APPENDIX B

FREE ENERGY OF A MAGNETIC BODY

SUPPOSE that a body of volume V is uniformly magnetized by an applied field H_a, so that its magnetic moment is M. The field is now increased to $H_a + dH_a$ and produces a change of magnetic moment dM. The total energy which must be supplied (e.g. by a battery driving a coil which generates the magnetic field) to make these increments in field and magnetic moment at constant temperature is†

$$dW_{\text{tot}} = V \,.\, d(\tfrac{1}{2}\mu_0 H_a^2) + \mu_0 H_a dM.$$

The first term on the right-hand side represents the work done in increasing the strength of the applied field over the volume occupied by the body; this work must be done to increase the field whether the body is present or not. The second term, $\mu_0 H_a dM$, is the energy which must be supplied to increase the magnetic moment of the body. Let us call this dW_M,

$$dW_M = \mu_0 H_a dM.$$

Comparison of this with the work done by an external pressure p in producing an infinitesimal change in volume dV of a body,

$$dW_V = -p dV,$$

shows that the expression for the work to magnetize the body has a similar form, if we suppose that $\mu_0 H_a$ corresponds to p and M to $-V$.‡

Now the Gibbs free energy for a body in the absence of a magnetic field is

$$G = U - TS + pV,$$

† See, for example, A. B. Pippard, *Classical Thermodynamics*, Cambridge University Press, 1957, p. 26.

‡ The signs are different because energy must be given *to* a body to increase its magnetization, whereas work is done *by* a body when it increases in volume against an external pressure.

where U and S are its internal energy and entropy. As we might expect from the above considerations, the effect of magnetization is to add a term $-\mu_0 H_a M$ analogous to the term $+pV$:

$$G = U - TS + pV - \mu_0 H_a M.$$

Small changes in the conditions will produce a change in G given by

$$dG = dU - TdS - SdT + pdV + Vdp - \mu_0 H_a dM - \mu_0 M dH_a.$$

If the applied field H_a and the magnetic moment M are changed while the temperature and pressure are kept constant ($dT = dp = 0$), we have

$$dG = dU - TdS + pdV - \mu_0 H_a dM - \mu_0 M dH_a.$$

But for a magnetic body under the same conditions of constant T and p,

$$dU = TdS \underbrace{- pdV + \mu_0 H_a dM}_{\text{work done on body}}.$$

Therefore $dG = -\mu_0 M dH_a$, and the change in free energy of a body when it is magnetized to a magnetic moment M by a field of strength H_a is

$$G(H_a) - G(0) = -\mu_0 \int_0^{H_a} M dH_a.$$