CHAPTER 7

TRANSPORT CURRENTS IN SUPERCONDUCTORS

7.1. Critical Currents

THE EARLY workers in superconductivity soon discovered that there is an upper limit to the amount of current that can be passed along a piece of superconductor if it is to remain resistanceless. We call this the *critical current* of that particular piece. If the current exceeds this critical value, some resistance appears.

We now show that the critical current is related to the critical magnetic field strength H_c. We saw in Chapter 3 that all currents in a superconductor flow at the surface within the penetration depth, the current density decreasing rapidly from some value $\mathcal{J}_a$ at the surface. It was pointed out in Chapter 4 that superconductivity breaks down if the supercurrent density exceeds a certain value which we call the critical current density $\mathcal{J}_c$.

In general there can be two contributions to the current flowing on the surface of a superconductor. Consider, for example, a superconducting wire along which we are passing a current from some external source such as a battery. We call this current the "transport current" because it transports charge into and out of the wire. If the wire is in an applied magnetic field, screening currents circulate so as to cancel the flux inside the metal. These screening currents are superimposed on the transport current, and at any point the current density $\mathbf{J}$ can be considered to be the sum of a component $\mathbf{J}_i$ due to the transport current and a component $\mathbf{J}_H$ which arises from the screening currents

$$\mathbf{J} = \mathbf{J}_i + \mathbf{J}_H.$$

We may expect that superconductivity will break down if the magnitude of the *total* current density $\mathbf{J}$ at any point exceeds the critical current density $\mathcal{J}_c$.

According to the London equation (3.17) there is a relation between the supercurrent density at any point and the magnetic flux density at that point, and this same relation holds whether the supercurrent is a screening current, a transport current or a combination of both. Hence, when a current flows on a superconductor, there will at the surface be a flux density B and a corresponding field strength $H(= B/\mu_0)$ which is related to the surface current density J_a.

If the total current flowing on a superconductor is sufficiently large, the current density at the surface will reach the critical value J_c and the associated magnetic field strength at the surface will have a value H_c. Conversely, a magnetic field of strength H_c at the surface is always associated with a surface supercurrent density J_c. This leads to the following general hypothesis: *a superconductor loses its zero resistance when, at any point on the surface, the total magnetic field strength, due to transport current and applied magnetic field, exceeds the critical field strength H_c.* The maximum amount of *transport* current which can be passed along a piece of superconductor without resistance appearing is what we call the critical current of that piece. Clearly the stronger the applied magnetic field the smaller is this critical current.

If there is no applied magnetic field the only magnetic field will be that generated by any transport current, so in this case, the critical current will be that current which generates the critical magnetic field strength H_c at the surface of the conductor. This special case of the general rule stated above is known as Silsbee's hypothesis† and was formulated before the concept of critical current density was appreciated. We shall call the more general rule for the critical current given in the previous paragraph the "generalized form" of Silsbee's hypothesis.

We saw in Chapter 4 that the critical magnetic field strength H_c depends on the temperature, decreasing as the temperature is raised and falling to zero at the transition temperature T_c. This implies that the critical current density depends on temperature in a similar manner, the critical current density decreasing at higher temperatures. Conversely, if a superconductor is carrying a current, its transition temperature is lowered.

7.1.1. Critical currents of wires

Let us consider a cylindrical wire of radius a. If, in the absence of any externally applied magnetic field, a current i is passed along the wire, a magnetic field will be generated at the surface whose strength H_i is given by

† F. B. Silsbee, *J. Wash. Acad. Sci.*, **6**, 597 (1916).

$$2\pi a H_i = i.$$

The critical current will therefore be

$$i_c = 2\pi a H_c. \tag{7.1}$$

This relation for the critical current can be tested by measuring the maximum current a superconducting wire can carry without resistance appearing, and it is found that, in the absence of any externally applied magnetic field, eqn. (7.1) predicts the correct value.

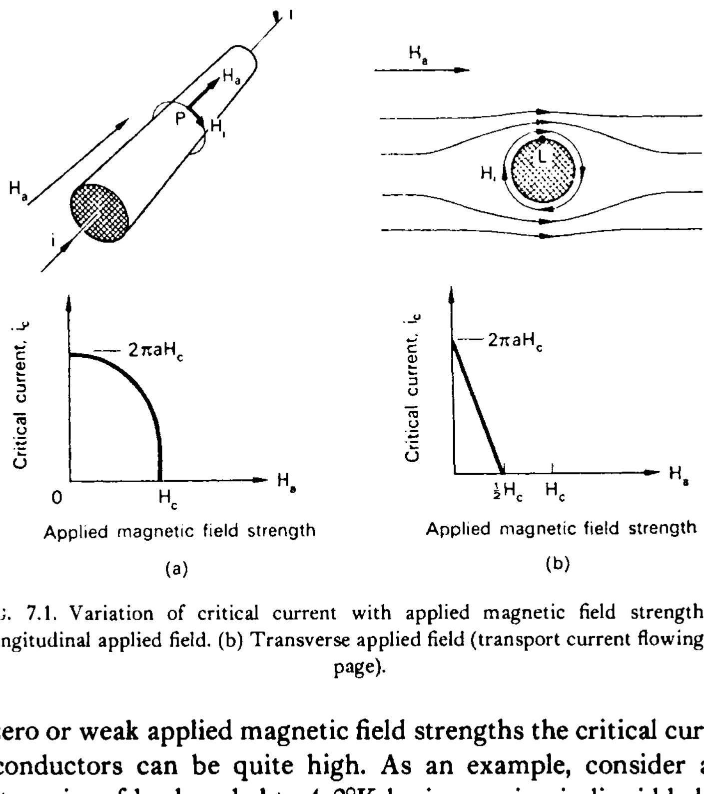

FIG. 7.1. Variation of critical current with applied magnetic field strength. (a) Longitudinal applied field. (b) Transverse applied field (transport current flowing into page).

In zero or weak applied magnetic field strengths the critical currents of superconductors can be quite high. As an example, consider a 1 mm diameter wire of lead cooled to 4·2°K by immersion in liquid helium. At this temperature the critical field of lead is about $4{\cdot}4 \times 10^4$ A m^{-1} (550 gauss) so, in the absence of any applied magnetic field, the wire can carry up to 140 A of resistanceless current.

Let us now consider to what extent the critical current is reduced by the presence of an externally applied magnetic field. First suppose that an applied magnetic field of flux density B_a and strength H_a ($= B_a/\mu_0$) is in a direction parallel to the axis of the wire (Fig. 7.1a). If a current i is passed along the wire it generates a field encircling the wire, and at the surface of the wire the strength of this field is $H_i = i/2\pi a$. This field and the applied field add vectorially and, because in this case they are at right angles, the strength H of the resultant field at the surface is given by $(H_a^2 + H_i^2)^{\frac{1}{2}}$ or

$$H^2 = H_a^2 + (i/2\pi a)^2.$$

The critical value i_c of the current occurs when H equals H_c:

$$H_c^2 = H_a^2 + \frac{i_c^2}{4\pi^2 a^2}. \tag{7.2}$$

H_c is a constant, and so this equation, which expresses the variation of i_c with H_a, is the equation of an ellipse. Consequently, the graph representing the decrease in critical current as the strength of a longitudinal applied magnetic field is increased has the form of a quadrant of an ellipse (Fig. 7.1a). In this configuration the magnetic flux density is uniform over the surface of the wire and the flux lines follow helical paths.

Another case of importance occurs when an applied magnetic field is normal to the axis of the wire (Fig. 7.1b). (We assume here that the applied field is not strong enough to drive the superconductor into the intermediate state.) In this case the total flux density is not uniform over the surface of the wire; the flux densities add on one side of the wire and substract on the other. The maximum field strength occurs along the line L. Here, because of demagnetization, as pointed out at the end of § 6.1, a field $2H_a$ is superimposed on the field H_i to give a total field

$$H = 2H_a + H_i = 2H_a + \frac{i}{2\pi a}.$$

The general form of Silsbee's rule states that resistance first appears when the total magnetic field strength at *any* part of the surface equals H_c, and so the critical current in this case is given by

$$i_c = 2\pi a(H_c - 2H_a). \tag{7.3}$$

In this case, therefore, the critical current decreases linearly with increase in applied field strength, falling to zero at $\frac{1}{2}H_c$.

It should be emphasized that the critical current of a specimen is defined as the current at which it ceases to have zero resistance, *not* as the current at which the full normal resistance is restored. The amount of resistance which appears when the critical current is exceeded depends on a number of circumstances, which we examine in the next section.

7.2. Thermal Propagation

The variation of critical current with applied magnetic field predicted by (7.2) and (7.3) has been confirmed by experiment, though measurement of critical currents, especially in low magnetic fields where the current values can be high, is not always easy. To see why there may be a difficulty, we now examine the processes by which resistance returns to a wire when the critical current is exceeded. Consider, for example, a cylindrical wire of superconductor. In practice no piece of wire can have perfectly uniform properties along its length; there may be accidental variations in composition, thickness, etc., or the temperature may be slightly higher at some points than others. As a result the value of critical

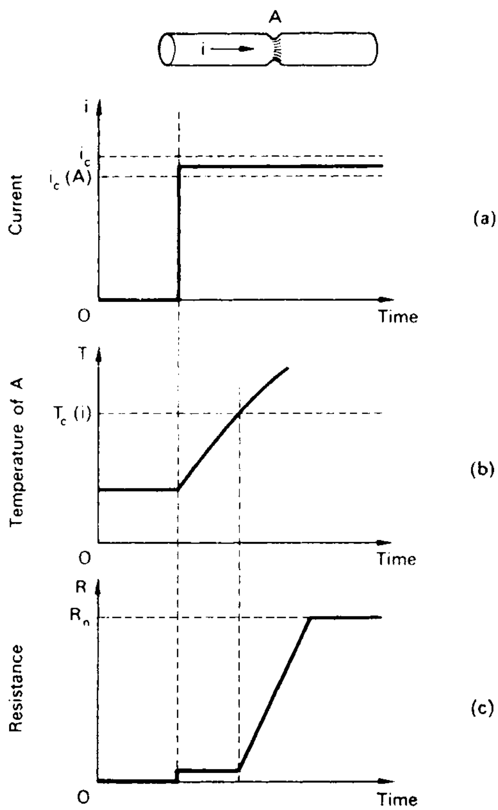

FIG. 7.2. Thermal propagation. (b) shows the temperature variation of the region A resulting from the current increase shown at (a). (c) shows the return of the wire's resistance when the normal region spreads from A.

current will vary slightly from point to point, and there will be some point on the wire which has a lower critical current than the rest of the wire. In Fig. 7.2 such a region is represented by the section A where the wire is slightly narrower. Suppose we now pass a current along the wire and increase its magnitude, until the current just exceeds the critical current $i_c(A)$ of the section A (Fig. 7.2a). This small section will become resistive while the rest of the wire remains superconducting, so a very small resistance r appears in the wire. At A the current i is flowing through resistive material and at this point heat is generated at a rate proportional to i^2r. Consequently the temperature at A rises, and heat flows away from A along the metal and into the surrounding medium at a rate which depends on the temperature increase of A, the thermal conductivity of the metal, the rate of heat loss across the surface, etc. The temperature of A will rise until the rate at which heat flows away from it equals the rate i^2r at which the heat is generated. If the rate of heat generation is low, the temperature of A rises only a small amount and the wire remains indefinitely in this condition. If, however, heat is generated at a high rate, either because the resistance of A is high or because the current i is large, the temperature of A may rise above the critical temperature of the wire (Fig. 7.2b). The presence of the current has in fact reduced the transition temperature of the superconducting wire from T_c to a lower value $T_c(i)$, and if, as a result of the heating of A, the regions adjacent to A are heated above $T_c(i)$ they will become normal. The current i is now flowing through these new normal regions and generates heat which drives the regions adjacent to them normal. Consequently, even though the current i is held constant, a normal region spreads out from A until the whole wire is normal and the full normal resistance R_n is restored (Fig. 7.2c). This process whereby a normal region may spread out from a resistive nucleus is called *thermal propagation*, and we see that it is more likely to occur if the critical current is large and if the resistivity of the normal state is high (e.g. in alloys).

On account of thermal propagation there can be difficulty in measuring the critical current of a specimen, especially in low or zero magnetic fields where the current value may be high. Consider a superconducting specimen of uniform thickness, as shown in Fig. 7.3a, whose critical current we are attempting to measure by increasing the current until a voltage is observed. If the current is less than the critical current there will be no voltage drop along the specimen and no heat will be generated in it. However, the leads carrying the current to the specimen are usually of ordinary non-superconducting metal and so heat is generated in these by the passage of the current. Consequently the ends of the specimen, where it makes contact with the leads, will be slightly heated and will have a lower critical current than the body of the specimen. As the current is increased, therefore, the ends go normal at a current less than the true critical current of the specimen, and normal regions may spread through the wire by thermal propagation. Consequently a voltage is observed at a current less than the true critical value. To lessen the risk of thermal propagation from the contacts one

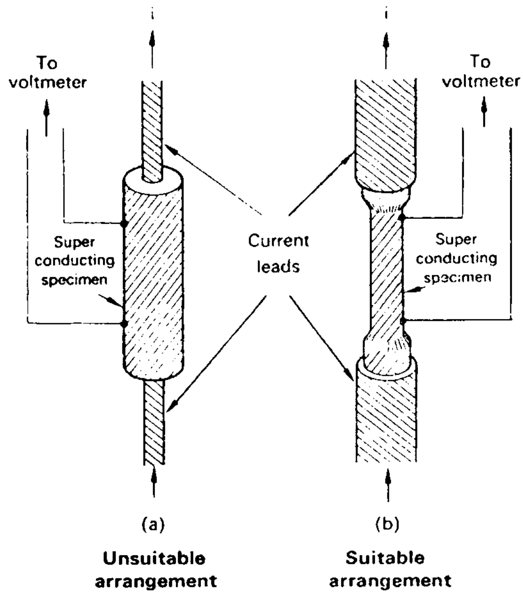

FIG. 7.3. Measurement of critical current.

should use as thick current leads as possible so that little heat is produced in them. It is also desirable to make the ends of the superconducting specimen thicker than the section whose critical current we are measuring, so that the critical current of the thinner section will be reached before thermal propagation starts from the ends (Fig. 7.3b).

A characteristic of the return of resistance by thermal propagation is the complete appearance of the full normal resistance once a certain current has been exceeded, as a result of the normal region spreading right through the specimen.

7.3. Intermediate State Induced by a Current

If thermal propagation does not occur, the full normal resistance does not appear at a sharply defined value of current but over a considerable current range. Consider a cylindrical wire of superconductor with a critical field strength H_c. If the radius of the wire is a, a current i produces a magnetic field strength $i/2\pi a$ at the surface. As we have seen, the greatest current the wire can carry while remaining wholly superconducting must be $i_c = 2\pi a H_c$, because, if the current were to exceed this, the magnetic field strength at the surface would be greater than H_c.

We might at first suppose that at i_c an outer cylindrical sheath is driven normal while the centre remains superconducting. However, this is not possible, as we shall now show. Suppose that an outer sheath becomes normal, leaving a cylindrical core of radius r in the superconducting state. The current will now flow entirely in this resistanceless core, and so the magnetic flux density it produces at the surface of the core will be $H_c\, a/r$. Since this is greater than H_c, the superconducting core will shrink to a smaller radius and this process will continue until the superconducting core contracts to zero radius, i.e. the whole wire is normal. However, it is not possible for the wire to become completely normal at a current i_c because if the wire were normal throughout, the current would be uniformly distributed over the cross section and the magnetic field strength inside the wire at a distance r from the centre would be less than the critical field, so the material could not be normal.

It therefore appears that, at the critical current, the wire can be neither wholly superconducting nor wholly normal, and that a state in which a normal sheath surrounds a superconducting core is not stable. In fact, at the critical current, the wire goes into an intermediate state of alternate superconducting and normal regions each of which occupies the full cross-section of the wire.† The current passing along the wire now has to flow through the normal regions, so at the critical current the resistance should jump from zero to some fraction of the resistance of the completely normal wire. Experiments show that a considerable resistance does indeed suddenly appear when the current is raised to the critical value (Fig. 7.4), the resistance jumping to between 0·6 and 0·8 of the full normal resistance. The exact value depends on factors such as the temperature and purity of the wire.

The detailed shapes of the normal and superconducting regions which appear when a current exceeding the critical current is passed along a

† F. London, *Superfluids*, vol. 1, Dover Publications Inc., New York, 1961.

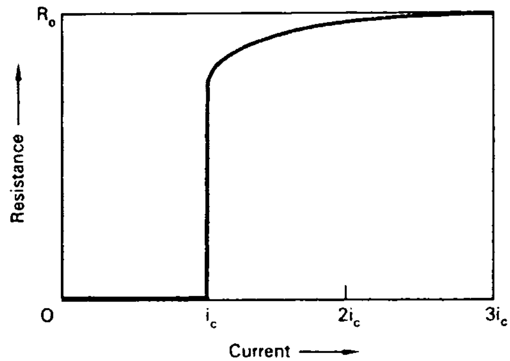

FIG. 7.4. Restoration of resistance to a wire by a current.

wire have not yet been determined experimentally, and it is a complicated problem to deduce them from theoretical considerations. The configuration shown in Fig. 7.5a is one which has been recently proposed on a theoretical basis, and for which there is some supporting experimental evidence.

It can be seen from Fig. 7.4 that as the current is increased above the critical value i_c the resistance of the wire gradually increases and approaches the full normal resistance asymptotically. London suggested that when the current is increased above the critical value i_c the in-

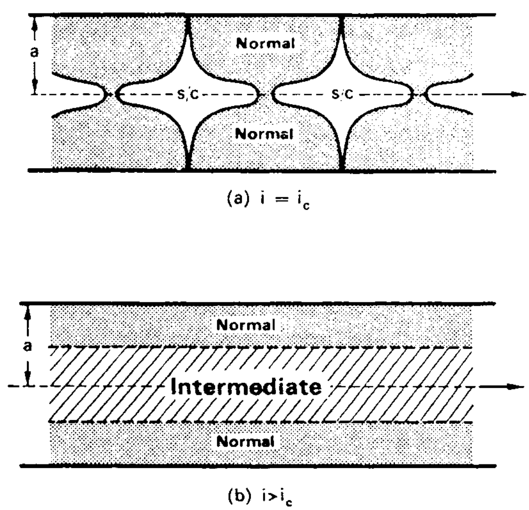

FIG. 7.5. Suggested cross-section of cylindrical wire carrying current in excess of its critical current (based on Baird and Mukherjee, and London).

termediate state contracts into a core surrounded by a sheath of normal material whose thickness increases as the current increases, so that the total current is shared between the fully resistive sheath and the partially resistive intermediate core (Fig. 7.5b). This model predicts a resistance increasing smoothly with the current in excess of i_c.

The sudden appearance of resistance, either by thermal propagation or by the appearance of an intermediate state when the critical current is exceeded, can make the measurement of the critical current of a conducting wire a rather precarious experiment. As the measuring current through the specimen is increased, a resistance R suddenly appears when the critical value i_c is exceeded. Power i_c^2R is then generated in the specimen, and if i_c is large and R is not very small, the heating may be enough to melt the wire, unless the current is reduced very quickly. In fact, superconducting wires can act like very efficient fuses with a sharply defined burn-out current.