

## GUIDANCE AND CONTROL

### INJECTION GUIDANCE ACCURACY AS APPLIED TO LUNAR AND INTERPLANETARY MISSIONS

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#### ABSTRACT

Studies that were performed at the Jet Propulsion Laboratory to determine the accuracy of a typical inertial guidance system as applied to future lunar and interplanetary missions are discussed. Errors in guidance systems are described and analytical techniques for converting these into injection and target errors are presented. The statistics of injection, target, and midcourse maneuver errors are briefly developed. The determination of midcourse maneuver fuel requirements, which is the primary purpose of the study, is then discussed. The effect of parking orbits on injection guidance accuracy was evaluated. These parking orbits (circular satellite coast periods) will be necessary for practical space missions of the near future in order to satisfy various geometrical constraints in an efficient manner. The technique for calculating the injection errors and the effect of the parking orbit on these errors is described. The results of studies of several specific trajectories are presented, illustrating the degree of accuracy to be expected for practical deep space missions of the immediate future. It will be seen that parking orbits do not necessarily reduce guidance accuracy, and in fact, that there is an optimum coast arc.

#### INTRODUCTION

Guidance is necessary in order to steer a vehicle to injection. The guidance system accomplishes this task by determining vehicle position and velocity with some measuring device and controlling the direction of the thrust vector until

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the guidance equations are satisfied, at which time thrust is terminated. If the guidance equations are such that all perturbations that are sensed are adequately compensated for, the vehicle will follow the equivalent of a standard trajectory unless the guidance equipment is inaccurate. In this case, the only sources of coordinate errors at injection are component errors, which lead to an incorrect computation of vehicle position and velocity. Since the vehicle's path is corrected to compensate for any error, true or false, which the guidance system measures, the coordinate errors at injection can be set equal to the measurement errors. This approach allows a system to be evaluated even though the specific guidance equations are not known. Nonstandard performance during burning leads to coordinate dispersions, which are not to be considered as guidance system errors. These coordinate dispersions can be included in the statistical analysis of injection errors, as indicated in the fourth section.<sup>2</sup>

Guidance system errors can be computed by integrating trajectories with the assumed component errors, or can be derived analytically. This paper derives an analytic method for computing these errors for an inertial guidance system. The analytic method gives a good first-order approximation that is quite adequate for error studies. By the use of this method a study can be carried out much faster, requiring less computer time than would be needed to actually integrate many trajectories.

### DESCRIPTION OF SYSTEM STUDIED

The guidance system postulated for this study is a vehicle-borne gyro-stabilized inertial platform on which are mounted three mutually perpendicular integrating accelerometers. A digital computer finds vehicle position and velocity and steers to shutoff such that it will compensate for measurable errors in the flight path.

The component error sources considered in this analysis are accelerometer errors and gyro errors. The accelerometer

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<sup>2</sup>The guidance system attempts to compensate for all dispersions which are sensed. Approximations in the guidance equations may permit some dispersions to be undetected and hence uncorrected. Fuel depletion before desired thrust termination is sensed but leads to dispersions which cannot be corrected. These dispersion sources may be minimized by proper design of the overall system.

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errors are considered to be scale factor, null shift, alignment, and integrator scale factor errors. The gyro errors are considered to be initial offset, random drift and acceleration-sensitive drift. It is assumed that these error sources are uncorrelated. Fig. 1 shows the accelerometer orientation, Fig. 2 shows one accelerometer computer loop, Fig. 3 shows the gyro orientation, and Table 1 lists the component errors used for this study. These values were taken from the open literature (1-4).<sup>3</sup> They represent reasonable values but do not reflect the performance of any specific system.

The pre-injection trajectory is considered to be divided into two powered flight phases, separated by a circular parking orbit coast period. (See Fig. 4.) The parking orbit will be discussed further in the fifth section. The coordinate errors contributed by each of the powered flight phases are computed in terms of quantities obtained from the standard trajectory and the results are combined at injection (see Appendix B). Certain assumptions are made in order to simplify the analysis, such that the vehicle is restricted to a plane (the thrust plane).

### DESCRIPTION OF COMPUTATION OF INJECTION COORDINATE ERRORS

At entry into the parking orbit the position and velocity errors arising from each error source are computed in an inertial Cartesian coordinate system (the plumb line system defined in Fig. 4) to obtain a six-dimensional error vector. The error vectors are transformed to the downrange point where the final burn terminates by a circular orbit B matrix (see Appendix A). This transformation is most simply carried out if the coordinate errors in the plane of motion are first put into polar coordinates (see Fig. 4). The total coordinate error vectors at injection are then obtained by adding the errors contributed by the final burn and those accumulated during the coast interval.

Error vectors in Cartesian coordinates are designated by  $\overline{\delta X}_i$  (the subscript  $i$  indicates the number of the error source, of which a total of 18 are considered). The components of  $\overline{\delta X}_i$  are designated by  $\delta x_{ij}$ ;  $j$  taking on the values one through six, corresponding respectively to  $\delta X$ ,  $\delta Y$ ,  $\delta \dot{X}$ ,  $\delta \dot{Y}$ ,  $\delta Z$ , and  $\delta \dot{Z}$  which are the displacement and velocity errors defined in the inertial plumb line system. Error vectors in polar coordinates are designated by  $\overline{\delta Z}_i$ , with elements  $\delta z_{ij}$ , where  $j$  takes the values

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<sup>3</sup>Numbers in parentheses indicate References at end of paper.

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one through six, corresponding respectively to  $\delta x$ ,  $\delta r$ ,  $\delta v$ ,  $\delta \Gamma$ ,  $\delta z$ , and  $\delta \dot{z}$ . Writing  $\delta \bar{x}_i$  and  $\delta \bar{z}_i$  as row vectors, it is convenient to define  $[\delta X]$  and  $[\delta Z]$  as  $18 \times 6$  matrices with elements  $\delta x_{ij}$  and  $\delta z_{ij}$ .

A  $[\delta X]$  matrix is obtained for each of the two burning periods. These are designated  $[\delta X]_1$  and  $[\delta X]_2$  corresponding to the burnout times  $t_1$  and  $t_2$ . The matrices  $[\delta Z]_k$  are computed from the transformation matrix  $E_k$  by:  $[\delta Z]_k = [\delta X]_k E_k$ , where  $E_k$  is the transformation from Cartesian to polar coordinates at time  $t_k$ ,  $k = 1$  or  $2$ . (See Appendix A.)

The matrix which describes the coordinate errors at injection due to first burn only is  $[\delta Z]_{I1} = [\delta Z]_1 B$ . For ease of computation, one coordinate error, due to integrator scale factor error, accumulated during the coast interval is calculated directly in polar coordinates. This computation results in a  $[\delta Z]_3$  matrix. The total coordinate error matrix at injection is then  $A = [\delta Z]_{I1} + [\delta Z]_2 + [\delta Z]_3$ . The elements of these matrices are derived in Appendix B.

### STATISTICAL CALCULATIONS

The six injection errors are random variables and must be described by a six-dimensional probability density function. If each error source is a Gaussian variable, and if a linear relationship exists between these error sources and the injection errors, then the injection error distribution is Gaussian.

An  $N$ -dimensional Gaussian distribution can be represented as

$$f(x_1, x_2, \dots, x_N) = (2\pi)^{-\frac{N}{2}} |\Lambda|^{-1/2} \exp \left\{ -\frac{1}{2} \bar{X} \Lambda^{-1} \bar{X}^T \right\}$$

where  $x_1, \dots, x_N$  are random variables,  $\bar{X} = (x_1, x_2, \dots, x_N)$ , the superscript  $T$  indicates the transpose of the matrix with that superscript, and  $\Lambda$  is the moment (or covariance) matrix which is real and symmetric. The elements of the  $\Lambda$  matrix are the ensemble averages of the products of the elements of the  $X$  vector

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$$\Lambda = \frac{\sim}{\vec{X}^T \vec{X}} = \begin{bmatrix} \sigma_{x_1}^2 & \rho_{x_1 x_2} \sigma_{x_1} \sigma_{x_2} & \rho_{x_1 x_3} \sigma_{x_1} \sigma_{x_3} & \dots \\ \rho_{x_2 x_1} \sigma_{x_2} \sigma_{x_1} & \sigma_{x_2}^2 & \rho_{x_2 x_3} \sigma_{x_2} \sigma_{x_3} & \dots \\ \vdots & \vdots & \vdots & \\ \vdots & \vdots & \vdots & \sigma_{x_N}^2 \end{bmatrix}$$

where  $\sigma_{x_i}^2$  is the variance of the  $i^{\text{th}}$  random variable, and  $\rho_{x_i x_j}$  is the correlation between the  $i^{\text{th}}$  and  $j^{\text{th}}$  random variables. Thus  $\Lambda$  is a complete statistical description of the probability distribution (5). The moment matrix of error sources  $\Lambda^{(s)}$  on a powered flight trajectory is

$$\Lambda^{(s)} = \begin{bmatrix} \sigma_{s_1}^2 & 0 & 0 & - \\ & \sigma_{s_2}^2 & 0 & - \\ 0 & 0 & \sigma_{s_3}^2 & - \\ - & - & - & \sigma_{s_N}^2 \end{bmatrix}$$

where  $\sigma_{s_i}^2$  is the variance of the  $i^{\text{th}}$  error source ( $i = 1, 2, \dots, N$ ). The off-diagonal terms are zero because it is assumed that there is no correlation between error sources. The same technique can be used if error sources are correlated, but in practice it is more convenient to use a set of uncorrelated error sources.

By using linear perturbation theory, an error source vector is mapped into an injection error vector by the transformation (using polar coordinates)

$$\vec{\delta q} = \vec{\delta S} L = (\delta x, \delta r, \delta v, \delta \Gamma, \delta z, \delta \dot{z})$$

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L is an  $N \times 6$  matrix with elements

$$l_{ij} = \partial q_j / \partial s_i \quad \left\{ \begin{array}{l} i = 1, 2, \dots, N \\ j = 1, 2, \dots, 6 \end{array} \right.$$

The moment matrix of injection coordinate deviations  $\Lambda^{(I)}$  is then

$$\Lambda^{(I)} = \overbrace{\delta \vec{q}^T \delta \vec{q}} = L^T \overbrace{\delta \vec{S}^T \delta \vec{S}} L = L^T \Lambda^{(s)} L$$

The A matrix, defined in the third section, corresponds to  $\delta \vec{S} L$  where the elements of  $\delta \vec{S}$  are the one-sigma values of component errors. Then  $A^T A = \Lambda^{(I)}$ . It may be desired to add more error sources to the analysis, or to include the effects of coordinate dispersions. This can be done by letting

$$A^* = \begin{bmatrix} A \\ \Delta \end{bmatrix}$$

where  $\Delta$  is an  $(N-18) \times 6$  matrix containing the additional terms to be included in the analysis. Then  $\Lambda = A^{*T} A^*$  is the required covariance matrix. It is noticed that  $A^{*T} A^* = A^T A + \Delta^T \Delta$ .

If the U matrix maps injection errors into target errors,  $\delta \vec{M} = \delta \vec{q} U = (\delta M_1, \delta M_2, \delta M_3)$ . U is a  $6 \times 3$  matrix with elements

$$u_{ij} = \partial M_j / \partial q_i \quad \left\{ \begin{array}{l} i = 1, 2, \dots, 6 \\ j = 1, 2, 3 \end{array} \right.$$

The moment matrix of target errors is then

$$\Lambda^{(M)} = \overbrace{\delta \vec{M}^T \delta \vec{M}} = U^T \Lambda^{(I)} U$$

The elements of M may be position deviations at a standard time or at closest approach, or other quantities of interest such as relative velocity or time of flight at closest approach.

The U matrix may be considered as a function of the initial and final values of some parameter defining position on the standard trajectory. The initial value refers to the point

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where  $\delta \mathbf{q}$  is evaluated and the final value to the point where  $\delta \mathbf{M}$  is evaluated. For convenience, the final point may be considered fixed, as in the previous paragraph in which the definition is given  $\overline{\delta \mathbf{q}}(\mathbf{I}) U(\mathbf{I}, \mathbf{T}) = \delta \mathbf{M}$ , I indicates injection and T the reference position at the target. If a midcourse maneuver is to be made, it will be made at some point C on the trajectory. At that point,  $\overline{\delta \mathbf{q}}(\mathbf{C}) U(\mathbf{C}, \mathbf{T}) = \delta \mathbf{M}$ . The midcourse maneuver changes the velocity such that all or some of the components of  $\delta \mathbf{M}$  are nulled.

The midcourse velocity maneuver required is then (6)

$$\overline{\delta \mathbf{q}}_v = - \delta \mathbf{M} F$$

where F is a 3 x 3 matrix with elements

$$f_{ij} = \frac{\partial \dot{q}_j}{\partial M_i} \left\{ \begin{array}{l} i = 1, 2, 3 \\ j = 1, 2, 3 \end{array} \right.$$

such that the velocity components are expressed in the appropriate coordinate system.

The moment matrix of midcourse velocity requirements is then

$$\Lambda^{(v)} = \overbrace{\delta \mathbf{q}_v^T \delta \mathbf{q}_v} = F^T \Lambda^{(M)} F$$

The amount of fuel necessary to perform the midcourse maneuver may then be calculated.

### UNITS OF VARIANCE

To determine the effect of each component error, the uncorrected rms value of the magnitude of the change in impact parameter, which is the distance from the center of the target to the incoming asymptote, was used as a figure of merit, FOM (7). The components of this FOM are represented by the elements of  $\delta \mathbf{M}$  which have been developed in the foregoing.

$$\overrightarrow{\delta \mathbf{M}} = \overrightarrow{\delta \mathbf{S}} \mathbf{L} \mathbf{U} = \overrightarrow{\delta \mathbf{S}} \mathbf{D}$$

where D = LU. The elements of D are

$$d_{ij} = \frac{\partial M_j}{\partial s_i} \left\{ \begin{array}{l} i = 1, 2, \dots, N \\ j = 1, 2, 3 \end{array} \right.$$

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$$\begin{aligned}
 \text{FOM}^2 &= |\vec{\delta M}|^2 = \overset{\sim}{\delta M_1^2} + \overset{\sim}{\delta M_2^2} + \overset{\sim}{\delta M_3^2} = \sum_{k=1}^3 \sum_{i,j=1}^N \frac{\partial M_k}{\partial s_i} \frac{\partial M_k}{\partial s_j} \overset{\sim}{s_i s_j} \\
 &= \sum_{k=1}^3 \sum_{i=1}^N \left( \frac{\partial M_k}{\partial s_i} \right)^2 \sigma_{s_i}^2 = \sum_{i=1}^N p_i^2 \sigma_i^2
 \end{aligned}$$

where

$$p_i^2 = \left( \frac{\partial M_1}{\partial s_i} \right)^2 + \left( \frac{\partial M_2}{\partial s_i} \right)^2 + \left( \frac{\partial M_3}{\partial s_i} \right)^2$$

The percentage of  $\text{FOM}^2$  due to the  $i^{\text{th}}$  error source, to be called the number of units of variance is

$$n_i = 100 p_i^2 \sigma_i^2 / \text{FOM}^2$$

The value of the  $i^{\text{th}}$  error source which produces one unit of variance is

$$u_i = \text{FOM} / 10 p_i$$

Values of  $n_i$  are listed in Table 2 for the trajectories studied.

### EFFECT OF PARKING ORBIT

Direct ascent trajectories lose payload rapidly as true anomaly at injection increases. In order to satisfy the necessary geometrical constraints and avoid large payload losses, the launching location must be moved. This is an impractical solution. By using a parking orbit as part of the ascent trajectory, the launcher is effectively moved, and a given mission may be accomplished with a resulting greater payload. In addition to this primary argument for the use of parking orbits, their use affords a simple mechanism of correcting for launch time delays (8). Thus it appears that a parking orbit will be used for most lunar and interplanetary missions.

A study was made to determine the effect of the parking orbit interval on guidance errors. The parking orbit determines the effects of the errors due to the first burning phase. The second burning phase errors, in polar coordinates, do not change for a given mission. This illustrates the utility of the polar coordinate system for the near Earth part of the trajectory. For a given parking orbit, the ascent trajectory does not change significantly with launch time delay so that  $[\delta Z]_1$  may be considered constant. Then the moment matrix at



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injection into the parking orbit,  $\Lambda^{(1)} = [\delta Z]_1^T [\delta Z]_1$  is also constant. To determine the effect of the parking orbit on the injection errors

$$\begin{aligned} \Lambda^{(I)} &= A^T A = \left\{ [\delta Z]_1 B + [\delta Z]_2 \right\}^T \left\{ [\delta Z]_1 B + [\delta Z]_2 \right\} \\ &= [\delta Z]_2^T [\delta Z]_2 + B^T \left\{ [\delta Z]_1^T [\delta Z]_1 + [\delta Z]_1^T [\delta Z]_2 B^{-1} \right. \\ &\quad \left. + B^{-1T} [\delta Z]_2^T [\delta Z]_1 \right\} B \\ &= \Lambda^{(2)} + B^T \left[ \Lambda^{(1)} + \Lambda^{(1,2)} + \Lambda^{(1,2)T} \right] B \end{aligned}$$

where  $\Lambda^{(2)} = [\delta Z]_2^T [\delta Z]_2$  is the contribution of the second burn phase.  $\Lambda^{(1,2)} = [\delta Z]_1^T [\delta Z]_2 B^{-1}$  is a matrix in the form of a sum of outer products of error vectors due to first burn and those of second burn rotated back around the coast arc to the point of injection into the parking orbit. The effect of the parking orbit on guidance accuracy is through the correlations between injection errors which are seen to be functions of the coast arc, through the B matrix.

When the post-injection trajectory is determined, launching from a given location at a certain time requires a definite coast arc in the parking orbit. For purposes of studying the effect of the parking orbit, it was assumed that the coast arc could be continuously varied for a given mission. This implies moving the launcher location. As previously pointed out such an implication is unrealistic, but it does point out the effects that are caused by the parking orbit. Another way to change the coast arc would be to launch at a different date, but this requires a different post-injection trajectory, and the effect of the coast interval would not be as clearly seen.

### RESULTS

The technique described has been used to determine the FOM for several representative lunar and interplanetary missions. Table 3 describes the trajectories and the FOM associated with each of them for the system described in the second section with the component errors listed in Table 1. The flow chart of computations performed is shown in Fig. 5.

Fig. 6 presents FOM vs. coast arc in the parking orbit  $\psi$

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for the interplanetary trajectories, and Fig. 7 shows data for the lunar trajectories. It is clear that there is an optimum value of the coast arc. This is because correlations between coordinate deviations change as a function of the parking orbit interval, and certain errors may cancel each other. Table 4 lists the standard deviations in polar coordinates and the correlation coefficients for the slow lunar trajectory. Data for the other six trajectories are similar.

Fig. 8 presents the FOM vs. flight time for the three lunar trajectories. These trajectories impact at roughly the same time, thus having similar geometrical properties. It is seen that the faster trajectories tend to have smaller target error (but, see Fig. 7 where there is a reversal for a coast arc greater than  $132^\circ$ ). A similar conclusion would be reached for the interplanetary cases. This does not mean that a larger midcourse maneuver would necessarily be required on the slower trajectories, for the higher error sensitivities mean that less correction is required for a given error.

As seen in Appendix B, the values of some of the error terms depend on certain parameters; specifically accelerometer erection angle  $\phi$  and gyro orientation angles  $\alpha$  and  $\beta$ . These parameters were varied for the fast Venus trajectory, and the results are shown in Fig. 9.

## CONCLUSIONS

It is seen from Table 2 that of all the error sources considered, only a few are of major significance. Any improvement in the design of the other components would not improve guidance accuracy much if the major error sources were unimproved.

As shown in Figs. 6 and 7, there is an optimum value of coast arc which minimizes the effects of guidance component errors. By scheduling launches appropriately, trajectories could be designed that would use near-optimum coast arc. Other considerations, such as post-injection trajectory characteristics determined by the positions of the planets will normally determine the scheduling of a launch. The value of the parking orbit study is that it shows that longer coast intervals do not necessarily require larger midcourse maneuver capabilities.

As shown in Fig. 9, there is an optimum set of values for the guidance parameters  $\alpha$ ,  $\beta$ , and  $\phi$ . This figure applies to a specific trajectory, but a similar result would apply to other trajectories. The method developed in this paper may be used to evaluate the guidance parameters for any specific trajectory.

APPENDIX A: DESCRIPTION OF THE MATRICES

The B matrix transforms errors in polar coordinates around the coast arc  $\psi$

$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{r_0}{r}(2 \sin \psi - 3\psi) & (2 - \cos \psi) & \frac{v}{r}(\cos \psi - 1) & \frac{\sin \psi}{r} & 0 & 0 & 0 \\
 \frac{r_0}{v}(4 \sin \psi - 3\psi) & 2 \frac{r}{v}(1 - \cos \psi) & (2 \cos \psi - 1) & \frac{2 \sin \psi}{v} & 0 & 0 & 0 \\
 2r_0(\cos \psi - 1) & r \sin \psi & -v \sin \psi & \cos \psi & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \cos \psi & \frac{-v}{r} \sin \psi \\
 0 & 0 & 0 & 0 & 0 & \frac{r}{v} \sin \psi & \cos \psi
 \end{bmatrix}$$

$v = \sqrt{\frac{\mu}{r}}$  the circular satellite velocity at altitude  $r - r_0$

$r = \sqrt{x_1^2 + (y_1 + r_0)^2}$  the radial distance from the center of Earth at injection into the parking orbit

$\psi = (\Delta r + t_2 - t_1) \frac{v}{r}$  where  $\Delta r$  is nominally zero and is used for arbitrarily changing the coast interval

The values of  $r_0$  and  $\mu$  are

|         | $r_0$         | $\mu$   |
|---------|---------------|---|
| English | 20,902,910 ft | $14.07689 \times 10^{15} \text{ ft}^3/\text{sec}^2$ |
| Metric  | 6,372,160 m   | $3.986135 \times 10^{14} \text{ m}^3/\text{sec}^2$  |

The  $E_i$  matrix transforms errors in Cartesian coordinates to errors in polar coordinates ( $i = 1$  at end of first burn phase,  $i = 2$  at end of second burn phase)

|  |                                    |   |  |    |    |
|--|------------------------------------|---|--|----|----|
| $\frac{r_0}{r_i} \cos\left(\frac{x_i}{r_0}\right)$ | $\sin\left(\frac{x_i}{r_0}\right)$ | 0   | $\frac{1}{r_i} \cos\left(\frac{x_i}{r_0}\right)$           | 0  | 0  |
| $\frac{r_0}{r_i} \sin\left(\frac{x_i}{r_0}\right)$ | $\cos\left(\frac{x_i}{r_0}\right)$ | 0   | $-\frac{1}{r_i} \sin\left(\frac{x_i}{r_0}\right)$          | 0  | 0  |
| 0  | 0                                  | $\cos\left(\Gamma_i - \frac{x_i}{r_0}\right)$ | $-\frac{1}{v} \sin\left(\Gamma_i - \frac{x_i}{r_0}\right)$ | 0  | 0  |
| 0  | 0                                  | $\sin\left(\Gamma_i - \frac{x_i}{r_0}\right)$ | $\frac{1}{v} \cos\left(\Gamma_i - \frac{x_i}{r_0}\right)$  | 0  | 0  |
| 0  | 0                                  | 0   | 0  | -1 | 0  |
| 0  | 0                                  | 0   | 0  | 0  | -1 |

$$r_i = \sqrt{x_i^2 + (y_i + r_0)^2}$$

$$\cos\left(\frac{x_i}{r_0}\right) = \frac{y_i + r_0}{r_i}$$

$$\dot{x}_i \frac{1}{V_i} = \cos\left(\Gamma_i - \frac{x_i}{r_0}\right)$$

$$V_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2}$$

$$\sin\left(\frac{x_i}{r_0}\right) = \frac{x_i}{r_0}$$

$$\dot{y}_i \frac{1}{V_i} = \sin\left(\Gamma_i - \frac{x_i}{r_0}\right)$$

The standard trajectory has all the necessary quantities in the inertial plumbline Cartesian coordinate system.

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### APPENDIX B: DERIVATION OF ERROR TERMS

One of the major aims of this analytical derivation is to avoid integrating perturbed trajectories. If the parking orbit interval is changed, the second burn phase produces different incremental Cartesian coordinates, although it produces identical incremental polar coordinates. (The incremental coordinates measured are of interest, since they correspond to the physical situation of setting all initial conditions equal to zero at the start of the second burn.) With a changed parking orbit interval, the local horizon at start of second burn will have rotated through  $V/r\Delta r$  radians with reference to the standard local horizon ( $\Delta r = 0$ ). The effect of this rotation can be duplicated by imagining that the accelerometers and gyros have been rotated by this amount and that the second burn occurs at the standard location on the coast arc. Then the transformation  $[\delta X]_2 E_2 = [\delta Z]_2$  uses the standard  $E_2$  matrix. In the following derivations the subscript  $k$  takes values 1 or 2 for first or second burn, and  $\xi_k = \xi + (k - 1)V/r\Delta r$ , where  $\xi$  indicates  $\phi$ ,  $\alpha$ , or  $\beta$ .

#### ACCELEROMETER ERRORS

##### Mathematical Model

It is assumed that the accelerometer axes A, B, and C are aligned relative to a fixed inertial reference as shown in Fig. 1 and that the computer loop is as shown in Fig. 2.

##### Effect of an Accelerometer Scale Factor Error

As may be shown (Fig. 2) the differential equation for the error in the A coordinate due to a scale factor error  $J_A$  only is

$$\delta \ddot{A} + \frac{\mu}{r^3} \delta A = J_{A_m} \ddot{A}_m$$

The solution of this equation is

$$\delta A = J_A A = J_A (X \cos \phi + Y \sin \phi)$$

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Therefore the error terms are<sup>4</sup>

$$\delta \dot{X} = J_A (\dot{X} \cos^2 \phi_k + \dot{Y} \sin \phi_k \cos \phi_k)$$

$$\delta X = J_A (X \cos^2 \phi_k + Y \sin \phi_k \cos \phi_k)$$

$$\delta \dot{Y} = \delta \dot{X} \tan \phi_k$$

$$\delta Y = \delta X \tan \phi_k$$

$$\delta \dot{Z} = \delta Z = 0$$

Similarly for the B accelerometer

$$\delta \dot{X} = J_B (\dot{X} \sin^2 \phi_k - \dot{Y} \sin \phi_k \cos \phi_k)$$

$$\delta X = J_B (X \sin^2 \phi_k - Y \sin \phi_k \cos \phi_k)$$

$$\delta \dot{Y} = -\delta \dot{X} \cot \phi_k$$

$$\delta Y = -\delta X \cot \phi_k$$

and no first-order errors arise from  $J_C$

### Effect of a Null Shift Error

As may be shown (Fig. 2) the differential equation for the error in A coordinates due to a null shift  $n_A$  only is

$$\ddot{A} + \frac{\mu}{r^3} \delta A = n_A$$

The solution of this equation is

$$\delta A = n_A \frac{r^3}{\mu} \left[ 1 - \cos \left( t \sqrt{\frac{\mu}{r^3}} \right) \right]$$

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<sup>4</sup>The subscript m denotes a measured coordinate, as distinguished from a true coordinate. It will be assumed throughout that the measured coordinate deviations are equal to the true coordinate deviations.

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Therefore the error terms are

$$\begin{aligned}\delta \dot{X} &= n_A \sqrt{\frac{r^3}{\mu}} \cos \phi_k \sin \left( t \sqrt{\frac{\mu}{r^3}} \right) \\ \delta X &= n_A \frac{r^3}{\mu} \cos \phi_k \left[ 1 - \cos \left( t \sqrt{\frac{\mu}{r^3}} \right) \right] \\ \delta \dot{Y} &= \delta \dot{X} \tan \phi_k \\ \delta Y &= \delta X \tan \phi_k \\ \delta \dot{Z} &= \delta Z = 0\end{aligned}$$

Similarly for the B accelerometer

$$\begin{aligned}\delta \dot{X} &= -n_B \sqrt{\frac{r^3}{\mu}} \sin \phi_k \sin \left( t \sqrt{\frac{\mu}{r^3}} \right) \\ \delta X &= -n_B \frac{r^3}{\mu} \sin \phi_k \left[ 1 - \cos \left( t \sqrt{\frac{\mu}{r^3}} \right) \right] \\ \delta \dot{Y} &= -\delta \dot{X} \cot \phi_k \\ \delta Y &= -\delta X \cot \phi_k \\ \delta \dot{Z} &= \delta Z = 0\end{aligned}$$

and for the C accelerometer

$$\begin{aligned}\delta \dot{X} &= \delta X = \delta \dot{Y} = \delta Y = 0 \\ \delta \dot{Z} &= n_C \sqrt{\frac{r^3}{\mu}} \sin \left( t \sqrt{\frac{\mu}{r^3}} \right) \\ \delta Z &= n_C \frac{r^3}{\mu} \left[ 1 - \cos \left( t \sqrt{\frac{\mu}{r^3}} \right) \right]\end{aligned}$$



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### Effect of an Alignment Error

As may be shown (Fig. 1) the differential equation for the error in the A coordinate due to the A accelerometer only is

$$\ddot{A}_m = \ddot{X}_m \cos(\phi_k + \epsilon_A) + \ddot{Y}_m \sin(\phi_k + \epsilon_A)$$

so that, since  $\epsilon_A$  is a small angle

$$\delta \ddot{A}_m = \epsilon_A (-\ddot{X}_m \sin \phi_k + \ddot{Y}_m \cos \phi_k)$$

Therefore the error terms are

$$\delta \dot{X} = \epsilon_A (-\dot{X}_m \sin \phi_k \cos \phi_k + \dot{Y}_m \cos^2 \phi_k)$$

$$\delta X = \epsilon_A (-X_m \sin \phi_k \cos \phi_k + Y_m \cos^2 \phi_k)$$

$$\delta \dot{Y} = \delta \dot{X} \tan \phi_k$$

$$\delta Y = \delta X \tan \phi_k$$

$$\delta \dot{Z} = \delta Z = 0$$

Similarly for the B accelerometer

$$\delta \dot{X} = \epsilon_B (\dot{X}_m \cos \phi_k \sin \phi_k + \dot{Y}_m \sin^2 \phi_k)$$

$$\delta X = \epsilon_B (X_m \cos \phi_k \sin \phi_k + Y_m \sin^2 \phi_k)$$

$$\delta \dot{Y} = -\delta \dot{X} \cot \phi_k$$

$$\delta Y = -\delta X \cot \phi_k$$

$$\delta \dot{Z} = \delta Z = 0$$

The C accelerometer alignment error was considered to consist of two components  $\epsilon_{CX}$  and  $\epsilon_{CY}$  such that

$$\delta \ddot{Z} = \epsilon_{CX} \ddot{X}_m + \epsilon_{CY} \ddot{Y}_m$$

## GUIDANCE AND CONTROL

Assuming that these two components are uncorrelated and have equal standard deviations  $\epsilon_C$  about zero mean

$$\sigma_{\dot{Z}}^2 = \epsilon_C^2 (\dot{X}_m^2 + \dot{Y}_m^2)$$

Therefore the error terms are

$$\delta \dot{Z} = \epsilon_C \sqrt{\dot{X}_m^2 + \dot{Y}_m^2}$$

$$\delta Z = \epsilon_C \sqrt{X_m^2 + Y_m^2}$$

$$\delta \dot{X} = \delta X = \delta \dot{Y} = \delta Y = 0$$

### Effect of Integrator Scale Factor Error (Clock Error)

This error arises from errors in the timing device that controls the integration interval in the digital integrator. From Fig. 2, the differential equation for the error in A coordinates due to a clock error ( $J_t$ ) only is

$$\delta \ddot{A} + (1 + J_t)^2 \frac{\mu}{r^3} \delta A = (2 + J_t) J_t \ddot{A} - (1 + J_t) J_t \ddot{A}_m$$

The solution of this equation is

$$\begin{aligned} \delta A &= J_t \cos \left( t \sqrt{\frac{\mu}{r^3}} \right) \int \sec^2 \\ &\quad \left( T \sqrt{\frac{\mu}{r^3}} \right) \int (2\ddot{A} - \ddot{A}_m) \cos \left( r \sqrt{\frac{\mu}{r^3}} \right) dr dT \\ &= J_t (2A - A_m) \text{ to first order}^5 \\ \delta \dot{A} &= J_t (\dot{A} - \dot{A}_m) \text{ to first order}^5 \end{aligned}$$

---

<sup>5</sup>Although all the terms required to evaluate the integrals are available on the standard trajectory, it was found that the first-order approximation is quite adequate for these terms.

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Therefore the error terms are

$$\delta \dot{X} = J_t (\dot{X} - \dot{X}_m)$$

$$\delta X = J_t (2X - X_m)$$

$$\delta \dot{Y} = J_t (\dot{Y} - \dot{Y}_m)$$

$$\delta Y = J_t (2Y - Y_m)$$

$$\delta \dot{Z} = \delta Z = 0$$

The clock error would also cause a change in the parking orbit interval. This effect, which is that of starting the second burn at the wrong time, can be most easily calculated directly in polar coordinates. It is simply an error in the downrange distance

$$\delta x = -J_t r_0 \frac{V}{r} (t_1 + r + \Delta r)$$

where  $r$  is the parking orbit interval. This element forms the only nonzero element in  $[\delta Z]_3$ .

### GYRO ERRORS

#### Mathematical Model

It is assumed that the gyro axes are oriented as shown in Fig. 3. This particular configuration is chosen to eliminate anisoelastic drift rate (i.e., drift rate proportional to the product of accelerations along spin and input axes) while maintaining orthogonality of the input axes. With no anisoelastic drift, the general expression for angular error  $\theta$  about the input axis of a gyro is

$$\theta = \theta_0 + \dot{\theta}_0 t + \mu_S \int a_I dt - \mu_I \int a_S dt$$

$$\theta_0 = \text{initial offset}$$

$$\dot{\theta}_0 = \text{random drift rate}$$

$$a_I = \text{measurable acceleration along the input axis}$$

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$a_S$  = measurable acceleration along the spin axis

$\mu_I, \mu_S$  = constants

The gyro error will cause the accelerometers to sense a false acceleration, and the acceleration error vector is

$$\vec{\delta a} = \vec{\theta} \times \vec{a}_m \text{ (standard)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \theta_1 & \theta_2 & \theta_3 \\ \ddot{X}_m & \ddot{Y}_m & 0 \end{vmatrix}$$

Input Axis Perpendicular to the Thrust Plane IA<sub>1</sub>

$$\delta a = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \theta \\ \ddot{X}_m & \ddot{Y}_m & 0 \end{vmatrix} = \vec{j} \theta \ddot{X}_m - \vec{i} \theta \ddot{Y}_m$$

$$\delta \ddot{X} = -\ddot{Y}_m \theta$$

$$= -\ddot{Y}_m \theta_0 + \dot{\theta}_0 t - \mu_I \int_0^t (\ddot{X}_m \cos \alpha + \ddot{Y}_m \sin \alpha) dt$$

$$\delta \ddot{Y} = \ddot{X}_m \theta$$

$$= \ddot{X}_m \left[ \theta_0 + \dot{\theta}_0 t - \mu_I \int_0^t (\ddot{X}_m \cos \alpha + \ddot{Y}_m \sin \alpha) dt \right]$$

The integral appearing in these two equations must be evaluated as

$$(k-1)(\dot{X}_{m1} \cos \alpha_1 + \dot{Y}_{m1} \sin \alpha_1) + \int_{t_{k-1}}^{t_k} (\ddot{X}_m \cos \alpha_k + \ddot{Y}_m \sin \alpha_k) dt$$

where the quantities multiplied by (k - 1) represent the acceleration sensitive drift effects of first burn as initial

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conditions for the second burn. Setting this quantity,  $\dot{X}_{m1} \cos \alpha_1 + Y_{m1} \sin \alpha_1 = W_1$ , and interpreting the integral to be over the first or second burning phase (for  $k = 1$  or  $2$ , respectively), the error terms are

$$\begin{aligned} \delta \dot{X} &= -\theta_0 \dot{Y}_{mk} - \dot{\theta}_0 \left[ \dot{I}_{1k} + (k-1) \dot{Y}_{m2}(t_1 + r + \Delta r) \right] \\ &\quad + \mu_I \left[ \dot{I}_{3k} \cos \alpha_k + \dot{I}_{4k} \sin \alpha_k + (k-1) \dot{Y}_{m2} W_1 \right] \end{aligned}$$

$$\begin{aligned} \delta X &= -\theta_0 Y_{mk} - \dot{\theta}_0 \left[ I_{1k} + (k-1) Y_{m2}(t_1 + r + \Delta r) \right] \\ &\quad + \mu_I \left[ I_{3k} \cos \alpha_k + I_{4k} \sin \alpha_k + (k-1) Y_{m2} W_1 \right] \end{aligned}$$

$$\begin{aligned} \delta \dot{Y} &= \theta_0 \dot{X}_{mk} + \dot{\theta}_0 \left[ \dot{I}_{2k} + (k-1) \dot{X}_{m2}(t_1 + r + \Delta r) \right] \\ &\quad - \mu_I \left[ \dot{I}_{5k} \cos \alpha_k + \dot{I}_{6k} \sin \alpha_k + (k-1) \dot{X}_{m2} W_1 \right] \end{aligned}$$

$$\begin{aligned} \delta Y &= \theta_0 X_{mk} + \dot{\theta}_0 \left[ I_{2k} + (k-1) X_{m2}(t_1 + r + \Delta r) \right] \\ &\quad - \mu_I \left[ I_{5k} \cos \alpha_k + I_{6k} \sin \alpha_k + (k-1) X_{m2} W_1 \right] \end{aligned}$$

$$\delta \dot{Z} = \delta Z = 0$$

where

$$\dot{I}_{1k} = \int_{t_{k-1}}^{t_k} (t - t_{k-1}) \ddot{Y}_m dt$$

$$\dot{I}_{2k} = \int_{t_{k-1}}^{t_k} (t - t_{k-1}) \ddot{X}_m dt$$

$$\dot{I}_{3k} = \int_{t_{k-1}}^{t_k} \ddot{Y}_m \dot{X}_m dt$$

$$\dot{I}_{4k} = \int_{t_{k-1}}^{t_k} \ddot{Y}_m \dot{Y}_m dt$$

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$$\dot{I}_{5k} = \int_{t_{k-1}}^{t_k} \ddot{X}_m \dot{X}_m dt$$

$$\dot{I}_{6k} = \int_{t_{k-1}}^{t_k} \ddot{X}_m \dot{Y}_m dt$$

$$I_{1k} = \iint_{t_{k-1}}^{t_k} (t - t_{k-1}) \ddot{Y}_m dt^2$$

$$I_{2k} = \iint_{t_{k-1}}^{t_k} (t - t_{k-1}) \ddot{X}_m dt^2$$

$$I_{3k} = \iint_{t_{k-1}}^{t_k} \ddot{Y}_m \dot{X}_m dt^2$$

$$I_{4k} = \iint_{t_{k-1}}^{t_k} \ddot{Y}_m \dot{Y}_m dt^2$$

$$I_{5k} = \iint_{t_{k-1}}^{t_k} \ddot{X}_m \dot{X}_m dt$$

$$I_{6k} = \iint_{t_{k-1}}^{t_k} \ddot{X}_m \dot{Y}_m dt^2$$

Spin Axis Perpendicular to the Thrust Plane SA<sub>2</sub>, SA<sub>3</sub>.

Gyro no. 2 is considered, in which the input axis is  $\beta$  deg above the X axis (the analysis for gyro no. 3 follows immediately by setting  $\beta' = \beta + 90$  deg).

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$$\begin{aligned} \vec{\delta a} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \theta \cos \beta_k & \theta \sin \beta_k & 0 \\ \ddot{X}_m & \ddot{Y}_m & 0 \end{vmatrix} \\ &= \vec{k} \theta (\ddot{Y}_m \cos \beta_k - \ddot{X}_m \sin \beta_k) \end{aligned}$$

$$\begin{aligned} \delta \ddot{Z} &= \theta_0 (\ddot{Y}_m \cos \beta_k - \ddot{X}_m \sin \beta_k) + \dot{\theta}_0 t (\ddot{Y}_m \cos \beta_k - \ddot{X}_m \sin \beta_k) \\ &+ \mu_S (\ddot{Y}_m \cos \beta_k - \ddot{X}_m \sin \beta_k) \int_0^t (\ddot{X}_m \cos \beta + \ddot{Y}_m \sin \beta) dt \end{aligned}$$

The integral must be evaluated as

$$\begin{aligned} &(k - 1) (\dot{X}_{m1} \cos \beta_1 + \dot{Y}_{m1} \sin \beta_1) + \\ &\int_{t_{k-1}}^{t_k} (\ddot{X}_m \cos \beta_k + \ddot{Y}_m \sin \beta_k) dt \end{aligned}$$

where the quantities multiplied by  $(k - 1)$  represent the acceleration sensitive drift effects of first burn as initial conditions for the second burn. Setting this quantity,  $\dot{X}_{m1} \cos \beta_1 + \dot{Y}_{m1} \sin \beta_1 = W_2$ , the error terms are

$$\begin{aligned} \delta \dot{X} &= \delta X = \delta \dot{Y} = \delta Y = 0 \\ \delta \dot{Z} &= \theta_0 (\dot{Y}_{mk} \cos \beta_k - \dot{X}_{mk} \sin \beta_k) + \dot{\theta}_0 \left[ \dot{I}_{1k} \cos \beta_k \right. \\ &- \dot{I}_{2k} \sin \beta_k + (k - 1) (\dot{Y}_{m2} \cos \beta_2 \\ &- \dot{X}_{m2} \sin \beta_2) (t_1 + r + \Delta r) \left. \right] + \mu_S \left[ \dot{I}_{3k} \cos^2 \beta_k \right. \\ &+ (\dot{I}_{4k} - \dot{I}_{5k}) \cos \beta_k \sin \beta_k - \dot{I}_{6k} \sin^2 \beta_k \\ &\left. + (k - 1) (\dot{Y}_{m2} \cos \beta_2 - \dot{X}_{m2} \sin \beta_2) W_2 \right] \end{aligned}$$

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$$\begin{aligned} \delta Z = & \theta_0 (Y_{mk} \cos \beta_k - X_{mk} \sin \beta_k) + \dot{\theta}_0 \left[ I_{1k} \cos \beta_k \right. \\ & - I_{2k} \sin \beta_k + (k-1) (Y_{m2} \cos \beta_2 \\ & - X_{m2} \sin \beta_2) (t_1 + r + \Delta r) \left. \right] + \mu_S \left[ I_{3k} \cos^2 \beta_k \right. \\ & + (I_{4k} - I_{5k}) \cos \beta_k \sin \beta_k - I_{6k} \sin^2 \beta_k \\ & \left. + (k-1) (Y_{m2} \cos \beta_2 - X_{m2} \sin \beta_2) W_2 \right] \end{aligned}$$

The twelve integrals used in the foregoing analysis can be reduced to seven, since, after integration by parts

$$\begin{aligned} \dot{I}_{1k} &= \dot{Y}_{mk}(t_k - t_{k-1}) - Y_{mk} \\ I_{1k} &= Y_{mk}(t_k - t_{k-1}) - 2I_{7k} \end{aligned}$$

where

$$I_{7k} = \int_{t_{k-1}}^{t_k} Y_m dt$$

$$\begin{aligned} \dot{I}_{2k} &= \dot{X}_{mk}(t_k - t_{k-1}) - X_{mk} \\ I_{2k} &= X_{mk}(t_k - t_{k-1}) - 2I_{8k} \end{aligned}$$

where

$$I_{8k} = \int_{t_{k-1}}^{t_k} X_m dt$$

$$I_{3k} = \dot{I}_{3k}(t_k - t_{k-1}) - I_{9k}$$



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where

$$I_{9k} = \int_{t_{k-1}}^{t_k} (t - t_{k-1}) \ddot{Y}_m \dot{X}_m dt$$

$$\dot{I}_{4k} = \frac{1}{2} \dot{Y}_{mk}^2$$

$$I_{4k} = \frac{1}{2} I_{10k}$$

where

$$I_{10k} = \int_{t_{k-1}}^{t_k} \dot{Y}_m^2 dt$$

$$\dot{I}_{5k} = \frac{1}{2} \dot{X}_{mk}^2$$

$$I_{5k} = \frac{1}{2} I_{11k}$$

where

$$I_{11k} = \int_{t_{k-1}}^{t_k} \dot{X}_m^2 dt$$

$$\dot{I}_{6k} = \dot{Y}_{mk} \dot{X}_{mk} - \dot{I}_{3k}$$

$$I_{6k} = I_{12k} - I_{3k}$$

where

$$I_{12k} = \int_{t_{k-1}}^{t_k} \dot{Y}_m \dot{X}_m dt$$

Thus the integrals  $\dot{I}_{3k}$ ,  $I_{7k}$ ,  $I_{8k}$ ,  $I_{9k}$ ,  $I_{10k}$ ,  $I_{11k}$ , and  $I_{12k}$  are needed, and these can be evaluated in terms of quantities available on the standard trajectory.

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### REFERENCES

- 1 Meisenholder, G.W., "An introduction to the operation and testing of accelerometers," TM 33-2, Jet Propulsion Laboratory, CIT, Pasadena, Calif., Oct. 3, 1960.
- 2 Jensen, L.K., Evans, B.H., and Clark, R.B., "Evaluation of precision gyros for space boost guidance applications," Preprint 1175-60, ARS Semi-Annual Meeting, Los Angeles, Calif., May 9-12, 1960.
- 3 "Preliminary descriptive material on the GG8001 B miniature integrating gyro," U-ED 9841 Minneapolis Honeywell Co., Minn., Aeronautical Division, March 28, 1960.
- 4 "Preliminary descriptive document, GG 177 hinged pendulous accelerometer," U-ED 9870, Minneapolis Honeywell Co., Minn., Aeronautical Division, Oct. 6, 1960.
- 5 Noton, A.R.M., "The statistical analysis of space guidance systems," T.M. 33-15, Jet Propulsion Laboratory, CIT, Pasadena, Calif., June 15, 1960.
- 6 Noton, A.R.M., Cutting, E., and Barnes, F.L., "Analysis of radio command mid-course guidance," TR 32-28, Jet Propulsion Laboratory, CIT, Pasadena, Calif., Sept. 8, 1960.
- 7 Kizner, W., "A method of describing miss distances for lunar and interplanetary trajectories," EP 674, Jet Propulsion Laboratory, CIT, Pasadena, Calif., Aug. 1, 1959.
- 8 Clarke, V.C., Jr., "Design of lunar and interplanetary trajectories," TR 32-30, Jet Propulsion Laboratory, CIT, Pasadena, Calif., July 26, 1960.

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Table 1 One sigma component errors (assuming Gaussian distribution)

| <u>k</u> | <u>Description</u>                      | <u><math>\sigma_k</math></u>          |
|----------|---|---------------------------------------|
| 1        | A accelerometer scale factor error      | $5 \times 10^{-4}$                    |
| 2        | B accelerometer scale factor error      | $5 \times 10^{-4}$                    |
| 3        | A accelerometer null shift              | $4 \times 10^{-3}$ m/sec <sup>2</sup> |
| 4        | B accelerometer null shift              | $4 \times 10^{-3}$ m/sec <sup>2</sup> |
| 5        | C accelerometer null shift              | $4 \times 10^{-3}$ m/sec <sup>2</sup> |
| 6        | A accelerometer alignment error         | 0 (See a)                             |
| 7        | B accelerometer alignment error         | $4 \times 10^{-4}$ radian             |
| 8        | C accelerometer alignment error         | $4 \times 10^{-4}$ radian             |
| 9        | Gyro no. 1 initial offset               | $5 \times 10^{-4}$ radian             |
| 10       | Gyro no. 2 initial offset               | $5 \times 10^{-4}$ radian             |
| 11       | Gyro no. 3 initial offset               | $5 \times 10^{-4}$ radian             |
| 12       | Gyro no. 1 random drift                 | $3 \times 10^{-6}$ radian/sec         |
| 13       | Gyro no. 2 random drift                 | $3 \times 10^{-6}$ radian/sec         |
| 14       | Gyro no. 3 random drift                 | $3 \times 10^{-6}$ radian/sec         |
| 15       | Gyro no. 1 acceleration-sensitive drift | $5 \times 10^{-7}$ rad-sec/m          |
| 16       | Gyro no. 2 acceleration-sensitive drift | $5 \times 10^{-7}$ rad-sec/m          |
| 17       | Gyro no. 3 acceleration-sensitive drift | $5 \times 10^{-7}$ rad-sec/m          |
| 18       | Clock error                             | 0 (See b)                             |

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<sup>a</sup> A accelerometer alignment error is taken to be zero, as it is considered that the A accelerometer alignment defines a reference direction for all other alignments.

<sup>b</sup> Clock error was found to have a truly negligible effect even when using pessimistic estimates.

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Table 2 Number of units of variance for error sources studied

| k | Description                           | 90 hr<br>lunar | 66 hr<br>lunar | 42 hr<br>lunar | 118 day<br>Venus | 88 day<br>Venus | 235 day<br>Mars | 216 day<br>Mars |
|---|---------------------------------------|----------------|----------------|----------------|------------------|-----------------|-----------------|-----------------|
| 1 | A accelerometer scale<br>factor error | 18.100         | 20.139         | 27.207         | 32.466           | 17.565          | 45.431          | 14.142          |
| 2 | B accelerometer scale<br>factor error | 9.867          | 5.211          | 5.736          | 18.411           | 14.118          | 9.124           | 11.555          |
| 3 | A accelerometer null<br>shift error   | 6.825          | 6.830          | 9.852          | 12.357           | 6.797           | 1.065           | 0.091           |
| 4 | B accelerometer null<br>shift error   | 1.709          | 1.144          | 0.756          | 2.316            | 2.208           | 3.156           | 4.147           |
| 5 | C accelerometer null<br>shift error   | 0.003          | 0.011          | 0.122          | 0.029            | 0.018           | 0.048           | 0.012           |
| 6 | A accelerometer alignment<br>error    | 0              | 0              | 0              | 0                | 0               | 0               | 0               |
| 7 | B accelerometer alignment<br>error    | 2.487          | 3.043          | 0.756          | 2.064            | 2.466           | 1.980           | 0.419           |
| 8 | C accelerometer alignment<br>error    | 0.008          | 0.027          | 0.287          | 0.062            | 0.038           | 0.103           | 0.026           |

Table 2 Number of units of variance for error sources studied (Continued)

| k  | Description                                 | 90 hr<br>lunar | 66 hr<br>lunar | 42 hr<br>lunar | 118 day<br>Venus | 88 day<br>Venus | 235 day<br>Mars | 216 day<br>Mars |
|----|---|----------------|----------------|----------------|------------------|-----------------|-----------------|-----------------|
| 9  | Gyro no. 1 initial offset                   | 7.102          | 6.719          | 2.707          | 7.965            | 6.175           | 3.761           | 3.640           |
| 10 | Gyro no. 2 initial offset                   | 0.002          | 0.009          | 0.117          | 0.045            | 0.037           | 0.047           | 0.012           |
| 11 | Gyro no. 3 initial offset                   | 0.011          | 0.036          | 0.461          | 0.112            | 0.074           | 0.267           | 0.067           |
| 12 | Gyro no. 1 random drift                     | 6.637          | 8.412          | 21.343         | 1.507            | 7.557           | 5.761           | 2.655           |
| 13 | Gyro no. 2 random drift                     | 0.000          | 0.002          | 0.479          | 0.350            | 0.367           | 0.782           | 0.213           |
| 14 | Gyro no. 3 random drift                     | 0.030          | 0.080          | 1.115          | 0.203            | 0.112           | 2.294           | 0.543           |
| 15 | Gyro no. 1 acceleration-<br>sensitive drift | 47.154         | 48.160         | 25.992         | 21.106           | 41.627          | 24.632          | 62.084          |
| 16 | Gyro no. 2 acceleration-<br>sensitive drift | 0.001          | 0.012          | 0.528          | 0.451            | 0.415           | 0.223           | 0.064           |
| 17 | Gyro no. 3 acceleration-<br>sensitive drift | 0.062          | 0.165          | 2.542          | 0.556            | 0.427           | 1.327           | 0.329           |

Table 2 Number of units of variance for error sources studied (Continued)

| k  | Description               | 90 hr<br>lunar | 66 hr<br>lunar | 42 hr<br>lunar | 118 day<br>Venus | 88 day<br>Venus | 235 day<br>Mars | 216 day<br>Mars |
|----|---------------------------|----------------|----------------|----------------|------------------|-----------------|-----------------|-----------------|
| 18 | Clock error               | 0              | 0              | 0              | 0                | 0               | 0               | 0               |
|    | Total for all Sources     | 100.000        | 100.000        | 100.000        | 100.000          | 100.000         | 100.000         | 100.000         |
|    | Total for gyro no. 1      | 60.893         | 63.291         | 50.042         | 30.578           | 55.359          | 34.154          | 68.389          |
|    | Total for A Accelerometer | 24.925         | 26.969         | 37.059         | 44.823           | 24.362          | 46.496          | 14.233          |

The two totals shown above illustrate that one high quality accelerometer (A) and one high quality gyro (no. 1) would reduce uncorrected target errors significantly whereas improving the other components would have a much smaller effect.

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Table 3 Trajectory description and uncorrected one sigma target error (FOM) due to injection errors

| Trajectory number | Trajectory type | Flight time | Parking orbit interval, sec | FOM, km |
|-------------------|-----------------|-------------|-----------------------------|---------|
| 1                 | slow lunar      | 90 hr       | 659.976                     | 15,650  |
| 2                 | nominal lunar   | 66 hr       | 721.648                     | 6,380   |
| 3                 | fast lunar      | 42 hr       | 859.978                     | 2,530   |
| 4                 | slow Venus      | 118 day     | 784.3957                    | 346,900 |
| 5                 | fast Venus      | 88 day      | 768.8225                    | 212,100 |
| 6                 | slow Mars       | 235 day     | 1575.6735                   | 454,800 |
| 7                 | fast Mars       | 216 day     | 1548.0000                   | 351,300 |

Table 4 Statistics of injection errors for trajectory no. 1<sup>a</sup>

Standard deviation

$$\begin{aligned}
 \sigma_x &= 16.000 \text{ km} & \rho_{xr} &= -0.86676 \\
 \sigma_r &= 10.942 \text{ km} & \rho_{xv} &= 0.98355 \\
 \sigma_y &= 14.443 \text{ m/sec} & \rho_{x\Gamma} &= -0.74795 \\
 \sigma_\Gamma &= 2.0501 \text{ millirad} & \rho_{rv} &= -0.91866 \\
 \sigma_z &= 10.268 \text{ km} & \rho_{r\Gamma} &= 0.95163 \\
 \sigma_{\dot{z}} &= 18.255 \text{ m/sec} & \rho_{v\Gamma} &= -0.81131 \\
 & & \rho_{z\dot{z}} &= 0.40330 \\
 & & \rho_{xz} = \rho_{x\dot{z}} = \rho_{rz} = \rho_{r\dot{z}} = \rho_{vz} = \rho_{v\dot{z}} = \rho_{\Gamma z} = \rho_{\Gamma\dot{z}} &= 0
 \end{aligned}$$

<sup>a</sup>This data is representative of the seven trajectories studied.

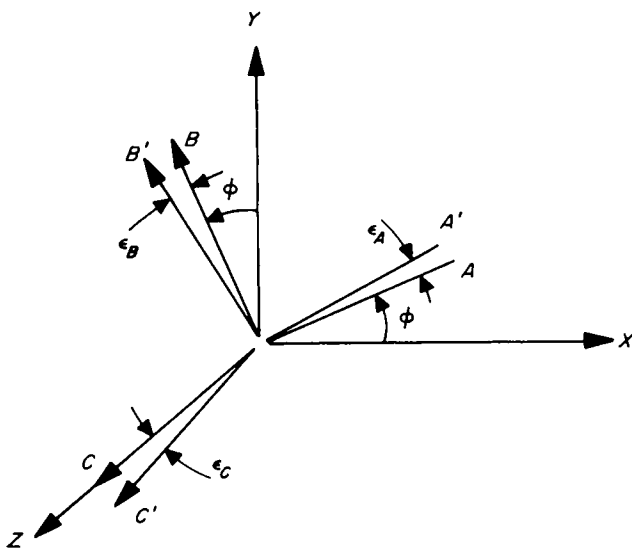


Fig. 1 Accelerometer orientation

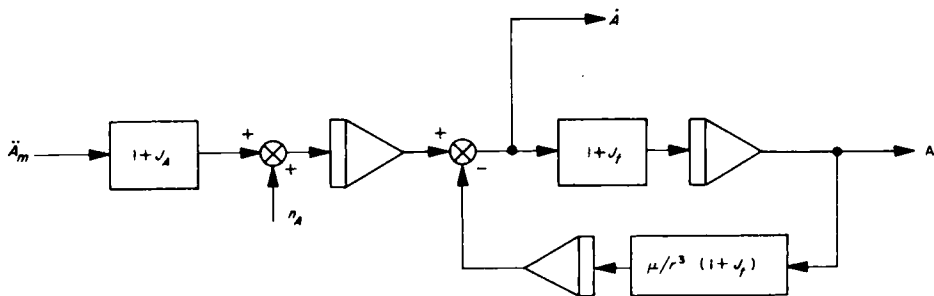


Fig. 2 Accelerometer computer loop



# GUIDANCE AND CONTROL

IA - INPUT AXIS  
 SA - SPIN AXIS  
 OA - OUTPUT AXIS

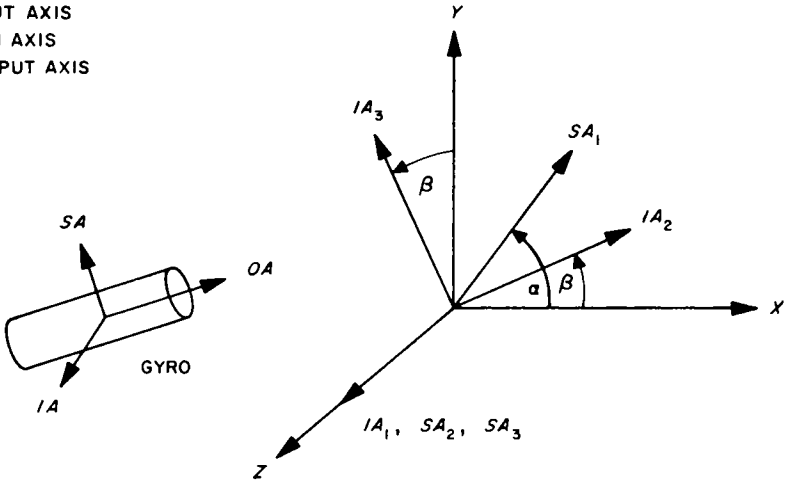


Fig. 3 Gyro orientation

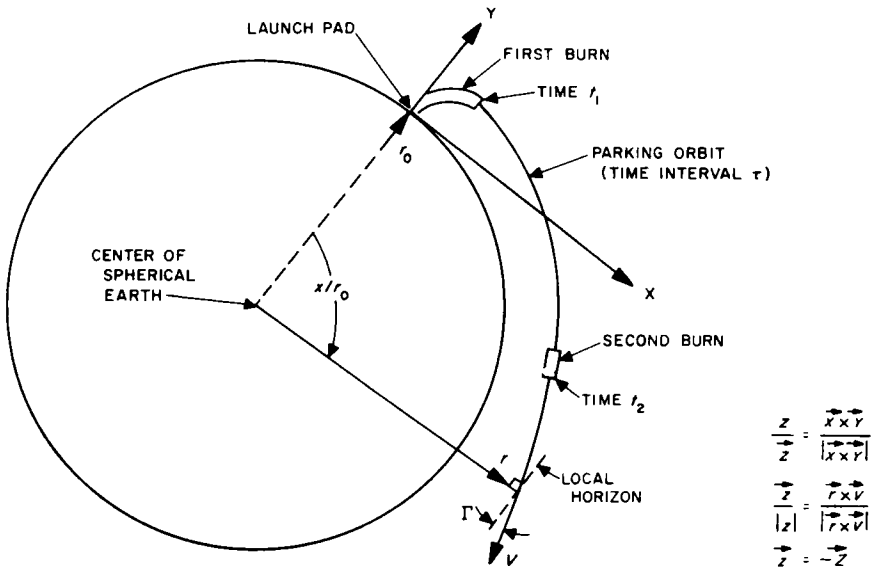


Fig. 4 Coordinate systems

# GUIDANCE AND CONTROL

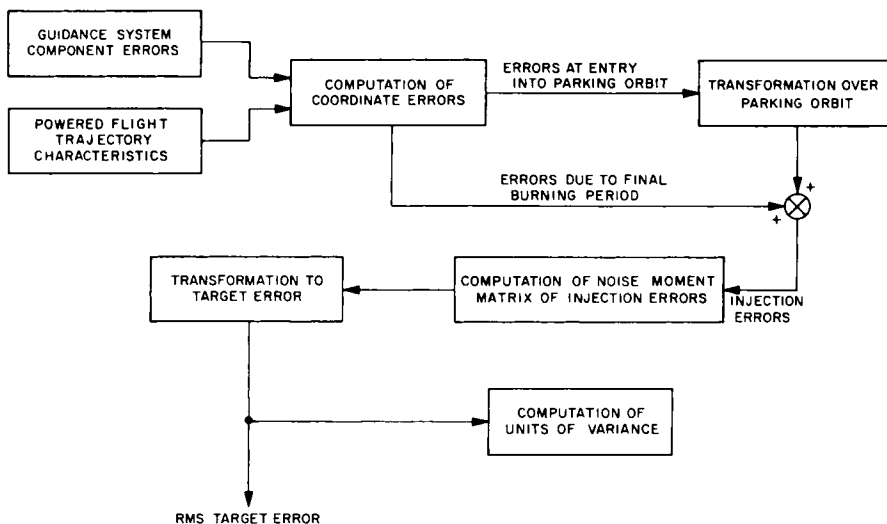


Fig. 5 Flow chart of error study

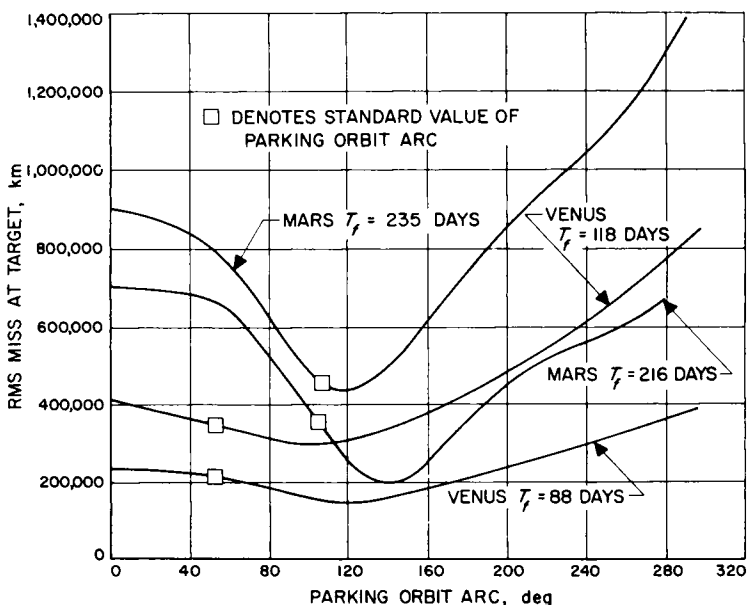


Fig. 6 RMS miss at target vs. parking orbit arc for typical trajectories

# GUIDANCE AND CONTROL

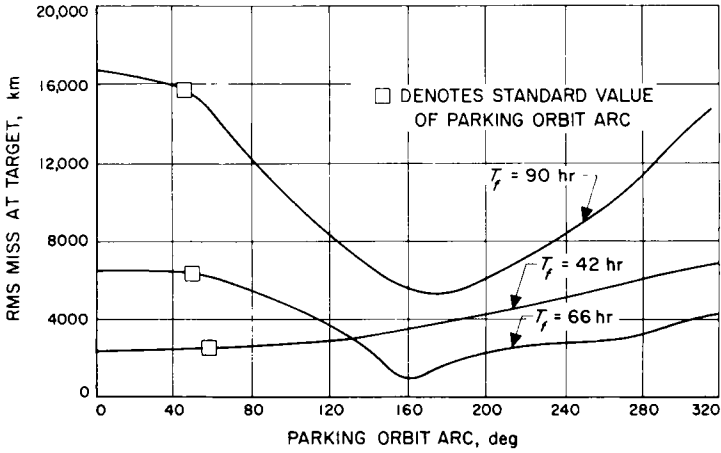


Fig. 7 RMS miss at target parking orbit arc for typical lunar trajectories

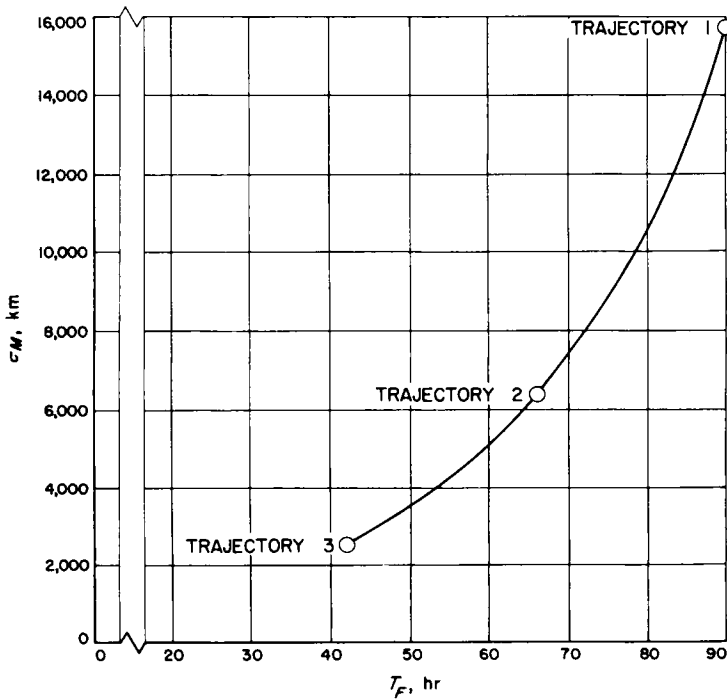


Fig. 8 RMS target miss vs. flight time for typical lunar trajectories

GUIDANCE AND CONTROL

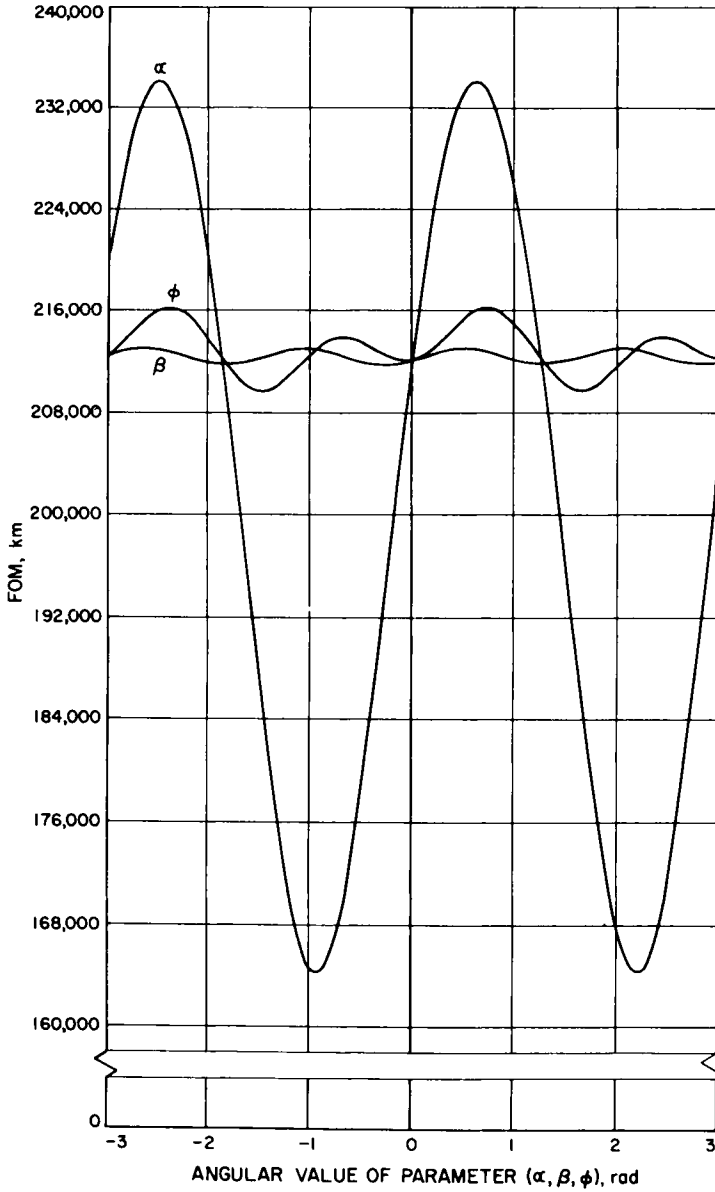


Fig. 9 RMS target error vs. value of guidance parameters for trajectory no. 5