickly from coil A into coil B. In the case of a diamagnetic sample, the flux threading coil A will therefore increase and the flux through B will decrease. Since the two coils are connected in series opposition the e.m.f.s. induced in them will add together and the ballistic galvanometer will swing by an amount proportional to the magnetization of the sample. The measurement is repeated at gradually increasing applied magnetic field strengths, and so we may construct a graph of sample magnetization as a function of applied field strength.

Because A and B are wound in opposite directions any e.m.f.s. due to unintended fluctuations of the applied field strength should cancel and not deflect the galvanometer.

A feature of measurements on superconductors is that they are self-calibrating. At applied field strengths below the critical field strength, the slope of the I versus H curve always equals -1.

4.4.3. Integrating method

In this method, illustrated in Fig. 4.10, we again have two identical pick-up coils A and B connected in series opposition, but the specimen is now permanently located in one of them. The current to the solenoid S is

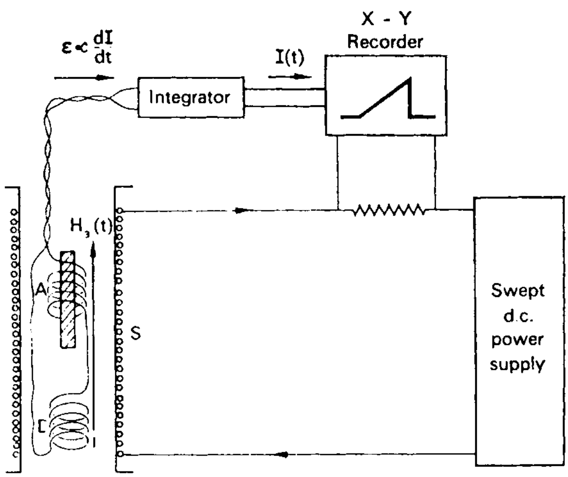

FIG. 4.10. Integration method of measuring magnetization.

gradually and smoothly increased so that a continuously increasing magnetic field is applied to the pick-up coils. An e.m.f. will be developed across each coil, proportional to the rate of change of the magnetic flux threading it, but, since one coil contains the sample and the other does not, and since they are in series opposition, the net e.m.f. $\mathscr{E}$ across the two coils will be proportional to the rate of change of magnetization of the sample:

$$\mathscr{E} = \mathscr{E}_A - \mathscr{E}_B \propto \frac{d}{dt}\mu_0(H + I) - \frac{d}{dt}\mu_0 H$$

or

$$\mathscr{E} \propto \frac{dI}{dt}.$$

This e.m.f. is fed into an electronic integrator, a circuit whose output voltage is related to its input voltage by $V_{out} \propto \int V_{in}\, dt$. The output of the integrator is therefore

$$V_{out} \propto \int_0^t \frac{dI}{dt}\, dt = I,$$

so at any time the output voltage of the integrator is proportional to the magnetization of the sample.

This method of measuring magnetization has the advantage that no movement of the specimen is required, but the electronic equipment is more complicated than the apparatus described in the previous two sections.