CHAPTER 12

THE MIXED STATE

FOR MANY years it was supposed that the behaviour which we have described in Part I of this book was characteristic of all superconductors. It had, indeed, been noticed that certain superconductors, especially alloys and impure metals, did not behave quite in the expected way, but this anomalous behaviour was usually ascribed to impurity effects, not considered to be of great scientific interest, and consequently little effort was made to understand it. However, in 1957 Abrikosov published a theoretical paper pointing out that there might be another class of superconductors with somewhat different properties, and it is now realized that the apparently anomalous properties of certain superconductors are not merely trivial impurity effects but are the inherent features of this other class of superconductor now known as "type-II".

One of the characteristic features of the type-I superconductors we considered in the first part of this book is the Meissner effect, the cancellation within the metal of the flux due to an applied magnetic field. We mentioned in § 6.7 that the occurrence of this perfect diamagnetism implies the existence of a surface energy at the boundary between any normal and superconducting regions in the metal. This surface energy plays a very important role in determining the behaviour of a superconductor; for example, as we shall now see, it determines whether the material is a type-I or a type-II superconductor.

Consider a superconducting body in an applied magnetic field of strength less than the critical value H_c, and suppose that within the material a normal region were to appear with boundaries lying parallel to the direction of the applied magnetic field. The appearance of such a normal region would change the free energy of the superconductor, and we may consider two contributions to this free energy change: a contribution arising from the bulk of the normal region and a contribution due to its surface. As we saw in Chapter 4, in an applied magnetic field of strength H_a, the free energy per unit volume of the normal state is greater than that of the superconducting, perfectly diamagnetic, state by an amount $\frac{1}{2}\mu_0(H_c^2 - H_a^2)$. Furthermore, as shown in Chapter 6, there is a surface energy associated with the boundary between a normal and a superconducting region. For the type-I superconductors we considered in the first part of this book this surface energy is positive. Hence, if a normal region were to form in the superconducting material, there would be an increase in free energy due both to the bulk and to the surface of the normal region. For this reason, the appearance of normal regions is energetically unfavourable, and a type-I superconductor remains superconducting throughout when a magnetic field of strength less than H_c is applied.

Suppose, however, that in certain metals the surface energy between normal and superconducting regions were *negative* instead of positive (i.e. energy is released when the interface is formed). In this case the appearance of a normal region would reduce the free energy, if the increase in energy due to the bulk of the region were outweighed by the decrease due to its surface. A material assumes that condition which has the lowest total free energy, so in the case of a sufficiently negative surface energy we would expect that, in order to produce the minimum free energy, a large number of normal regions would form in the superconducting material when a magnetic field is applied. The material would split into some fine-scale mixture of superconducting and normal regions whose boundaries lie parallel to the applied field, the arrangement being such as to give the maximum boundary area relative to volume of normal material. We shall call this the *mixed state*. In the next section it will be shown that the conditions in some superconductors are such that the surface energy is indeed negative. These metals are therefore able to go into the mixed state, and these are the type-II superconductors.

It is important to distinguish clearly between the *mixed* state which occurs in type-II superconductors and the *intermediate* state which occurs in type-I superconductors. The intermediate state occurs in those type-I superconducting bodies which have a non-zero demagnetizing factor, and its appearance depends on the shape of the body. The mixed state, however, is an intrinsic feature of type-II superconducting material and appears even if the body has zero demagnetizing factor (e.g. a long rod in a parallel field). In addition, the structure of the intermediate state is relatively coarse and the gross features can be made visible to the naked eye [Figs. 6.6 and 6.7 (pp. 73 and 74)]. The structure of the mixed state is, as we shall see, on a much finer scale with a periodicity less than 10^{-5} cm (see *frontispiece*).

12.1. Negative Surface Energy

In § 6.9 we showed that, as a result of the existence of the penetration depth and coherence length, there is a surface energy associated with the boundary between a normal and superconducting region. It was shown that, if the coherence range is *longer* than the penetration depth, as it is in most pure metallic elements, the total free energy is increased close to the boundary [Fig. 6.9 (p. 78)], that is to say there is a *positive* surface energy.

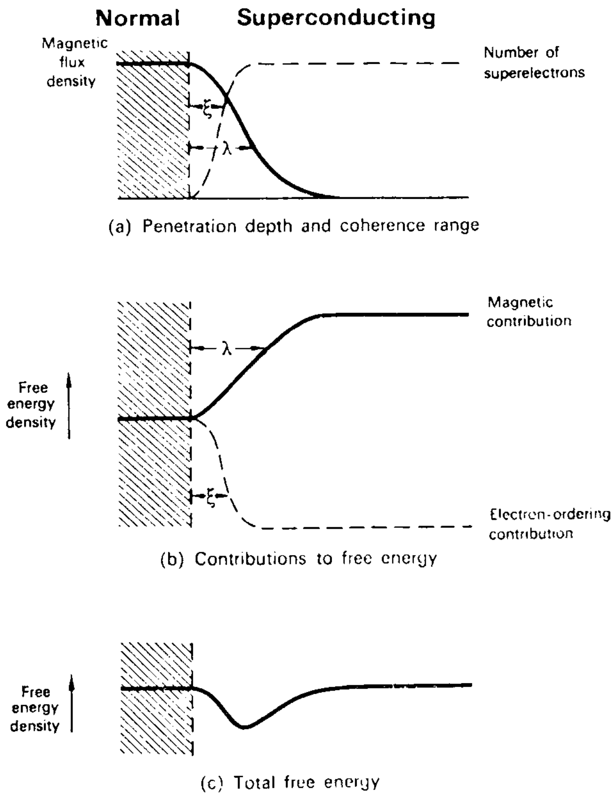

FIG. 12.1. Negative surface energy; coherence range less than penetration depth. (Compare this with Fig. 6.9.)

The relative values of the coherence length ξ and the penetration depth λ vary for different materials. In many alloys and a few pure metals the coherence range is greatly reduced, as was pointed out in Chapter 6. A similar argument to that used in § 6.9 shows that, if the coherence length is ***shorter*** than the penetration depth, the surface energy is ***negative***, as Fig. 12.1 illustrates, and therefore such a superconductor will be type-II. In most pure metals the coherence length has a value ξ_0 of about 10^{-4} cm. This is considerably greater than the penetration depth, which is about 5×10^{-6} cm, so in such metals the surface energy is positive and they are type-I. A reduction in the electron mean free path, however, reduces the coherence length and increases the penetration depth (§ 6.9, § 2.4.1). Impurities in a metal reduce the electron mean free path, and in an impure metal or alloy the coherence range can easily be less than the penetration depth. Alloys or sufficiently impure metals are, therefore, usually type-II superconductors.

12.2. The Mixed State

We have seen that it may be energetically favourable for superconductors with a negative surface energy between normal and superconducting regions to go into a mixed state when a magnetic field is applied. The configuration of the normal regions threading the superconducting material should be such that the ratio of surface to volume of normal material is a maximum. It turns out that a favourable configuration is one in which the superconductor is threaded by cylinders of normal material lying parallel to the applied magnetic field (Fig. 12.2). We shall refer to these cylinders as *normal cores*. These cores arrange themselves in a regular pattern, in fact a triangular close-packed lattice (Fig. 12.2 and *frontispiece*).

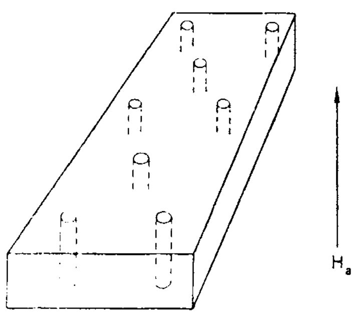

FIG. 12.2. The mixed state.

We might expect the normal cores to have a very small radius because the smaller the radius of a cylinder the larger the ratio of its surface area to its volume. The picture of the mixed state which emerges from these considerations is as follows. The bulk of the material is diamagnetic, the flux due to the applied field being opposed by a diamagnetic surface current which circulates around the perimeter of the specimen. This diamagnetic material is threaded by normal cores lying parallel to the applied magnetic field, and within each core is magnetic flux having the same direction as that of the applied magnetic field. The flux within each core is generated by a vortex of persistent current that circulates around the core with a sense of rotation opposite to that of the diamagnetic surface current. (We saw in § 2.3.1 that any normal region containing magnetic flux and enclosed by superconducting material must be encircled by such a current.) The pattern of currents and the resulting flux are illustrated in Fig. 12.3.

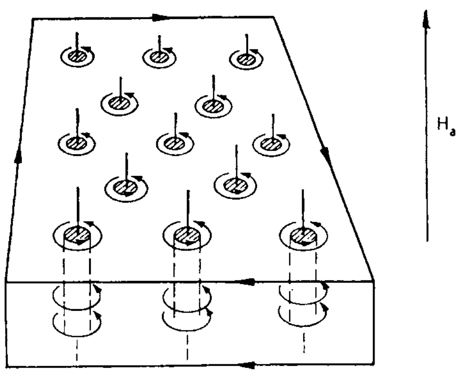

FIG. 12.3. The mixed state, showing normal cores and encircling supercurrent vortices. The vertical lines represent the flux threading the cores. The surface current maintains the bulk diamagnetism.

The vortex current encircling a normal core interacts with the magnetic field produced by the vortex current encircling any other core and, as a result, any two cores repel each other. This is somewhat similar to the repulsion between two parallel solenoids or bar magnets. Because of this mutual interaction the cores threading a superconductor in the mixed state do not lie at random but arrange themselves into a regular periodic hexagonal array† as shown in Fig. 12.3. This array is usually known as the *fluxon lattice*. The existence of the normal cores and their arrangement in a periodic lattice has been revealed by two experimental techniques. The decoration technique of Essmann and

† Occasionally a square lattice may be formed, but this is very uncommon and only occurs under special circumstances.

Träuble reveals the pattern of the normal cores by allowing very small (500 Å) ferromagnetic particles to settle on the surface of a type-II superconductor in the mixed state. The particles locate themselves where the magnetic flux is strongest, i.e. where the normal cores intersect the surface. The resulting pattern can then be examined by electron microscopy. The frontispiece shows the pattern of normal cores in the mixed state revealed by this method. An alternative method, developed by Cribier, Jacrot, Rao and Farnoux, makes use of neutron diffraction. Neutrons, because of their magnetic moment, interact with magnetic fields. The regular arrangement of current vortices in the mixed state produces a periodic magnetic field which acts as diffraction grating, and scatters a beam of neutrons shone through the specimen into preferential directions given by the Bragg law. Observation of the directions into which the beam of neutrons is scattered shows that the cores are arranged in a hexagonal periodic array.

12.2.1. Details of the mixed state

The picture of the mixed state we have just given, with thin cylindrical normal cores threading the superconducting material, is a good enough approximation for many purposes, but it does not accurately describe the details of the structure. For one thing, the cores are not sharply defined. We saw in § 6.9 that there cannot be a sharp boundary between a superconducting and a normal region; the transition is spread out over a distance which is roughly equal to the coherence range ξ. Furthermore, the magnetic flux associated with each core spreads into the surrounding material over a distance about equal to the penetration depth λ.

A detailed analysis of the free energy of the mixed state, which we shall not attempt here, shows that the normal cores should have an exceedingly small radius. However, as the size of a normal cylinder is reduced so that it approaches ξ it becomes progressively more difficult to define its radius exactly on account of the diffuseness of the boundary. Because it is not possible to define exactly the volume and surface area of such a small core, we cannot properly divide its free energy into distinct volume and surface contributions, and we must consider its free energy as a whole. A detailed consideration of free energy gives the following structure for the mixed state.

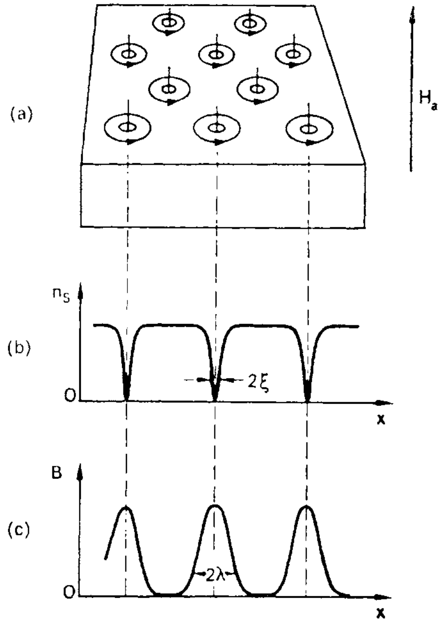

FIG. 12.4. Mixed state in applied magnetic field of strength just greater than H_{c1}. (a) Lattice of cores and associated vortices. (b) Variation with position of concentration of superelectrons. (c) Variation of flux density.

The properties of the material vary with position in a periodic manner. Towards the centre of each vortex the concentration n_s of superelectrons falls to zero, so along the centre of each vortex is a very thin core (strictly a line) of normal material (Fig. 12.4b). The dips in the superelectron concentration are about two coherence-lengths wide. The flux density due to the applied magnetic field is not cancelled in the normal cores and falls to a small value over a distance about λ away from the cores (Fig. 12.4c). The total flux generated at each core by the encircling current vortex is just one fluxon (see § 11.2).

We shall now confirm that, when a magnetic field is applied to a type-II superconductor, the appearance of cores of the form we have just described does result in a lowering of the free energy. At each core the number n_s of superelectrons decreases and energy must be provided to split up the pairs. As an approximation we may think of each core as equivalent to a cylinder of normal material with radius ξ. The appearance of a normal core will therefore result in a local increase in free energy of $\pi\xi^2.\frac{1}{2}\mu_0 H_c^2$ per unit length of core due to the decrease in electron order. However, over a radius of about λ the material is not diamagnetic so there is a local decrease in magnetic energy approximately equal to $\pi\lambda^2.\frac{1}{2}\mu_0 H_a^2$ per unit length, where H_a is the strength of the applied field. If there is to be a net reduction in free energy by the formation of such cores, we must have

$$\pi\xi^2.\tfrac{1}{2}\mu_0 H_c^2 < \pi\lambda^2.\tfrac{1}{2}\mu_0 H_a^2. \tag{12.1}$$

According to this relation, if the mixed state is to appear in applied fields less than H_c (a necessary condition, otherwise an applied field would drive the whole superconductor into the normal state before the mixed state could establish itself), we must have $\xi < \lambda$. This is the same condition as that we derived for negative surface energy. So, as predicted by the simple arguments on page 184, the mixed state is produced by the application of a magnetic field to superconductors which would have negative surface energy between superconducting and normal regions.

12.3. Ginzburg–Landau Constant of Metals and Alloys

Let us write the ratio of the penetration depth λ to the coherence length ξ as a parameter κ:

$$\kappa = \lambda/\xi.$$

κ varies for different superconductors and is known as the Ginzburg–Landau constant of the material.† It is an important parameter because its value determines several properties of the superconductor; for example, according to the considerations of the previous sections, a superconductor is type-I or type-II depending on whether its value of κ is less or greater than unity.

A more detailed treatment than we have given shows that the sign of the surface energy and the possibility of the formation of a mixed state depends, strictly, not on whether the κ of the material is less or greater than unity, but on whether κ is less or greater than $1/\surd 2$:

$\kappa < 0{\cdot}71$ surface energy positive (type-I),
$\kappa > 0{\cdot}71$ surface energy negative (type-II).

† The constant κ appears in the theoretical treatment of superconductivity by Ginzburg and Landau, which is an extension of the London treatment and explicitly includes the surface energy between normal and superconducting regions. In the Ginzburg–Landau treatment κ is defined by $\kappa = (\surd 2)2\pi\lambda^2\mu_0 H_c/\Phi_0$ where Φ_0 is the quantum of magnetic flux (see § 11.2). If the electron mean free path is very short, λ increases and so κ is large in alloys. For our purposes, however, we can consider κ to be the ratio of the penetration depth to the coherence range.

This correct critical value of κ is, however, not very different from the value of unity we obtained by simple considerations.

It was pointed out in § 12.1 that in alloys and impure metals the coherence range is shorter than in pure metals; consequently κ can have a large value and these superconductors are usually type-II. It is, however, possible for even pure metals to be type-II superconductors. It can be shown that superconductors with high transition temperatures can be expected to have relatively short coherence ranges and, in fact, three superconducting metals (niobium, vanadium, and technetium) have κ greater than 0·71 even in the absence of impurities. These are called *intrinsic* type-II superconductors. Pure niobium, vanadium, and technetium have κ values of 0·78, 0·82 and 0·92, respectively.† However, pure metals are usually type-I and alloys are usually type-II.

The Ginzburg–Landau constant κ of a superconductor which contains impurities is related to its resistivity in the normal state because the scattering of electrons by the impurities shortens the coherence range ξ and also increases the normal resistivity ρ. For a given metal, therefore, κ increases with the normal state resistivity.

12.4. Lower and Upper Critical Fields

12.4.1. Lower critical field, H_{c1}

We have seen that when a magnetic field is applied to a type-II superconductor it may be energetically favourable for it to go into the mixed state whose configuration has been described in previous sections. However, a certain minimum strength of applied field is required to drive a type-II superconductor into the mixed state. This can be seen by examination of (12.1), which gives the condition for the free energy to be lowered by the appearance of the mixed state. For a given value of ξ relative to λ (remembering that, in a type-II superconductor, $\xi < \lambda$), we see that H_a must be greater than a certain fraction of H_c. Therefore a certain minimum strength of applied field is required to drive a type-II superconductor into the mixed state, and this is known as the *lower critical field*, H_{c1}. We can get an approximate value for H_{c1} from eqn.

† In fact, for intrinsic type-II superconductors and dilute alloys κ varies slightly with temperature, increasing as the temperature falls. In vanadium, for example, κ at the transition temperature (5·4°K) is 0·82 but rises to 1·5 at 0°K. Similarly the alloy $Pb_{.99}Tl_{.01}$ has a κ-value of 0·58 at its transition temperature, 7·2°K, and so is type-I, but on cooling to 4·3°K the κ-value rises to 0·71, so below this temperature the alloy is type-II.

(12.1) (approximate because eqn. (12.1) was based on a simplified model of a core). From this equation we see that the mixed state will be energetically favoured if the strength of the applied field exceeds $H_c\xi/\lambda$, i.e.

$$H_{c1} \simeq H_c/\kappa.$$

Clearly the value of H_{c1} relative to H_c decreases as the value of κ increases.

12.4.2. Upper critical field, H_{c2}

In the previous section we have shown that if a gradually increasing magnetic field is applied to a type-II superconductor it goes into the mixed state at a "lower critical field" H_{c1} which is less than H_c. Now, in a type-I superconductor, H_c is the field strength at which the magnetic free energy of the superconductor has been raised to such an extent that it becomes energetically favourable for it to go into the normal state. A type-II superconductor in the mixed state has, however, in an applied field, a lower free energy than if it were type-I and perfectly diamagnetic. Consequently we may expect that a magnetic field stronger than H_c must be applied to drive a type-II superconductor normal. (This is similar to the argument used in § 8.1 to show that the critical field of a thin superconductor is greater than the critical field of the bulk material.) Furthermore, we may note that an argument similar to that on p. 189 shows that in fields *above* H_c the mixed state can have a lower free energy than the completely normal state. The high magnetic field strength up to which the mixed state can persist is called the *upper critical field*, H_{c2}.

At the lower critical field strength H_{c1} a type-II superconductor goes from the completely superconducting state into the mixed state and a lattice of parallel cores is formed. As the strength of the applied magnetic field is increased above H_{c1} the cores pack closer together and, because each core is associated with a fixed amount of flux, the average flux density B in the superconductor increases. At a sufficiently high value of applied magnetic field the cores merge together and the mean flux density in the material due to the cores and the diamagnetic surface current approaches the flux density $\mu_0 H_a$ of the applied magnetic field (Fig. 12.5). At the upper critical field H_{c2} the flux density becomes equal to $\mu_0 H_a$ and the material goes into the normal state.

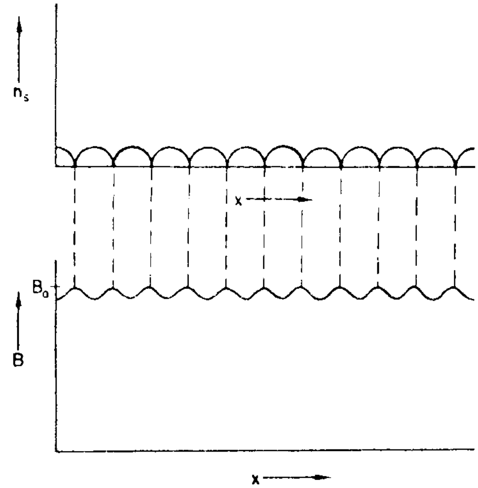

FIG. 12.5. Mixed state at applied magnetic field strength just below H_{c2}.

We have now seen that, whereas type-I superconductors can exist in one of two states, superconducting or normal, type-II superconductors can be in one of *three* states, superconducting, mixed or normal. The phase diagrams of the two types are compared in Fig. 12.6. In a type-II superconductor, the larger the value of κ the smaller will be H_{c1} but the larger will be H_{c2} relative to the critical field H_c.

12.4.3. Thermodynamic critical field, H_c

In Chapter 4 we saw that for a type-I superconductor the critical field has a value given by

$$H_c = \left[\frac{2}{\mu_0}(g_n - g_s)\right]^{\frac{1}{2}}, \tag{12.2}$$

where $(g_n - g_s)$ is the difference in free energy densities of the normal and superconducting states in the absence of an applied magnetic field. We may *define* the critical field for *all* types of superconductor by means of (12.2), which can apply equally to type-I and type-II, since for each superconductor there must be, in the absence of an applied magnetic field, a characteristic energy difference $(g_n - g_s)$ between the completely superconducting and completely normal states. H_c is a measure of this energy difference and, to distinguish it from the upper and lower critical fields, it may be called the *thermodynamic critical field*. Only in a type-I

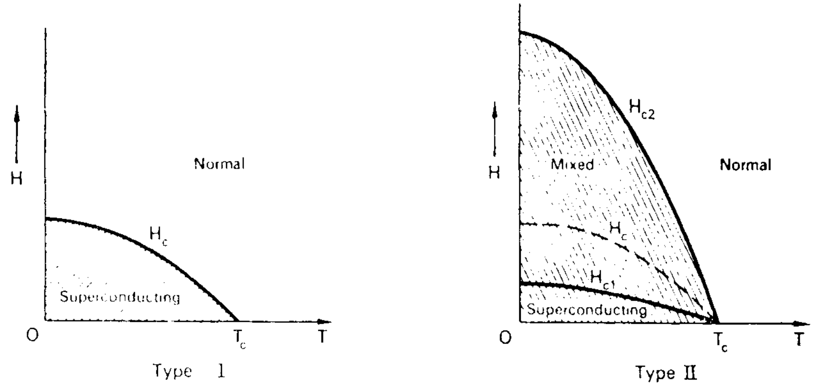

FIG. 12.6. Phase diagrams of type-I and type-II superconductors.

superconductor does the material become normal in a field strength equal to H_c.

When, in describing a type-II superconductor, we say that the upper or lower critical fields have certain values relative to the thermodynamic critical field H_c, we may loosely think of H_c as being about the critical field that would be characteristic of an equivalent type-I superconductor, i.e. one with the same transition temperature.† As will be seen in § 12.5.1, the value of H_c for a type-II superconductor can nevertheless be determined indirectly from its experimentally measured magnetization curve.

12.4.4. Value of the upper critical field

It can be shown that for a type-II superconductor the upper critical field has a value

$$H_{c2} = (\sqrt{2})\kappa H_c, \tag{12.3}$$

so materials with a high value of κ remain in the mixed state and do not go normal until strong magnetic fields are applied.

The ability of type-II superconductors with high values of κ to resist being driven normal until strong magnetic fields are applied is of considerable technical importance, especially in the construction of super-

† We saw in Chapter 9 that the BCS theory of superconductivity predicts a law of corresponding states for different superconductors, from which it follows that superconductors with the same transition temperature should have the same value of H_c at any temperature. This law of corresponding states is fairly well obeyed in practice.

conducting solenoids to generate strong magnetic fields. Whereas at 4·2°K type-I superconductors have critical fields of only a few times 10^4 A m^{-1} (i.e. a few hundred gauss), type-II superconductors can have upper critical field strengths exceeding a million A m^{-1}. Some typical examples are shown in Table 12.1.

TABLE 12.1. UPPER CRITICAL FIELD H_{c2} AT 4·2°K, κ-VALUE AND TRANSITION TEMPERATURE OF SOME TYPE-II ALLOYS COMPARED TO LEAD (TYPE-I)

		H_{c2} (4·2°K)		κ	T_c (°K)
		A m^{-1}	Gauss		
Type-II	Mo_3–Re	$6·7 \times 10^5$	8,400	4	10
	Ti_2–Nb	$\sim 8 \times 10^6$	~100,000	20	9
	Nb_3Sn	$\sim 1·6 \times 10^7$	~200,000	34	18
		H_c (4·2°K)			
Type-I	Pb	$4·4 \times 10^4$	550	0·4	7·2

12.4.5. Paramagnetic limit

The question may be asked, if κ is made indefinitely large, is there any limit to the strength of magnetic field required to drive a type-II superconductor normal? To answer this question, consider a material with a high transition temperature and a large value of κ. At temperatures well below the transition temperature the thermodynamic critical field will be fairly high and so at these temperatures, according to (12.3), we should have a very large value of H_{c2}. For example, at 1·2°K an alloy of 60 atomic per cent titanium and 40 atomic per cent niobium is predicted to have an upper critical field strength H_{c2} of about 20×10^6 A m^{-1}. Experiment shows, however, that for such materials with a very high predicted value of H_{c2}, resistance in fact returns at a considerably weaker field. In the case of the titanium–niobium alloy, the normal state is restored by a magnetic field of strength 10×10^6 A m^{-1}, about half the predicted value of H_{c2}.

This reduced critical field has been ascribed to paramagnetism arising from the spins of the conduction electrons. A magnetic field applied to a normal metal tends to align parallel to itself the spins of the electrons near the Fermi level (Pauli paramagnetism). For moderate magnetic field strengths the degree of alignment and the resulting lowering of the free energy is small, and until now we have neglected it, considering a normal metal to be non-magnetic. But in very strong magnetic fields there may be a considerable reduction of magnetic free energy if the spins of the electrons align parallel to the applied field. Such an alignment is, however, incompatible with superconductivity, which requires that in each Cooper pair the spins of the two electrons shall be anti-parallel. Consequently, in a sufficiently strong magnetic field it may be energetically favourable for the metal to go into the normal paramagnetic state with electrons near to the Fermi level aligned parallel to the field, rather than to remain superconducting with electrons in anti-parallel pairs.

A calculation by Clogston suggests that as a result of the electron paramagnetism a superconductor must go into the normal state if the applied field strength exceeds a strength H_p, equal to about $1{\cdot}4 \times 10^6\, T_c$ A m^{-1}. Consequently the mixed state of a type-II superconductor cannot persist in fields above this value, no matter how large the value of κ.

The same limitation should apply to the increased critical field of a very thin specimen of type-I superconductor. Equation (8.8) implies that the critical field of a thin film could be made indefinitely high if the film were thin enough. In fact, however, the critical field will not increase above the value H_p.

It must be remembered that this effect of the normal electron paramagnetism is only important in superconductors with a very high upper critical field, say 5×10^6 A m^{-1} or more. The effect of electron paramagnetism is negligible in superconductors to which we only apply a relatively weak magnetic field.

To summarize: subject to the considerations which have been discussed in the previous paragraphs, the values of the lower and upper critical fields for a type-II superconductor with Ginzburg–Landau constant κ are given approximately by

$$H_{c1} \simeq \frac{1}{\kappa} H_c,$$

$$H_{c2} \simeq \kappa H_c.$$

12.5. Magnetization of Type-II Superconductors

We now examine the magnetic properties of type-II superconductors. At applied magnetic field strengths H_a below H_{c1}, a type-II superconductor behaves exactly like a type-I superconductor, exhibiting perfect diamagnetism and a magnetization equal to $-H_a$ (Fig. 12.7). When the

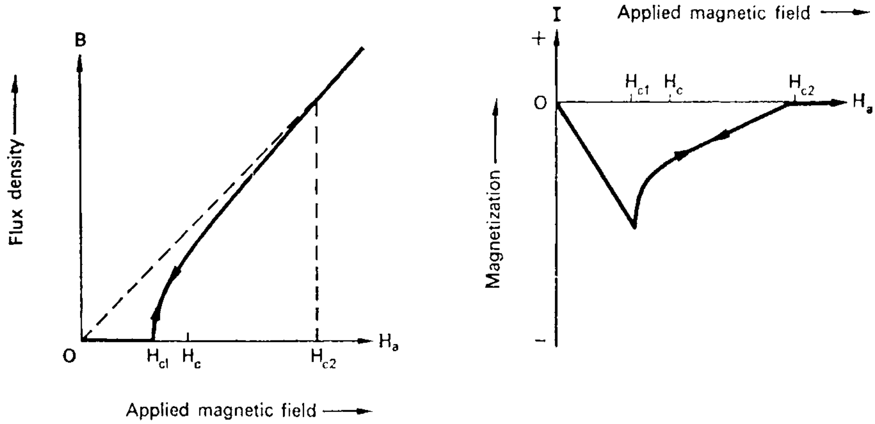

FIG. 12.7. Magnetization of a type-II superconductor.

applied field strength reaches H_{c1}, normal cores with their associated vortices form at the surface and pass into the material. The flux threading the vortices is in the same direction as that due to the applied magnetic field, so the flux in the material is no longer equal to zero and the magnitude of the magnetization suddenly decreases (Fig. 12.7). In fields between H_{c1} and H_{c2} the number of vortices which occupy the sample is governed by the fact that vortices repel each other. The number of normal cores per unit area for a given strength of applied magnetic field is such that there is equilibrium between the reduction in free energy of the material due to the presence of each non-diamagnetic core and the existence of the mutual repulsion between the vortices. As the strength of the applied field is increased, the normal cores pack closer together, so the average flux density in the material increases and the magnitude of the magnetization decreases smoothly with increasing H_a. Near to the upper critical value H_{c2} the flux density and magnetization change linearly with applied field strength. At H_{c2} there is a discontinuous change in the slope of the flux density and magnetization curves, and above H_{c2} the material is in the normal state with flux density equal to $\mu_0 H_a$ and zero magnetization.

We pointed out in § 4.1 that the total area enclosed by the magnetization curve is always equal to the difference between G_n and G_s, i.e. to $\frac{1}{2}\mu_0 H_c^2 V$, and this remains true for a type-II superconductor. In Fig. 12.8

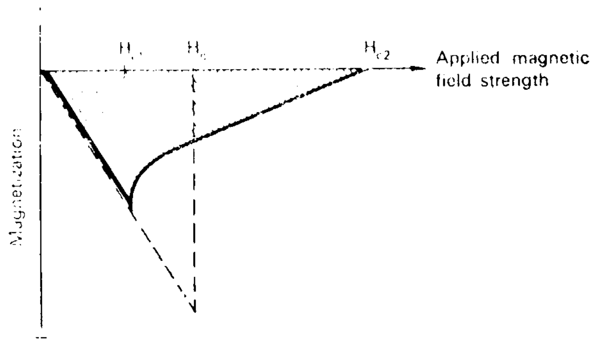

FIG. 12.8. Illustration of thermodynamic critical field H_c of type-II superconductor. The dotted right-angled triangle is drawn to have an area equal to the shaded area within the magnetization curve.

the relationship between H_{c2} and the thermodynamic critical field is illustrated. The ratio between H_{c2} and H_c is such that the area enclosed by the dashed curve equals the area enclosed by the magnetization curve of the type-II superconductor.

12.5.1. Determination of κ

The value of κ of a type-II superconductor can be determined if a magnetization curve has been obtained. If the area under the magnetization curve is measured, the ratio of H_{c2} to H_c can be deduced by use of the construction shown in Fig. 12.8. Formula (12.3) then gives $\kappa = (1/\sqrt{2})(H_{c2}/H_c)$. Furthermore, it has been shown that the slope of the magnetization curve where it cuts the applied field axis at H_{c2} is given by

$$\left[\frac{dI}{dH}\right]_{H_{c2}} = \frac{-1}{1\cdot 16\,(2\kappa^2 - 1)},$$

so we can also determine κ from the slope of the measured magnetization curve. Note, however, that these procedures to determine κ are only valid if the magnetization is reversible (i.e. the same curve is traced in both increasing and decreasing fields). As we shall see in the next section, the magnetization is often not reversible, and the greater the degree of hysteresis the less accurate are the values of κ determined from the magnetization curve.

12.5.2. Irreversible magnetization

If a type-II superconductor is perfectly homogeneous in composition, its magnetization is reversible, i.e. the curves in Fig. 12.7 are the same whether the applied field H_a is increased from zero or decreased from some value greater than H_{c2}. Real samples, however, usually show some irreversibility in their magnetic characteristics (Fig. 12.9). Irreversibility

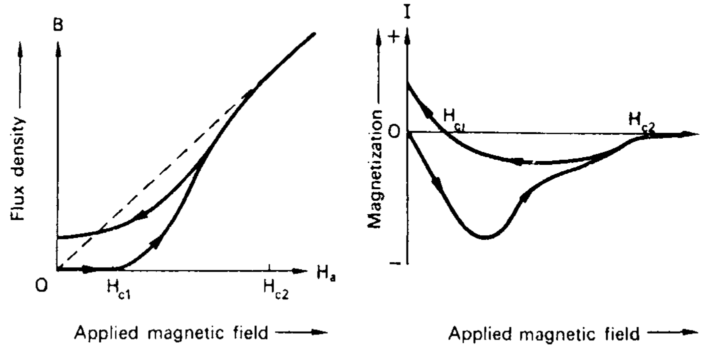

FIG. 12.9. Irreversible type-II magnetization curves.

is attributed to the fact that the normal cores which thread the superconductor in the mixed state, can be "pinned" to imperfections in the material and so are prevented from being able to move freely. Consequently, on increasing the applied field strength from zero there is no sudden entry of flux at H_{c1} because the cores formed at the surface are hindered from moving into the interior. Similarly, on reducing the applied field strength from a value greater than H_{c2}, there is a hysteresis, and flux may be left permanently trapped in the sample, because some of the normal cores are pinned and cannot escape. It appears that almost any kind of imperfection whose dimensions are as large or larger than the coherence length can pin normal cores. For example, both the long chains of lattice faults, called dislocations, and particles of chemical impurities, such as oxides, can give rise to magnetic irreversibility. But type-II materials which are very carefully prepared and purified so as to be free of such defects can show very little irreversibility and can exhibit magnetization curves almost as "ideal" as those in Fig. 12.7. In general, however, specimens contain a number of imperfections and their magnetization curves exhibit some irreversibility and permanently trapped flux. We shall see in the next chapter that the pinning of normal cores by imperfections plays a very important part in determining the critical currents of type-II superconductors.

12.6. Specific Heat of Type-II Superconductors

In § 5.2 we saw that if a type-I superconductor is heated there is, in general, a sudden change in the specific heat as the metal goes from the superconducting into the normal state; the magnitude of this change being given by (5.4). If the heating is done in a constant applied magnetic field H_a, the transition is of the first order, and latent heat is absorbed.

A type-II superconductor, however, has two critical fields, H_{c1} and H_{c2}. On heating a sample in a constant applied magnetic field H_a (e.g. from A to B in Fig. 12.10) we may expect to observe two changes in the specific heat, first at T_1 and then at T_2.

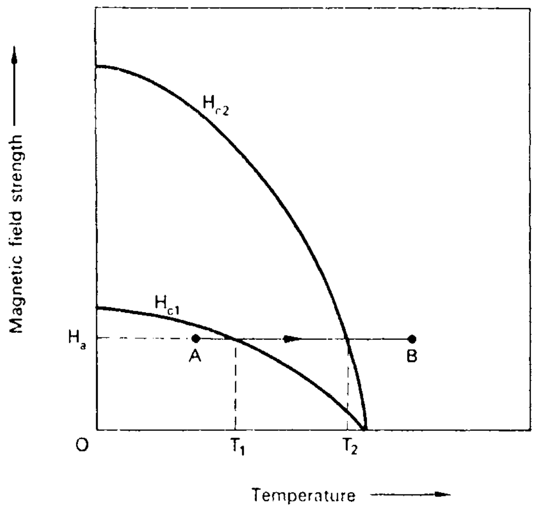

FIG. 12.10. Type-II superconductor heated in a magnetic field.

At temperature T_1 the metal passes from the superconducting to mixed state. It can be seen from Fig. 12.11 that this transition is associated with a large narrow peak in the specific heat curve. According to the Abrikosov model of a type-II superconductor, the magnetization curve at H_{c1} has an infinite slope (see Fig. 12.7), from which it can be shown that at T_1 the entropy is continuous but has an infinite temperature derivative in the mixed state. The lack of discontinuity in the entropy leads to a second-order transition and the infinite temperature derivative should produce a "λ-type" specific heat anomaly. The sharp peak observed at T_1 in Fig. 12.11 is consistent with such a

λ-type specific heat anomaly. At T_2 the specimen passes from the mixed to normal state. According to the description of the mixed state given in § 12.4.2 we would expect that at T_2 the nature of the mixed state approaches that of the normal state. We do not therefore expect any sudden increase in entropy as the metal is heated through T_2 but expect that the entropy of the mixed state will rise towards that of the normal state as the temperature is raised towards T_2. Hence this transition should also be of second order, and we expect at T_2 a sudden drop in the specific heat similar to that observed in type-I superconductors in the absence of a magnetic field. It can be seen in Fig. 12.11 that at T_2 such a specific heat drop is indeed observed.

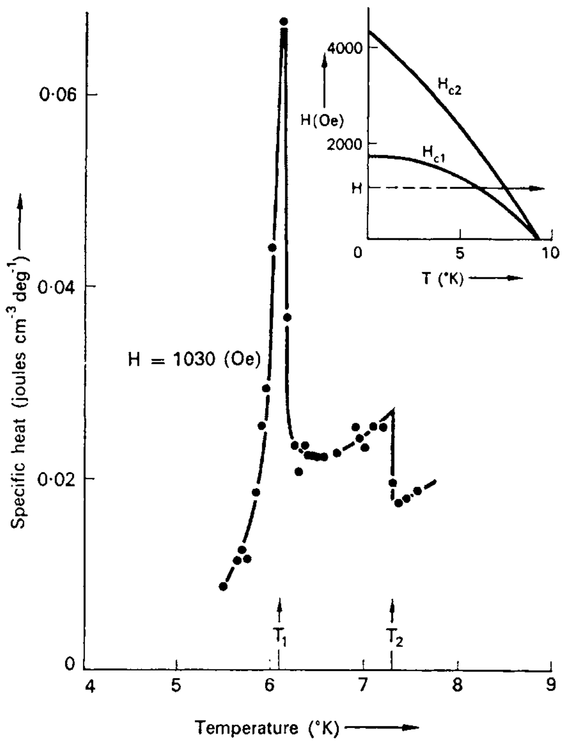

FIG. 12.11. Specific heat of type-II superconductor (niobium) measured in a constant applied magnetic field. (Based on McConville and Serin.)