CHAPTER 13

CRITICAL CURRENTS OF TYPE-II SUPERCONDUCTORS

THE CRITICAL currents of type-II superconductors are of considerable practical interest. We have mentioned previously that electromagnets capable of generating strong magnetic fields can be wound from wires of type-II superconductors, and clearly the more current that can be passed through the windings of such an electromagnet without resistance appearing the stronger will be the magnetic field that can be generated without heat being produced.

In Chapter 7 we saw that, provided the specimen is considerably larger than the penetration depth, the critical current of a type-I superconductor is successfully predicted by Silsbee's hypothesis, i.e. if the resistance is to remain zero, the total magnetic field strength at the surface, due to the current and applied magnetic field together, must not exceed H_c. The situation in type-II superconductors is, however, more complicated, because the state of the material changes at two field strengths, H_{c1} and H_{c2}, not at a single field strength H_c.

It should be pointed out that at present (1977) the behaviour of currents in type-II superconductors is by no means fully understood. Consequently we shall only discuss rather general aspects of the current-carrying capacity and we shall not try to present any detailed treatment, because present ideas are almost bound to be modified by future developments.

13.1. Critical Currents

In a magnetic field whose strength is less than H_{c1} a type-II superconductor is in the completely superconducting state and behaves like a type-I superconductor, whereas in field strengths greater than H_{c1} it goes into the mixed state. We shall see later that, contrary to what one might expect, the mixed state does not necessarily have zero resistance. We might guess, therefore, that, so long as the applied magnetic field is not by itself strong enough to drive the material into the mixed state, the critical current should be determined by a criterion like Silsbee's rule (§ 7.1) for a type-I superconductor, but with H_{c1} substituted for H_c; i.e. the metal will be resistanceless so long as the magnetic field generated by the transport current does not bring the total field at the surface to a value above H_{c1}.

Experiments show that for weak applied magnetic field strengths this modified Silsbee's rule is obeyed, but only in the case of extremely perfect samples, i.e. those with reversible magnetization curves. In the case of a field applied at right angles to a wire of pure type-II superconductor, the critical current falls linearly with increasing field strength, as would be expected (compare curve *a* Fig. 13.1 and Fig. 7.1*b*, p. 84). However, the critical current does not fall to zero at $\frac{1}{2}H_{c1}$, but there remains a small critical current; and even above H_{c1}, where the applied field is by itself strong enough to drive the metal into the mixed state, a type-II superconductor can still carry some resistanceless current. Curve *a* on Fig. 13.1 shows this small critical current extending up to about

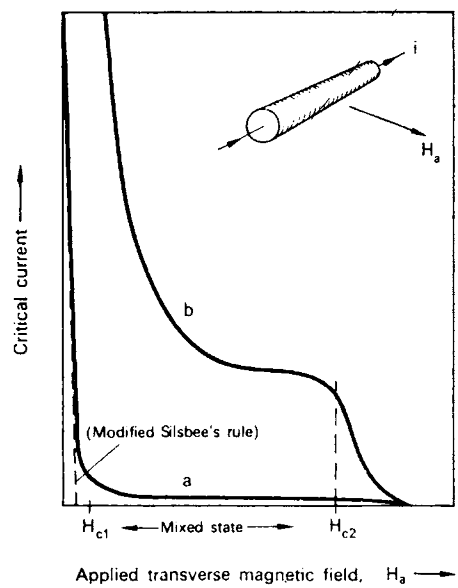

FIG. 13.1. Typical variation of critical currents of wires of (*a*) highly perfect, (*b*) imperfect type-II superconductors in transverse applied magnetic field.

H_{c2}. Most samples are not, however, extremely perfect, and for such imperfect samples the critical current is increased considerably both above and below H_{c1} (curve *b*, Fig. 13.1).

In this chapter we shall be chiefly concerned with the critical currents when the applied magnetic field is perpendicular to the current flow. There is particular interest in this configuration because it is the one which occurs in electromagnets. In a solenoid, for example, the field generated is perpendicular to the coil windings. The critical current curves of Fig. 13.1 refer to this situation. The shape of curve *b*, with a plateau extending up to about H_{c2}, is characteristic of type-II superconductors which are imperfect,† and is quite unlike that which would be predicted by any form of Silsbee's rule. It is found that when a superconductor is in the mixed state its critical current is almost completely controlled by the perfection of the material; the more imperfect the material the greater is the critical current, i.e. the higher and more pronounced is the plateau (Fig. 13.1). A highly imperfect wire may be able to carry about 10^3 A mm^{-2} of its cross-section. Conversely, a rather perfect specimen has a very small critical current, perhaps a few tens of microamps per mm^2, when it is in the mixed state. It is extremely important to understand that the critical current of a type-II superconductor in the mixed state is entirely determined by imperfections and impurities and not by any form of Silsbee's rule. This dependence of the critical current on the perfection of the material is of considerable technical importance because superconducting electromagnets require resistanceless wire of high current-carrying capacity. If Silsbee's rule applied, with H_{c2} as the appropriate magnetic field, the critical currents would be orders of magnitude greater than those which are actually found.

It sometimes happens that at high applied magnetic field strengths the critical current *increases* with increasing field strength, rising to a maximum near H_{c2}. This is known as the "peak effect". However, the reason for the occasional appearance of this effect is not yet understood, and we shall not consider it further in this book.

13.2. Flow Resistance

Before attempting an explanation of what determines how much resistanceless current can flow through a type-II superconductor when it is in the mixed state, we must draw attention to an important experimental observation. Suppose we take a length of wire of type-II

† When speaking of the perfection of a material we mean the lack of both "chemical" impurities (i.e. foreign atoms) and "physical" impurities (i.e. faults in the periodic arrangement of the atoms in the crystal lattice).

superconductor and apply perpendicular to it a magnetic field H_a of sufficient strength to drive the material into the mixed state. We now pass a current along the wire and observe how the voltage V developed between the ends varies as the magnitude i of the current is altered (Fig. 13.2). So long as the current is less than the critical value i_c no voltage is

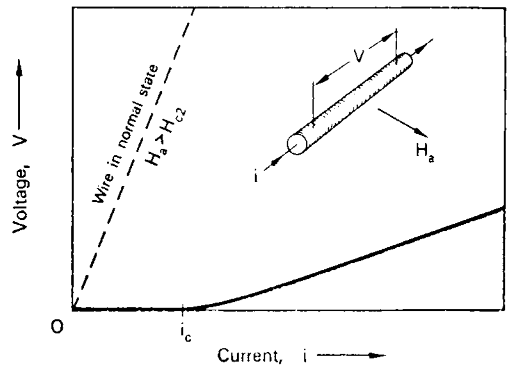

FIG. 13.2. Voltage–current characteristic of type-II superconductor in transverse magnetic field ($H_{c1} < H_a < H_{c2}$).

observed along the wire, but when the current is increased above i_c a voltage appears which, at currents somewhat greater than i_c, approaches a linear increase with increasing current. It should be noted that the voltage developed is considerably less than that which would be observed if the wire were in the normal state. Figure 13.3 shows the

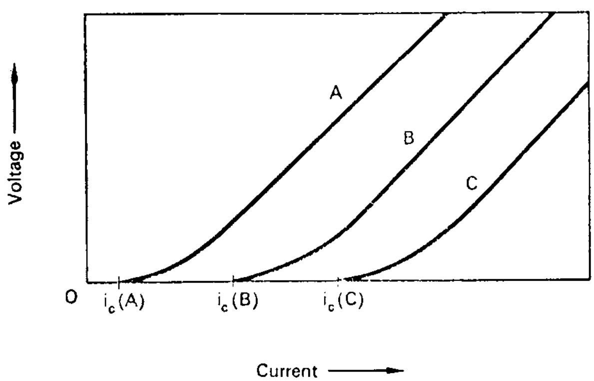

FIG. 13.3. Voltage–current characteristics of three wires of the same type-II superconductor in the mixed state in the same transverse applied magnetic field. Curves A, B, and C refer to specimens which are progressively less perfect.

voltage–current characteristics, measured at the same strength of applied magnetic field, of three wires of equal diameter of the same type-II superconductor but of different degrees of perfection. The critical current is different for each wire, the purer or more perfect wires having lower critical currents; but the slope of the characteristic is the same for all three specimens. We see, therefore, that, though the value of the critical current of a specimen depends on the perfection of the material, the rate at which voltage appears when the critical current is exceeded is an innate characteristic of the particular material and does not depend on how perfect it is.

The value of the slope dV/di which the characteristic approaches at currents well above i_c is known, for reasons which will become apparent later, as the *flow resistance* R' of the specimen. The *flow resistivity* ρ' of

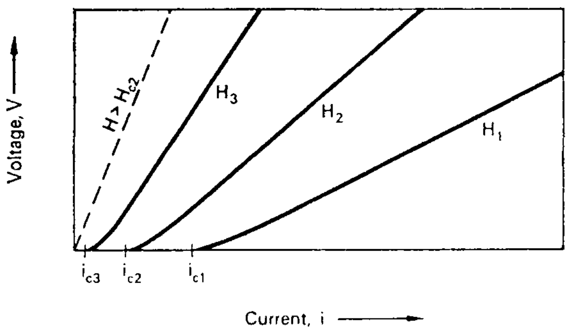

FIG. 13.4. Effect of applied magnetic field strength on V–i characteristic of a type-II superconductor in mixed state in a transverse magnetic field ($H_{c1} < H_1 < H_2 < H_3 < H_{c2}$).

the material from which the specimen is made may be defined by $R' = \rho' l/\mathscr{A}$, where l is the length of the specimen and $\mathscr{A}$ its cross-sectional area. It is found that, for a given strength of applied magnetic field, the flow resistivity is proportional to the normal resistivity of the metal. Furthermore, the flow resistivity increases with increasing strength of applied magnetic field (Fig. 13.4), approaching the normal resistivity as the applied field strength approaches H_{c2}.

13.3. Flux Flow

13.3.1. Lorentz force and critical current

We have seen that a type-II superconductor in the mixed state is able to carry some resistanceless current and that the critical current cannot be determined by any modification of Silsbee's rule. Furthermore, the manner in which voltage appears when the critical current is exceeded is quite different from the case of a type-I superconductor. We now ask what determines the magnitude of the critical current of a type-II superconductor in the mixed state, and what is the source of the voltage which appears at currents greater than the critical current.

The current-carrying properties of type-II superconductors can be qualitatively explained if it is supposed that when a current is passed along a type-II superconductor, which has been driven into the mixed state by an applied magnetic field, the current flows not just at the surface, as in a type-I superconductor, but *throughout the whole body of the metal.*

Consider a length of type-II superconductor in an applied transverse magnetic field of strength greater than H_{c1} (Fig. 13.5). If a current is

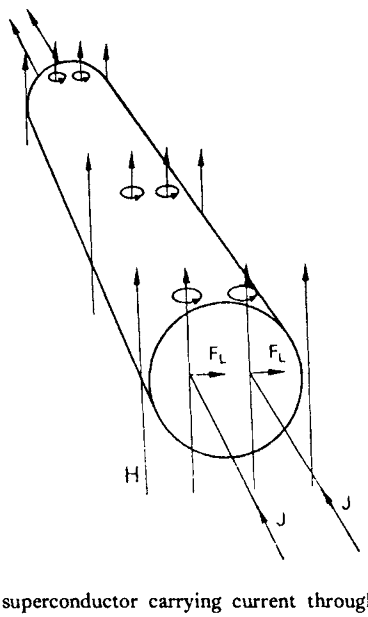

FIG. 13.5. Type-II superconductor carrying current through the mixed state. For stationary cores the Lorentz force F_L is perpendicular both to the axes of the cores and to the current density J.

passed through this specimen, there will be at every point a certain transport current density J. (The "transport current" is the current flowing along a specimen. We use the term to distinguish this motion of the electrons from the circulating vortex currents around the cores.)

However, because the metal is in the mixed state, it is threaded by the magnetic flux associated with the normal cores. There will therefore be an electromagnetic force (Lorentz force) between this flux and the current. When speaking of a Lorentz force between a moving charge and a magnetic field one is in fact referring to the mutual force which exists between the moving charge and the source of the magnetic field. In this case, therefore, the force acts between the electrons carrying the transport current and the vortices generating the flux in the cores. Hence there will be on each vortex a Lorentz force F_L acting at right angles to both the direction of the transport current and to the direction of the flux.

Suppose the specimen is of length l, cross-sectional area $\mathscr{A}$ and carries a current i in an applied magnetic field which is at an angle θ to the direction of the current. If the flux density of the field is B the Lorentz force on the specimen is $liB \sin\theta$. However, since each vortex encloses an amount of flux Φ_0, the mean flux density is $B = n\Phi_0$, where n is the number of vortices per unit area perpendicular to B, and the Lorentz force is therefore $lin\ \Phi_0 \sin\theta$. The total length of all the vortices threading the specimen is $nl\mathscr{A}$, so the mean force per unit length of vortex is $(i/\mathscr{A})\ \Phi_0 \sin\theta$. Though the current density varies between the cores, the average current density J equals $i/\mathscr{A}$, so the Lorentz force on unit length of each vortex can be seen to be

$$F_L = J\Phi_0 \sin\theta. \tag{13.1}$$

In the special case where the applied magnetic field is perpendicular to the direction of the current, $\theta = 90°$ and the force is just

$$F_L = J\Phi_0. \tag{13.2}$$

In the previous chapter we have seen that the cores tend to be pinned at imperfections in the material. Consequently, if the Lorentz force is not too great, the cores remain stationary and do not move under its action. (The electrons which carry the transport current cannot move sideways in the opposite direction, because there can be no component of current across the specimen.) Not every individual core is directly pinned to the material, but the interaction between the vortex currents is sufficient to give the lattice of cores a certain rigidity, so that if only a few cores are pinned the whole pattern is immobilized. What matters, therefore, is the average pinning force per core. Let the average pinning force per unit length of core be F_P. So long as the transport current density J produces a Lorentz force per unit length of core which is less than F_p, the core lattice will not move, i.e. there will be a stable situation if

$$J\Phi_0 < F_p.$$

If, however, the transport current is increased, so that the Lorentz force exceeds F_p, the core lattice is no longer prevented from moving through the specimen. If the cores are set in motion,† and if there is some viscous force opposing their motion through the metal, work must be done in maintaining this motion. This work can only be supplied by the transport current, and so energy must be expended in driving this current through the material. In other words, if the current sets the cores into motion, and if their motion is impeded, *there will be a voltage drop along the material.* This motion of the cores (and the fluxons they contain) through the material is known as "flux flow" and is the source of the flow resistivity observed at currents greater than the critical current.

This motion of the cores when the current exceeds the critical current has been observed directly by the effect it has on neutron diffraction from the mixed state‡ (see p. 188).

The mechanisms producing the viscous force that opposed the motion of the cores through the metal are complicated and we shall not discuss them here. One contribution to this viscous drag arises because the cores contain magnetic flux and, as each core moves, this flux moves with it through the metal. This flux motion induces an e.m.f. which drives a current across the core, the current returning via the surrounding superconducting material. These currents may be thought of as eddy currents set up by the flux motion. Since the core is normal, work is dissipated in driving the current across it, and this is one reason why energy must be provided to keep the cores in motion.

It should be stressed that the situation with regard to the voltage is different in type-I and type-II superconductors. In a type-I superconductor, if the critical current is exceeded, the voltage is due to the transport current flowing through normal regions which span the whole specimen. When flux flow is occurring in a type-II superconductor, the material is still in the mixed state and there are still continuous superconducting paths threading the whole specimen.

The critical current will be that which creates just enough Lorentz

† When the cores are moving the forces acting on them are different. The velocity and direction of the core motion will be discussed in § 13.3.2.

‡ Schelten, Ullmaier and Lippmann, *Phys. Rev.* **B12**, 1772 (1975).

force to detach the cores from the pinning centres, i.e. the critical current density $\mathcal{J}_c$ will be given by

$$\mathcal{J}_c \Phi_0 = F_p.$$

We can now see why the more imperfect specimens have higher critical currents: if there are many imperfections, a greater fraction of cores will be pinned to the material and the mean pinning force per core will be greater.

In the previous chapter we saw that the presence of pinning centres gives rise to irreversible magnetization of imperfect type-II superconductors. If the above explanation of the critical current is correct, the critical current in the mixed state should be greater in those materials which have more irreversible magnetization curves. This indeed is found to be the case. Figure 13.6 shows magnetization and critical current curves of a wire of type-II superconductor (an alloy of tantalum and

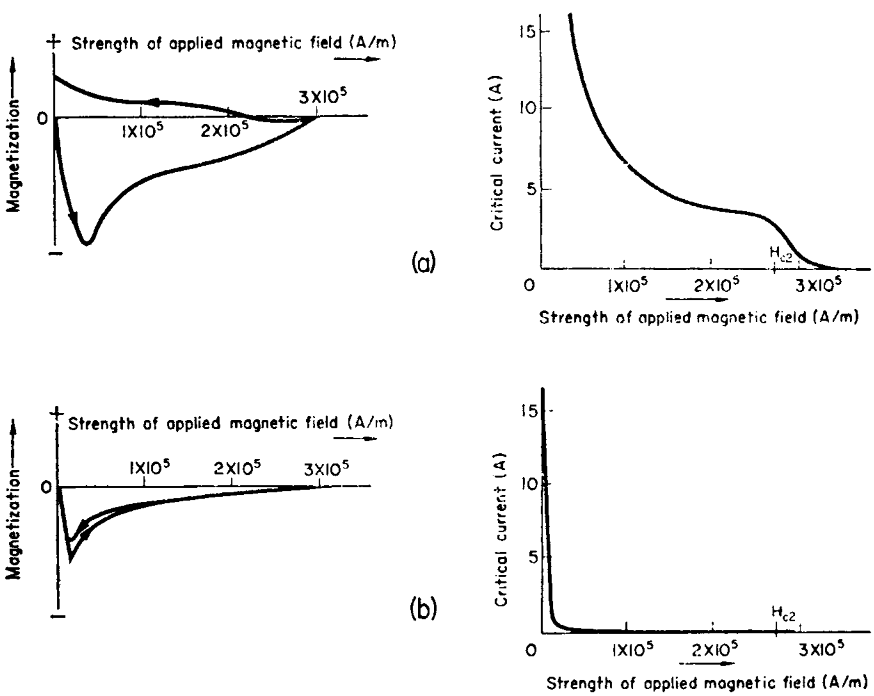

FIG. 13.6. Magnetization and critical current of imperfect (a) and nearly perfect (b) type-II superconductor (tantalum-niobium alloy measured at 4·2°K; Heaton and Rose-Innes).

niobium) before and after it had been purified.† The top two curves show the magnetization and critical current of the wire after it had been drawn from an ingot and so contained many imperfections due to the drawing process. The magnetization curve is very irreversible and there is a plateau of high critical current extending to H_{c2}. The lower curves show the properties of the *same* piece of wire after it had been carefully purified and the imperfections eliminated by heating for several days in a very good vacuum. It can be seen that the magnetization has become almost perfectly reversible and, though the mixed state still persists up to H_{c2}, the high current-carrying capacity has been lost.

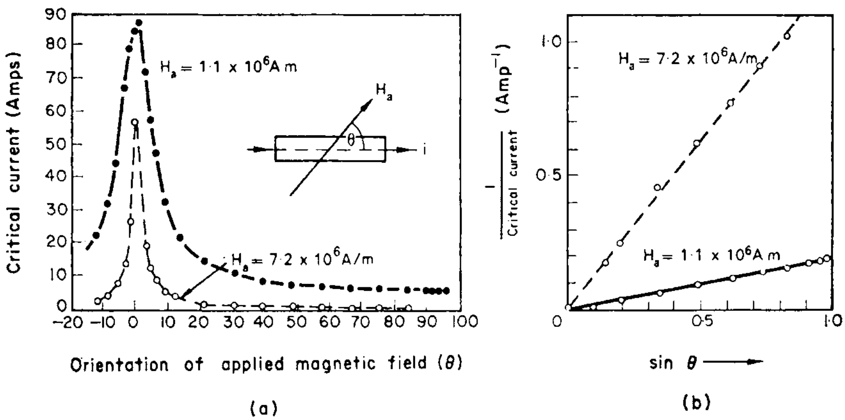

FIG. 13.7. Variation of critical current of Nb_3Sn strip in mixed state with orientation of applied magnetic field. (After G. D. Cody and G. W. Cullen, RCA Laboratories.)

If the critical current is that current which produces a Lorentz force strong enough to move the vortices off any pinning centres, we should expect it to depend markedly on the angle between the magnetic flux and the current. According to (13.1), if θ is the angle the applied magnetic field makes with the current, the critical current should be inversely proportional to $\sin\theta$. Figure 13.7 shows the results of an experiment to test this relationship. It can be seen that, except when the applied field is nearly parallel to the current, the inverse of the critical current does vary

† It may be noticed that many of the experiments used to illustrate the properties of type-II superconductors have been performed on tantalum–niobium or niobium–molybdenum alloys. This is because these alloys are two of the few type-II superconductors which can be prepared in a really pure state so that they show reversible magnetization. Though there are many type-II superconducting alloys, most of these cannot, for metallurgical reasons, be produced in very perfect form.

as sin θ in accordance with the Lorentz force model. If the inverse of the critical current were to vary as sin θ for *all* values of θ, the critical current would tend to infinity as the direction of the applied magnetic field is rotated to lie parallel to the current. Clearly the critical current cannot be infinite, and when the field is parallel to the wire the critical current has a certain maximum value. The factors which limit the critical current is parallel applied magnetic fields are not yet, however, properly understood.

13.3.2. Flux flow

We have associated the voltage, which appears when the transport current is increased above the critical value, with the work required to drive the cores through the metal. For a given current the voltage is independent of time, from which it may be deduced that the core lattice is not accelerating under the forces which act on it but moves with constant velocity. This implies that the metal behaves as though it were a viscous medium as far as motion of the cores is concerned. We have seen

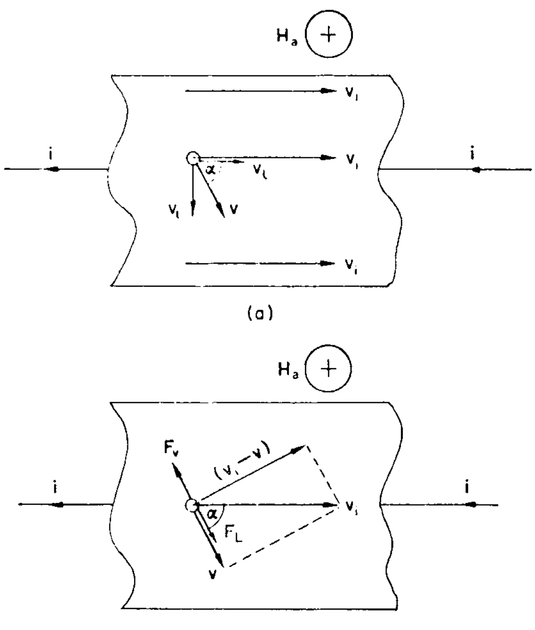

FIG. 13.8. Velocity and force diagrams for vortex motion through mixed state. v_i is the velocity of the electrons carrying the transport current i, and v is the velocity of a vortex through the material. The applied magnetic field is directed into the plane of the page. (Based on Volger, Stass, and van Vijfeiken.)

that the electrical flow resistivity does not depend on the purity of the superconductor, so to each material we can ascribe a "viscosity constant" η such that, if there is a net force F per unit length acting on a core, the core will acquire a velocity $v = F/\eta$.

We now consider what motion the vortices take up when the transport current is raised above the critical value, so that the vortices become detached from the pinning centres. It is important to realize that when the cores are in motion, the magnitude and direction of the forces acting on them are different from when they are held stationary in the material. Consider the special case when the applied magnetic field is perpendicular to the direction of transport current flow. If the cores are held stationary in the metal, the relative velocity between the cores and the electrons carrying the transport current has a certain value, and, as we have seen, there is a Lorentz force tending to detach the vortices from their pinning sites. This force is perpendicular both to the transport current and to the axis of the cores. When, however, the cores have been set in motion, there is a different relative velocity between them and the electrons carrying the current, so the force acting on them is changed. We must remember that the vortices encircling the cores represent a circulatory motion of the superelectrons. This motion is superimposed on the linear motion produced by the transport current, and *in the absence of any other forces* the vortices would be carried along by the transport current with the velocity v_l of its electrons (Fig. 13.8a). If this were to happen there would be no relative motion of the vortices and transport current electrons, and so there would be no Lorentz force on the vortices. However, as we have seen, the appearance of a voltage suggests that there is opposition to the motion of the vortices through the metal, and we now show that, as a result, the vortices acquire a component of velocity at right angles to the transport current.

Because of the viscous drag exerted by the material of the metal on the vortices they will not move as fast as the transport current electrons, but only with some lower velocity v_l. In other words, in the direction of the transport current there will be a relative velocity between the vortices and the electrons carrying the transport current. Consequently there will now be a Lorentz force on the flux threading the vortices, and this gives them a component of velocity v_t sideways across the conductor. The resultant velocity $\mathbf{v}$ of the vortices is therefore at an angle α to the direction of the transport current (Fig. 13.8a).

We can find the direction of the vortex motion by the following self-consistent argument. If in the steady state the vortices move through the metal with a velocity **v** which is constant, the forces on them must balance. These forces are illustrated in Fig. 13.8b. The metal exerts a viscous drag on vortices moving through it, so there will be a force $\mathbf{F}_v$ acting in a direction opposite to the vortex velocity **v**:

$$\mathbf{F}_v = -\eta \mathbf{v}. \tag{13.3}$$

Now the electrons of the transport current have a velocity $(\mathbf{v}_t - \mathbf{v})$ relative to the core of the vortex and this produces, at right angles to $(\mathbf{v}_t - \mathbf{v})$, the Lorentz force $\mathbf{F}_L$ which drives the vortices in the direction **v** (see Fig. 13.8b). The magnitude of $\mathbf{F}_L$ is given by

$$F_L = |\mathbf{v}_t - \mathbf{v}| n_s e \Phi_0, \tag{13.4}$$

where n_s is the number of superelectrons per unit volume. For the velocity of the vortices to be constant, $\mathbf{F}_L$ must equal $-\mathbf{F}_v$, which from (13.3) and (13.4) gives

$$|\mathbf{v}_t - \mathbf{v}| = \frac{\eta v}{n_s e \Phi_0},$$

and the angle α which the vortex motion makes with the transport current is

$$\alpha = \tan^{-1}\left\{\frac{\eta}{n_s e \Phi_0}\right\}.$$

This shows that the greater the viscous drag that the metal exerts on the vortices (i.e. the bigger the value of η) the more nearly will the vortices move at right angles to the transport current. Measurements on type-II superconductors have shown that α is close to 90°. (This result has been obtained from Hall effect measurements. The Hall angle equals $90° - \alpha$.) The fact that α is nearly 90° implies that the viscous drag is large so that when flux flow occurs the vortices move virtually at right angles to the direction of the transport current.

13.3.3. E.M.F. due to core motion

In the previous sections we have seen that a sufficiently strong current i passed through a superconductor in the mixed state sets the cores into motion, and we have supposed that the metal exerts a viscous drag on cores moving through it. The energy required to keep the cores moving can only come from the current and this means that, irrespective of the detailed mechanism of the process, work is required to maintain the current; in other words, there will be a voltage difference V between the ends of the specimen, and the specimen shows resistance. If P is the power required to keep the cores in motion, i.e. the power dissipated in the specimen, the voltage will be simply $V = P/i$.

The above argument as to why a voltage appears at currents greater than the critical current is quite general but gives no insight into the mechanism by which the voltage is generated. In fact, the voltage may be ascribed to an induced e.m.f. generated by the motion of the magnetic flux in the moving cores. Consider, for example, the circuit shown in Fig. 13.9, where M is a piece of type-II superconductor driven into the mixed state by a magnetic field applied perpendicular to the plane of the page. If the cores move across the sample, as the result of a Lorentz force due to a current passed through the sample *or for any other reason*, a voltage V will be recorded on the voltmeter. This voltage is induced by the motion of the magnetic flux contained in the cores. The rate at which flux crosses between the contacts to the voltmeter is $n\Phi_0 v_t d$ where n is the number of cores per unit area, Φ_0 is the magnetic flux within each core (i.e. the fluxon), v_t the transverse velocity of core motion, and d the separation of the contacts. The voltage measured is

$$V = n\Phi_0 v_t d.$$

This "induced voltage" is the same voltage as the "resistive voltage" due to the passage of the current i.

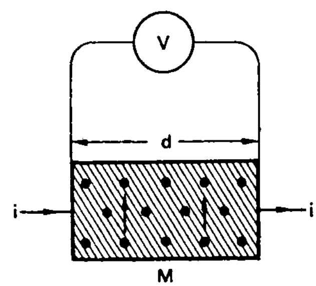

FIG. 13.9. Generation of e.m.f. by flux flow in mixed state.

If it is the motion of the fluxons across a type-II superconductor which produces the e.m.f. in the mixed state, an e.m.f. should appear whenever the cores are set in motion, and the appearance of the e.m.f. should not depend on the cause of the motion. That this is so has been demonstrated by Lowell, Muñoz, and Sousa, who were able to set the fluxons in motion through a specimen of niobium–molybdenum alloy, in which no transport current was flowing, by heating one end of the specimen. If we consider the variation with temperature of the magnetization curve of a type-II superconductor we see that, in a uniform applied magnetic field, there will be a greater density of fluxons in hotter regions than in colder ones. Consequently in a temperature gradient, because of the mutual repulsion of the fluxons, there will be a force driving the fluxons from the hotter to the colder parts. Figure 13.10 shows the arrangement. When a temperature difference was established between the ends of the specimen, a voltage difference V appeared between the two edges. Since no transport current was present, the observed voltage cannot have any "ohmic" source. The voltage changed sign when the direction of the applied magnetic field H_a was reversed. This experiment is the most convincing demonstration that motion of fluxons through the mixed state can generate an e.m.f. The experiment can only be performed on very perfect specimens, otherwise the motion of the fluxon pattern is prevented by the pinning due to imperfections.

We can sum up the current-carrying behaviour of the mixed state as follows: it is supposed that when a current is passed through a type-II superconductor in the mixed state this current flows throughout the metal. This current exerts a Lorentz force on the cores which thread the mixed state. These cores may be anchored to imperfections but if the current exceeds a certain value, the critical current, the cores may be driven across the material. When this "flux flow" occurs a voltage appears perpendicular to the direction of flux motion and heat is generated in the material.

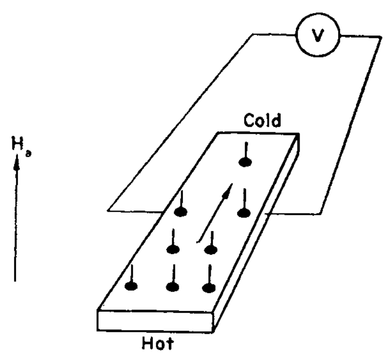

FIG. 13.10. E.M.F. generated by fluxon motion due to a temperature gradient.

It should be made clear that at present (1977) the details of these processes are by no means fully understood. In particular, it is not clear why imperfections pin the core lattice against motion, nor can the form of the variation of critical current with applied magnetic field strength (e.g. Fig. 13.1) be properly explained.

13.4. Surface Superconductivity

It can be seen from Fig. 13.1 that though the critical current of a type-II superconductor falls rapidly when the applied magnetic field strength exceeds H_{c2}, the material is nevertheless able to carry a small, though rapidly decreasing, resistanceless current in fields greater than H_{c2}. This is surprising, because above H_{c2} the metal should be in the normal state and not able to carry any resistanceless current at all. For a number of years, attempts were made to explain this and similar anomalies by supposing that the material was inhomogeneous. It might, for example, be threaded by a network of regions whose critical field was higher than that of the bulk material. It has recently been realized, however, that these "anomalies" are a manifestation of a property which is possessed even by perfectly pure and homogeneous superconductors. This property is that of *surface superconductivity.*

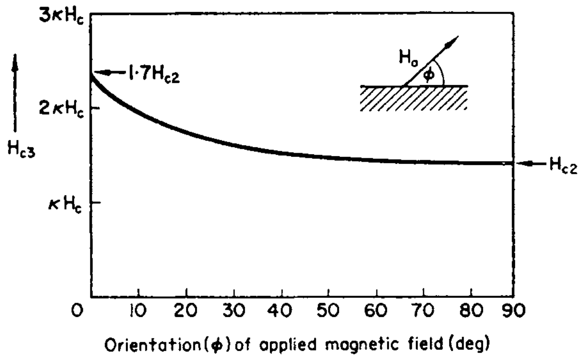

FIG. 13.11. Variation of H_{c3} with angle of applied magnetic field to surface.

In 1963 St. James and De Gennes deduced from theoretical considerations that superconductivity can persist close to the surface of a superconductor in contact with an insulator (including vacuum), even in a magnetic field whose strength is sufficient to drive the bulk material normal. This superconducting surface layer can occur in materials whose

Ginzburg–Landau parameter κ exceeds 0·42. Surface superconductivity is not a special property of type-II superconductors ($\kappa > 0{\cdot}71$) but can occur in any superconductor whose κ-value exceeds 0·42. However, because its effects are usually observed in type-II superconductors, we have delayed its discussion till this chapter.

The surface superconducting layer† can persist in applied magnetic fields up to a certain maximum strength which we call H_{c3}. The value of H_{c3} depends on the angle the applied magnetic field makes with the surface, and H_{c3} is a maximum when the applied field is parallel to the surface. In this case $H_{c3} = 2{\cdot}4\,\kappa H_c$ (i.e. $1{\cdot}7\,H_{c2}$ for a type-II superconductor). If the angle the applied field makes with the surface is increased, the value of H_{c3} decreases (Fig. 13.11) reaching a minimum value of $\sqrt{2}\kappa H_c$ (i.e. H_{c2} in the case of a type-II superconductor) when the field is perpendicular to the surface.

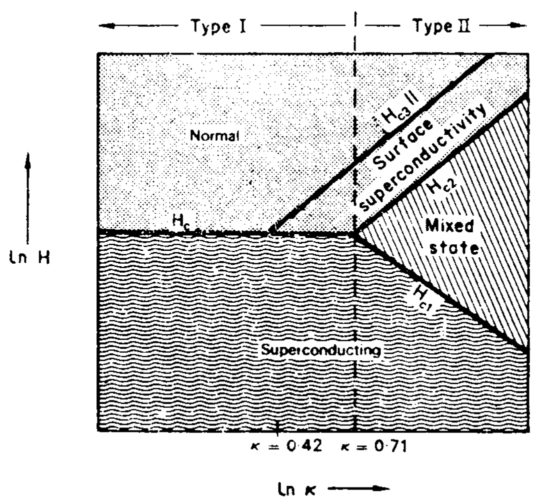

FIG. 13.12. Dependence of characteristics of superconductors on value of Ginzburg–Landau constant κ. $H_{c3}\|$ is value of limiting field strength for surface superconductivity when field is parallel to surface.

We are now in a position to produce a diagram (Fig. 13.12) which shows how the nature of superconductors depends on the value of their Ginzburg–Landau parameter κ. For $\kappa < 0{\cdot}42$ the superconductor is type-I and can exist in one of two states, superconducting or normal,

† This surface layer is often referred to as the superconducting surface "sheath". We, however, prefer to speak of the superconducting surface "layer" to emphasize that, as will be soon shown, this layer may appear only on parts of the surface and does not necessarily enclose the specimen.

depending on whether the magnetic field strength is below or above the thermodynamic critical field H_c. However, if κ exceeds 0·42 a thin superconducting layer can exist on the surface in fields up to H_{c3}. When κ is greater than 0·71 a superconductor is type-II and can exist in four possible conditions—superconducting, mixed, normal with surface superconductivity, and completely normal.

It was pointed out in the previous chapter than the κ of pure metals varies slightly with temperature, increasing as the temperature falls. Hence it is possible for a metal to change its type of superconductivity if the temperature is changed. Lead, for example, has a κ-value of about 0·37 at 7·2°K, but its κ-value increases to 0·58 on cooling to 1·4°K. The value of κ becomes equal to 0·42 at 5·8°K, so at temperatures below this a superconducting surface layer can appear on a lead specimen.

Surface superconductivity occurs only at an interface between a superconductor and a dielectric (including vacuum), and does not occur at an

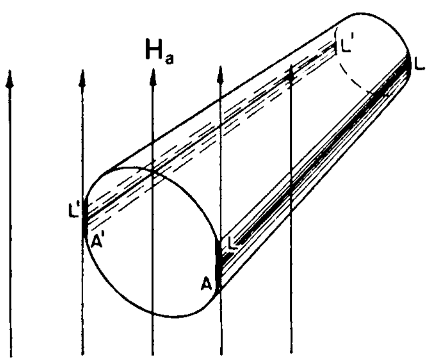

FIG. 13.13. Bands of surface superconductivity along a cylindrical specimen in a transverse magnetic field.

interface between a superconductor and a normal metal. Hence, surface superconductivity can be prevented by coating the surface of a specimen with normal metal. We can, for example, test if an observed behaviour is due to surface superconductivity by seeing whether the behaviour persists after the specimen has been copper-plated.

In general only part of the surface of a specimen is parallel to the applied magnetic field, and consequently, in a magnetic field of strength between H_{c2} and H_{c3}, the superconducting layer will only cover this part of the surface. Figure 13.13 illustrates the important special case of a cylindrical rod or wire in a transverse magnetic field. At an applied field strength equal to H_{c2} the superconducting layer extends right round the surface; but as the field strength is increased, this layer contracts into two bands A and A' running along the sides of the specimen. As the field strength approaches H_{c3} these bands contract to zero along the two lines LL and $L'L'$ where the surface is parallel to the applied field. It is these superconducting bands which account for the small resistanceless current that a type-II superconductor can carry in applied field strengths exceeding H_{c2}.