CHAPTER 10

TUNNELLING AND THE ENERGY GAP

WE DESCRIBE in this chapter a technique introduced by Giaever in 1960 for the direct measurement of the energy gap in a superconducting metal by studying the tunnelling of electrons into the metal through a very thin insulating film. The discovery of this technique soon after the publication of the BCS theory made a great contribution towards our understanding of superconductivity because of the ease with which it enabled energy gaps to be measured experimentally.

10.1. The Tunnelling Process

To understand the method, consider first what happens if two plates made from a normal metal are separated by a very thin gap. The energy band diagram for this situation is shown in Fig. 10.1. Under ordinary

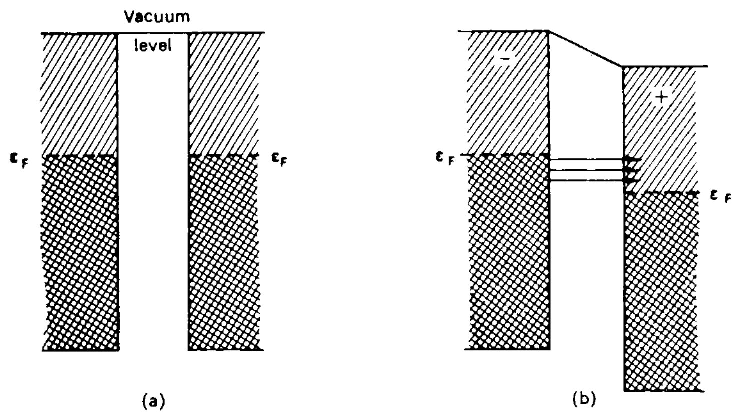

FIG. 10.1. Tunnelling between normal metals. The single-hatching represents empty states and the double hatching occupied states. At absolute zero, with no bias between the plates, as in (a), tunnelling is completely forbidden by the Pauli principle. If a positive voltage is applied to the right-hand plate, as in (b), so that the Fermi levels no longer coincide, there are occupied states on the left opposite to unoccupied states on the right, and tunnelling can take place as shown by the arrows.

conditions an electron in either plate cannot leave the metal because its energy is considerably less than the potential energy of a free electron in the vacuum outside. But if the separation between the metals is extremely small, an electron in one metal can cross to the other by virtue of a quantum-mechanical phenomenon known as tunnelling. In this process the electron is represented outside the metal by an exponentially attenuated standing wave whose amplitude falls off as $e^{-x/\zeta}$ instead of the usual travelling wave which represents the electron inside the metal. The length ζ is typically of the order of 10^{-8} cm, so the wave is attenuated very quickly indeed. However, if the gap is very thin (of the order of 10^{-7} cm) there is a small but significant chance that an electron may pass through the potential barrier separating the plates, and exchange of electrons between the two metals becomes possible. This "tunnelling" process is also possible if the two electrodes are separated by a very thin insulating film, such as the film of oxide which often forms on the surface of metals exposed to the atmosphere, and this is the situation which is important in practice.

There are two conditions which must be fulfilled for tunnelling to take place, apart from the obvious one that the separation between the metals must not be large compared with the attenuation length ζ of the tunnelling wavefunction. First, energy must be conserved in the process, i.e. the total energy of the system, including the metals on both sides of the insulating film, must be the same before and after tunnelling. Second, tunnelling can take place only if the states to which the electrons tunnel are empty, otherwise the process is forbidden by the Pauli principle. To take as an example the situation shown in Fig. 10.1a, in which we have for simplicity assumed that the metals are separated by a vacuum, at absolute zero there could be no tunnelling from one metal to the other because all the states which satisfy the first condition are occupied on both sides. However, if there is a small voltage difference between the metals (say the left-hand one negative with respect to the right), the energy levels will be shifted with respect to each other as in Fig. 10.1b. There will now be empty states on the right opposite to the topmost occupied states on the left, and tunnelling can take place from left to right. The number of states uncovered in this way is proportional to the voltage difference, and if the tunnelling probability is constant, as it is for very small bias voltages, the resulting current is linearly proportional to the voltage.

With superconductors, a number of situations may be envisaged. We may have one of the electrodes superconducting and the other normal, or both electrodes made of the same superconductor, or the electrodes made from two different superconductors. All of these have current–voltage characteristics which are peculiar to the particular combination of metals involved.

10.2. The Energy Level Diagram for a Superconductor

The difference between tunnelling involving superconductors and tunnelling involving normal metals can be explained in terms of the difference between their energy level diagrams. When we draw a band diagram for an ordinary metal, as in Fig. 10.1, we are showing the range of energies allowed to an individual electron. The electrons are independent of each other, so the energy of a particular electron is not affected by whether or not another level happens to be occupied. In the case of a superconductor this is no longer true. The electrons in the condensed

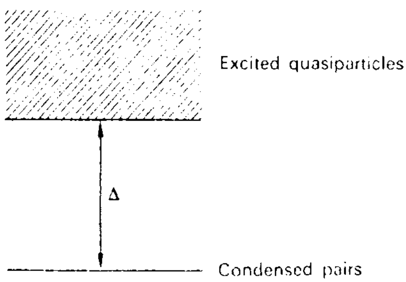

FIG. 10.2. Energy level diagram for superconductor.

state are not independent of each other, and their contribution to the total energy depends very much on whether or not they have a partner with equal and opposite momentum. We cannot therefore draw a band diagram in the usual way. As we have already pointed out, *all* the pairs have the same energy because they are all represented by the same wavefunction Φ [eqn. (9.5)], and we can therefore draw a single level as in Fig. 10.2 to represent the average energy *per electron* (or one half the energy of a Cooper pair) in the condensed state. This level can only contain paired electrons. As we saw in § 9.3.2, this level can contain many pairs because the Pauli principle in its usual form does not apply to Cooper pairs. If any of the pairs is split up, the energy of the system is increased in accordance with (9.8), namely

$$E_i + E_j = \{(\varepsilon_i - \varepsilon_F)^2 + \Delta^2\}^{\frac{1}{2}} + \{(\varepsilon_j - \varepsilon_F)^2 + \Delta^2\}^{\frac{1}{2}},$$

so that each of the resulting quasi-particles can be regarded as contributing an amount of energy $\{(\varepsilon - \varepsilon_F)^2 + \Delta^2\}^{\frac{1}{2}}$, where $\varepsilon = p^2/2m$. Since the quasi-particles behave almost like independent electrons, their allowed energy values can be represented by a continuum of levels separated by an interval Δ from the level representing the pairs, as in Fig. 10.2. (Notice that the quantity we have called the energy gap is 2Δ. This is because if quasi-particles are produced by splitting up a pair they are always produced two at a time. It is, however, possible to inject a single quasi-particle by tunnelling, and if this is done the minimum energy added to the system is Δ.)

At absolute zero there are no quasi-particles and the continuum states are empty. At temperatures above absolute zero, the continuum levels are partly filled in accordance with the laws of statistical mechanics.

10.3. Tunnelling Between a Normal Metal and a Superconductor

Consider first tunnelling between a normal metal and a superconductor at absolute zero. The energy level diagrams are as shown in Fig. 10.3. With the plates at the same potential, the Fermi level in the normal metal coincides with the level representing the condensed pairs in the superconductor as in Fig. 10.3a. Again tunnelling is impossible if there is no potential difference between the plates.

If a positive bias of V volts is applied to the superconductor, all its energy levels are lowered† relative to those of the normal metal, but no tunnelling processes which conserve energy are possible until V reaches the value Δ/e, when the bottom of the continuum of quasi-particle levels coincides with the Fermi level in the normal metal, as shown in Fig. 10.3b. It now becomes possible for electrons in the normal metal to tunnel into the quasi-particle states, and the number of electrons which may tunnel increases steadily as the bias increases, so that the resulting current increases monotonically with V as shown in Fig. 10.3d. If the superconductor is given a negative bias voltage, so that all its energy levels are raised relative to those of the normal metal, no tunnelling can occur until the voltage becomes equal to $-\Delta/e$, when a completely new process becomes possible which involves the splitting up of a Cooper pair. It should be appreciated that if only one electron is involved in a

† An increase in electrostatic potential lowers the potential energy because the charge on the electrons is negative.

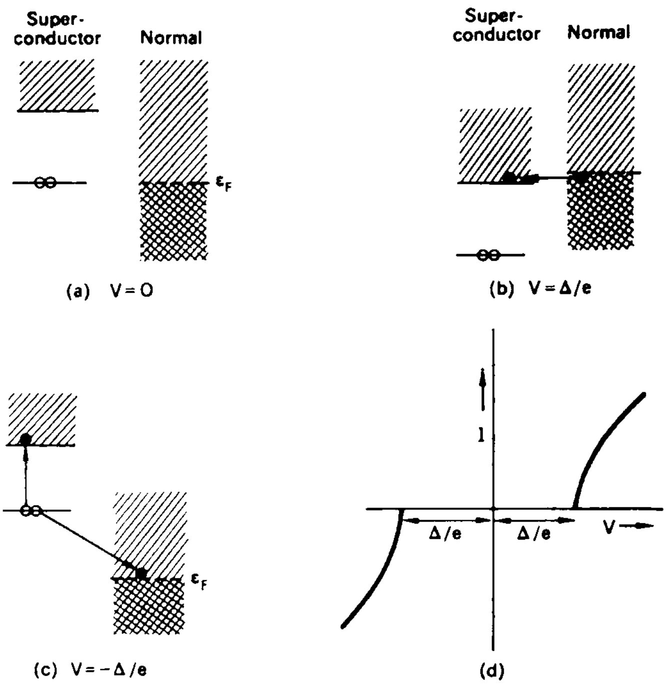

FIG. 10.3. Tunnelling between a normal metal and a superconductor. (a), (b), and (c) show the energy levels at 0°K for various differences in voltage V between a superconductor and a normal metal. V is taken as positive if the superconductor is biased positively relative to the normal metal. The singly-hatched areas denote empty states and the double-hatched areas denote occupied states. (d) Shows the resulting current–voltage characteristic.

transition the only processes which can conserve energy are those in which the electron moves horizontally on the energy band diagram between states of the same energy. However, if two electrons are involved it becomes possible for one to gain energy and the other to lose it as long as the total energy is conserved. Such a process is shown in Fig. 10.3c. In this, a Cooper pair splits up, one of the electrons tunnelling to an empty state just above the Fermi level of the normal metal with a loss of energy Δ, while the second electron, having lost its partner, is converted into a quasi-particle and occupies the lowest excited state in the superconductor with a gain in energy Δ. Thus the total energy of the complete system before the transitions indicated by arrows have taken place is equal to the total energy after, and the process is allowed. The number of pairs that may split up in this way increases with the bias voltage, because more quasi-particle states and states in the normal metal become accessible, and the resulting negative current increases with $-V$ as shown in Fig. 10.3d. Δ is thus given directly by the voltage at which the tunnelling current between the superconductor and any normal metal shows a sudden increase. At temperatures above absolute zero, a very small current may flow at voltages between the values $\pm\Delta/e$, because there are a few electrons excited to states above the Fermi level in the normal metal which may, if there is positive bias, tunnel into the quasi-particle states, and there are a few empty states below the Fermi level of the normal metal to which one of the members of a pair may tunnel if there is negative bias. However, there will still be a sharp rise in tunnelling current when $V = \pm\Delta/e$. Therefore, from the tunnelling $I - V$ characteristic we can directly determine the energy gap ($= 2\Delta$) of the superconductor. Tunnelling is a relatively easy experiment and is a most useful way of measuring the energy gap.

10.4. Tunnelling Between Two Identical Superconductors

The energy level diagram for two identical superconductors with no applied bias is as shown in Fig. 10.4, where it is assumed that the temperature is above absolute zero so that the quasi-particle states are partially occupied. It is possible for quasi-particles to tunnel in either direction, because the states to which they may tunnel are not completely full, but at zero bias the current due to tunnelling from left to right will be the same as that from right to left, so that no net current flows.

Now suppose the left-hand electrode is biased positively by a voltage V relative to the right-hand electrode. This means that the energy level diagram on the left will be shifted downwards relative to that on the right by an amount eV, as shown in Fig. 10.4b. There will now be a net flow of electrons from right to left, because the lowest quasi-particles on the left have no states on the right to tunnel into, while all the quasi-particles on the right are able to tunnel. The current increases with V until the occupied quasi-particle states on the left are below the bottom of the continuum states on the right, so that quasi-particles can no longer tunnel from left to right. Since the quasi-particles will all lie within about kT of the lowest level, this stage is reached when $eV \simeq kT$ or $V \simeq 10^{-4}$ V. If V is increased further the current remains more or less constant because none of the quasi-particles on the left can tunnel, and those on the right, which are able to tunnel, are constant in number. However, when V reaches the magnitude $2\Delta/e$, an additional process

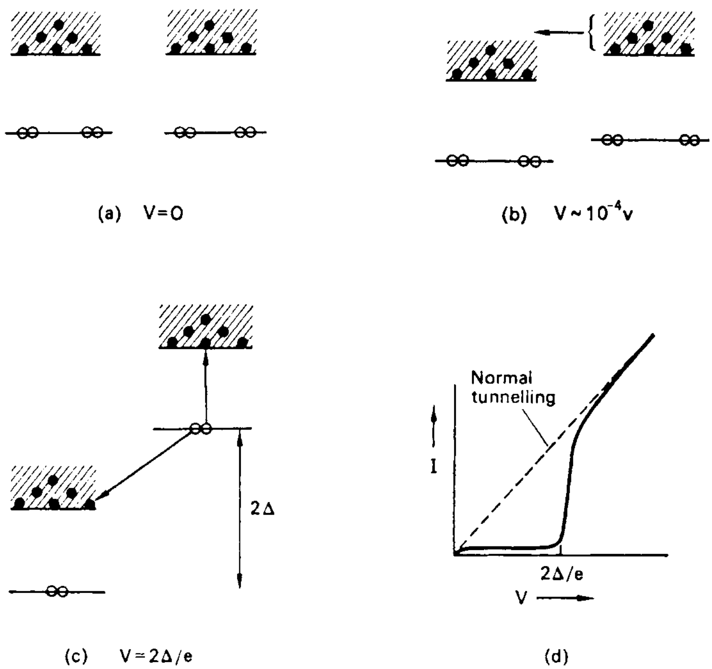

FIG. 10.4. Tunnelling between identical superconductors. (a), (b), and (c) energy levels. ⊖⊖ denotes a Cooper pair, ● denotes a quasi-particle. (d) Current–voltage characteristic.

becomes possible which involves the splitting up of a Cooper pair, as shown in Fig. 10.4c. One of the electrons tunnels into the left-hand superconductor so as to occupy the lowest quasi-particle state. This process is accompanied by a loss of energy Δ. The second electron, having lost its partner, is converted into a quasi-particle and occupies the lowest excited state on the right with a gain in energy Δ. Thus the total energy before the transitions indicated by the arrows have taken place is identical with the total energy after, and the process is allowed. As a result of this process, extra electrons flow from right to left and the current increases. If V is increased beyond $2\Delta/e$, a process of this sort continues to be possible except that the quasi-particles do not go into the lowest excited states on each side. There is now a large number of combinations of excited states which can serve as final states, and the tunnelling current increases rapidly, as shown in Fig. 10.4d. In this case also we can determine the energy gap from the voltage at which the tunnelling current increases rapidly.

10.5. The Semiconductor Representation

The reader is warned that there is another way of representing energy levels in a superconductor which is widely used in the literature. In this model the energy levels are represented by two bands, one of which is completely full at absolute zero and the other completely empty, as in a semiconductor. The width of the gap between the bands is 2Δ, not Δ as in Fig. 10.2, though, as we shall see, the two representations are equivalent.

To see how this model arises, consider a superconductor at a temperature above absolute zero, so that there are quasi-particles present, and suppose a single electron is added to it. Then one of two things may happen. The added electron may go into a momentum state $\mathbf{p}\uparrow$ whose partner $-\mathbf{p}\downarrow$ is empty. In this case it behaves as an excited quasi-particle. Alternatively, it may join up with another unpaired electron and form a Cooper pair. Compared with the first case, the second gives rise to a total energy which is lower by at least 2Δ. Since the superconductor is in the same condition before the electron is added in both instances, the energy of the extra electron before it enters the superconductor must be lower by at least 2Δ in the second case if energy is to be conserved overall. We may therefore regard the added electron as belonging to one of two bands which are separated in energy by 2Δ. An electron can only enter the superconductor with overall conservation of energy if, *before entering*, its energy lies above the bottom of the upper band or below the top of the lower band. These bands therefore define the ranges of energy which an electron must have if it is to tunnel into the superconductor. It is important to realize that in this model both the bands are *single electron* bands. It should not be thought that the levels in the upper band correspond to quasi-particles with $p > p_F$ and the lower to quasi-particles with $p < p_F$; a full range of momentum values is present in each case.

The semiconductor representation is most easily illustrated by considering tunnelling between a normal metal and a superconductor. In Fig. 10.5a,b we describe this process in terms of the excited quasi-particle representation, as we did in § 10.3 and Fig. 10.3. In Fig. 10.5c,d a description is given in terms of the semiconductor representation. The physics of the process is much more clearly brought out by the quasi-particle representation. The advantage of the semiconductor representation is that the allowed transitions always correspond to horizontal arrows which represent transitions made by a single electron, and this

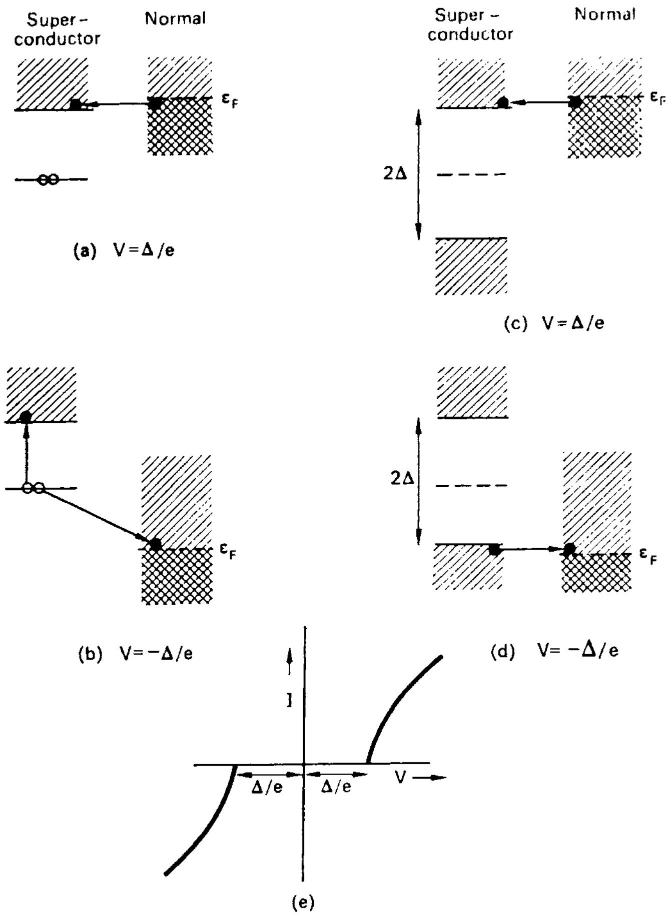

FIG. 10.5. Tunnelling between a superconductor and a normal metal at 0°K. (a) and (b) Excited quasi-particle representation. (c) and (d) Semiconductor representation. (e) Current–voltage characteristic.

makes the detailed interpretation of tunnelling phenomena somewhat easier; probably for this reason, it has been much more commonly used in the literature of tunnelling. It will be noted that an electron can only be injected into the lower band if there is a quasi-particle with which it can combine to form a Cooper pair, so the number of empty states in the lower band is equal to the number of quasi-particles in the upper band, rather like an intrinsic semiconductor. The mechanism by which the two bands come about is, however, totally different from that responsible for the band structure of a semiconductor and great care must be used in applying this representation in the case of a superconductor. The difference between the two representations has been discussed by Schrieffer† and Adkins.‡

10.6. Other Types of Tunnelling

There are other, more complicated, types of tunnelling. For example, if the two metals are dissimilar superconductors, a characteristic as shown in Fig. 10.6 is obtained. There is a negative resistance region between $V = (\Delta_2 - \Delta_1)/e$ and $V = (\Delta_2 + \Delta_1)/e$. The explanation of this negative resistance region depends on the way in which the density of states in the quasi-particle band varies with energy, and we shall not discuss it here. For further details the reader is referred to the original paper by Giaever and Megerle.§

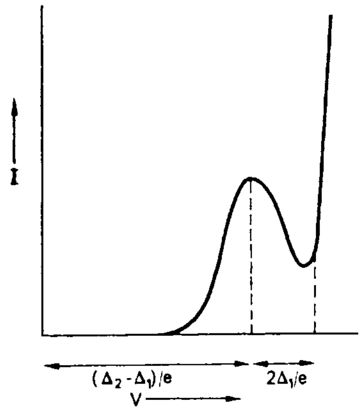

FIG. 10.6. Tunnelling between two superconductors with energy gaps Δ_1 and Δ_2 ($\Delta_2 > \Delta_1$).

There is also the possibility, in the case of two superconductors, of two electrons which form a pair tunnelling *as a pair*, so that they maintain their momentum pairing after crossing the gap. This type of tunnelling, known as Josephson tunnelling, is possible because the superconducting ground state can contain many pairs. It only appears under very special circumstances, namely exceptionally thin insulating layers ($<10^{-7}$ cm). Josephson tunnelling, the consequences of which are discussed at length in the following chapter, takes place when there is no

† J. R. Schreiffer, *Rev. Mod. Phys.* **36**, 200 (1964).
‡ C. J. Adkins, *Rev. Mod. Phys.* **36**, 211 (1964).
§ I. Giaever and K. Megerle, *Phys. Rev.*, **122**, 1101 (1961).

difference in voltage between the superconductors, so that a current may flow without any accompanying voltage drop. We might call it a tunnelling supercurrent. This supercurrent has a critical current density j_c which is characteristic of the junction.

If the Josephson tunnelling current density exceeds the value j_c, a voltage difference V appears across the junction and two processes occur. Some electrons tunnel individually, as in § 10.4, with an I–V characteristic as shown in Fig. 10.4d or Fig. 10.6 (depending on whether the superconductors are identical or not) and, at the same time, some electrons continue to tunnel in the form of electron-pairs. However, the condensed states are no longer opposite each other on an energy-level diagram, so pairs cannot tunnel from one condensed state to the other with conservation of energy if the energy of the electron pairs alone is considered. The energy balance is made up by the emission of a photon of electromagnetic radiation of frequency ν such that

$$h\nu = 2eV. \tag{10.1}$$

The factor 2 can be considered as arising either because the pair can be considered as a particle with charge $2e$, or because two electrons each with charge e are involved in the transition. Whichever way one looks at it, the energy required to make up the balance is $2eV$. This process, which involves the emission of radiation, is known as the a.c. Josephson effect. Since V is normally of the order of 10^{-3} V, the radiation is in the short wavelength microwave part of the spectrum. The emission of such radiation from very thin tunnel junctions has been detected by Langenburg, Scalapino, and Taylor.

Josephson tunnelling will be considered more fully in the next chapter (§ 11.3.1 *et seq.*).

10.7. Practical Details

Tunnelling is a very useful phenomenon, because we can use it to measure simply and directly the energy gap of a superconductor. The majority of superconducting tunnelling experiments have been carried out using evaporated films of the two metals. In a typical experiment a thin film of one of the metals is evaporated on to a glass plate (such as a microscope slide) in the shape of a strip a millimetre or so wide. This film is then oxidized by exposure to air or oxygen until a layer of oxide a few tens of Ångstrom units thick is built up on the surface. A film of the second metal is then evaporated, usually as a strip which crosses the first one at right angles, so that the area through which tunnelling can take place is a few square millimetres. Electrical contact is then made to the films, as a rule with indium solder, so that the current–voltage characteristic can be observed. Once the *I–V* characteristic has been observed, the energy gap can be determined from the voltage at which the curve shows a pronounced change in slope. This occurs at a voltage equal to Δ/e for superconductor–normal tunnelling and to $2\Delta/e$ for superconductor–superconductor tunnelling (see Figs. 10.3d and 10.4d).

Tunnelling currents are usually less than 10^{-3} A for applied voltages around 10^{-3} V, so care must be taken with the electrical measurements. It is common practice to superpose a small alternating component on the steady voltage V applied to the sandwich; the current then contains an alternating component which is proportional to the value of the differential conductance dI/dV. This a.c. component can be amplified, using a tuned amplifier. In this way a very sensitive direct measurement can be made of the slope of the *I–V* curve.

Figure 10.7 shows some measurements of the temperature dependence of the energy gap of indium, tin and lead, measured by Giaever and

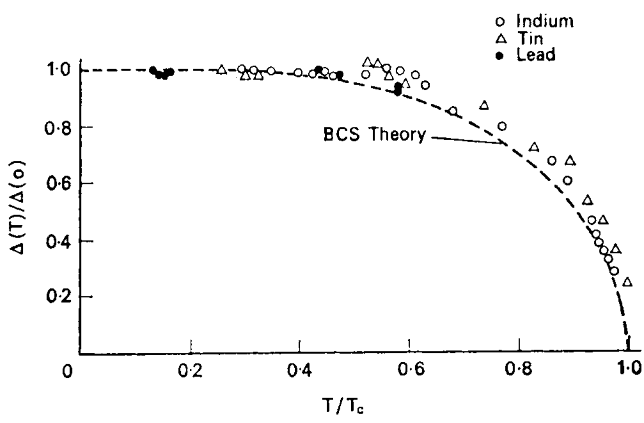

FIG. 10.7. The energy gap of lead, tin, and indium versus temperature, as determined by tunnelling experiments. (After Giaever and Megerle.)

Megerle who studied tunnelling from these metals into a normal metal. The fit with the temperature dependence predicted by the BCS theory is shown to be quite good. We should not expect the BCS theory to make ***absolute*** predictions of the energy gap and transition temperature of a superconductor with any great accuracy, so the figure shows the ratio of the energy gap at temperature T to the energy gap at 0°K, plotted against T expressed as a fraction of the transition temperature T_c. The curve obtained in this way should be the same for all superconductors.

One of the most impressive examples of the power of the tunnelling technique in measuring energy gaps is its use to measure the dependence of the energy gap on an applied magnetic field. As we saw in Chapter 5, the superconducting to normal transition in the presence of a magnetic field is a first-order transition, which implies that the energy gap should go abruptly to zero as the specimen is driven normal by the field. The data represented by circles in Fig. 10.8, which show the energy gap of an

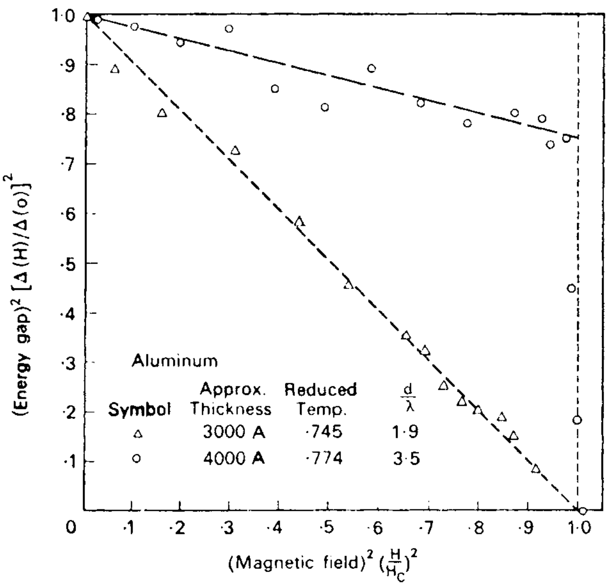

FIG. 10.8. Energy gap of aluminium as a function of magnetic field for films of thickness 3000 Å and 4000 Å. (After Douglass.)

aluminium film 4000 Å thick, obtained from tunnelling into superconducting lead, show the expected behaviour. The small decrease of energy gap with increasing field before H_c is reached can be explained in the same way as the field-dependence of the penetration depth discussed in §8.4. However, for a film which is 3000 Å thick, represented by triangles in Fig. 10.8, the energy gap falls linearly to zero as H increases to H_c. This gradual disappearance of the energy gap is indicative of a second-order transition, and is effective confirmation of the prediction by Ginzburg and Landau, mentioned in § 8.5, that below a certain film thickness the transition in a magnetic field should become of second order.