PRECISION TRACKING OF SPACE VEHICLES

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ABSTRACT

Radar techniques are described which are capable of producing measurement accuracy in near real-time of .1 ppm, which is about the limit in the present art. Particular emphasis is placed on long baseline techniques, which are required for the accurate angle measurements. Practical problems encountered in designing long baseline systems are discussed. The timing problem is particularly significant. Methods for producing sufficient timing accuracy across the baseline are considered, and the feasibility and suitability of very long baselines (hundreds of miles) are discussed. By the use of one, or preferably several, of these precision tracking systems, it should be possible to measure satellite positions to a degree of accuracy that will yield information on the fine grained structure of Earth's gravitational field. The possibility opens up of obtaining the longitude-dependent or tesseral harmonics by accurately observing the short period variations in a satellite's ephemeris parameters. Calculations are given relating estimates of the higher harmonics to the radio observations.

INTRODUCTION

Radio tracking systems that are being planned or are in use today have accuracy capabilities that range over several orders of magntiude, yet they all have their place in this space age. One reason for this is that accuracy is not the only criterion for judging the suitability of a

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particular radar system. Another reason is that crude accuracy is sometimes all than can presently be obtained (e.g., at very long ranges). Also, even with only modest measurement capability, under certain conditions data smoothing can produce considerable improvement in the overall accuracy.

Since different space missions pose different requirements for position and velocity accuracy, there will be a need for many kinds of radars with a wide range of accuracy capabilities. This paper deals with the radars on the far upper end of the spectrum, i.e., the precision radars. First the overall system accuracy and methods of obtaining it will be discussed, and the role for the precision radar will be clarified. Next, some techniques required for implementing such precision radars are described, particularly long baselines. This is followed by two examples of precision radar applications.

SYSTEM ACCURACY

The term precision radar deserves further clarification. However, before doing this, some of the applications of radar to space problems and the accuracy that may be required will be examined.

The problems may be separated into three broad categories: 1) orbit determination; 2) model studies; and 3) guidance. It is agreed that these categories overlap (e.g., both guidance and model studies might employ orbit determination as a means to an end). However, there are some fundamental differences, and this classification provides a convenient means of describing them.

In orbit determination the entire result is usually a set of numbers such as satellite orbit elements or initial conditions of position and velocity. These numbers remain relatively constant during the entire measurement period, which may be very long, and all of the data contribute to the final answer.

The second category, model studies, tends toward fine detail. Thus, instead of the numbers describing the entire orbit, for example, a sought after result might be the fine detail perturbation in an orbit caused by a gravitational anomaly. In such a case the result is most sensitive to orbit measurements near the perturbation; measurements made on another part of the orbit have less importance.

The third category, guidance, is like the second in that the fine detail is significant. During powered flight guidance, the perturbations can be severe; long smoothing is usually not feasible. Only the data near the end of powered flight are significant to the final result.

This comparison may be stated in another equivalent way. In the first case a good model is known which mathematically expresses the dynamics of the vehicle that is being tracked. This model is valid during the entire course of the problem and allows widely separated data to be combined effectively. Thus the advantage is gained of smoothing over a long time to reduce the random errors and also of obtaining data from widely different geometries, which allows a reduction in the systematic measurement errors. In the second and third cases the model is poor and contains unaccounted-for anomalies, possibly of short duration. The smoothing must be shorter than the duration of the anomalies if they are to be discovered and measured. Consequently only small changes in geometry can be expected during the observation time.

An attempt has been made to illustrate these facts in Fig. 1, where the overall errors are on the vertical scale and observation time on the horizontal. The upper curve shows the results obtained using a typical, present-day, 100 to 1000 ppm radar of the type used for many space missions. As indicated, when the model is good, long smoothing is effective, and, given sufficient tracking time, the errors can be made quite small. On the other hand, if the model is poor, smoothing is less effective, and the accuracy will reach a limit as indicated. In fact, excessive smoothing can degrade the net result if the data are not properly weighted; this fact is illustrated by the dotted curve.

Results obtained with a precision radar (1 to 10 ppm) are shown by the lower curves. Even with a poor model, when smoothing is not effective, the overall errors can be quite small. Of course, with a good model the errors are even smaller. However, since sufficient accuracy can often be obtained with little smoothing, events that do not last long can be examined accurately and in detail.

Besides the accuracy of a radar, another important consideration is the set of quantities which the radar actually measures. It is well known that, as far as orbit determination is concerned, virtually any set of radar measurements (range, range rate, angles, etc.) is adequate to determine the orbit so long as enough (six or more) independent measurements are made. Roughly speaking this determination amounts to solving a number of simultaneous equations, which state the constraints imposed by the model, for the orbit constants. It is never really necessary to determine directly with the radar the position of the vehicle. Usually best results are obtained with this approach when large changes in geometry occur during the interval between measurements of the same quantity.

In guidance and in model studies it is often necessary to determine uniquely the vehicle position with each radar measurement. One reason for this is to avoid the extensive calculations that would be necessary otherwise. Another reason is that only small changes in the geometry might be expected over regions where the model is good, and under such conditions the results are usually extremely sensitive to measurement errors.

RADAR CONFIGURATION

Precision radars are capable of producing 1 to 10 ppm accuracy. This accuracy is based on three-dimensional determinations, and samples are supplied at high data rates (20 or more per minute). The radars that will produce these results are likely to be long baseline, microwave, continuous wave systems using interferometer measurement techniques. A typical configuration is shown in Fig. 2. Measurement of three ranges completely determines the position of the vehicle. The accuracy of the determination depends on how accurately the ranges are measured and how accurately the three stations are known with respect to some fixed frame of reference.

In space work, where ranges are great, a transponder is invariably used in the vehicle to provide adequate signal strength for tracking. In considering the many ways of implementing the system, it is usually very important to try to keep the transponder equipment small and light. For this reason a logical configuration employs only one transmitter. The transmitter replies to signals received from the ground station, and the replies are received at the central station, where the range is measured. The replies are also received at each of the two outlying stations, where the total time delay in each case is equivalent to the total path (central station vehicle outlying station). The measurement of this time delay will be discussed later.

The quantities measured in this radar configuration are range (from the central station to the vehicle) and some combinations of the three ranges from which the angles may be obtained; these are often referred to loosely as the angle measurements. Rates are not measured directly, but they may be obtained easily by numerical differentiation of the position data. The high data rates will allow this to be done accurately.

Range measurement of 1 ppm is within the state of the art today. Using interferometer techniques, range can be measured to a fraction of a wavelength at microwave frequencies; this amounts to less than a foot. The effects of the troposphere on radio propagation (effects of the ionosphere are rendered practically negligible by the use of microwave frequencies) can cause range errors of hundreds of feet, but these errors can be accounted for in part. Residual errors would be less than a few feet if this compensation is done carefully. The uncertainty in the speed of light in free space causes an additional error in range measurement of about 1 ppm. The foregoing statements apply to total range measurement accuracy; incremental range can be measured to hundredths of a foot or better.

The angle measurements are more important, or at least accuracy of 1 ppm (1 μ radian) is usually more difficult to achieve. It turns out that radars having very long baselines are required to attain this accuracy because of the propagation errors arising in the troposphere. The remainder of this section will deal with these long baselines, how their use surmounts propagation errors, and how they may be implemented.

Need for Long Baselines

One leg of a typical baseline system is shown in Fig. 3. The length of the baseline is b, and the ranges to the vehicle from the central station and from the outlying station are R and R_1 respectively. The elevation angle E is determined by these two ranges and is given approximately by

$$\cos E = \frac{R - R_1}{b} \qquad [1]$$

This approximation is valid when the range is large compared to the length of the baseline. Thus in this typical situation the angle is determined by the range difference and by the length of the baseline.

The error in the elevation angle is

$$\delta E = \frac{\delta R_1 - \delta R}{b \sin E} \qquad [2]$$

The rms elevation angle is

$$\sigma_E = \frac{\sqrt{2}\,\sigma_R}{b \sin E}\left[1 - \rho(b)\right]^{1/2} \qquad [3]$$

where $\rho(b)$ is the correlation between the range errors δR and δR_1, and σ_R is the rms range error on each path.

A little reflection shows that the range errors, which are due mainly to the troposphere, are highly correlated for short baselines and weakly correlated for long baselines. The latter is the case of interest when the angle errors are small, and in this case

$$\sigma_E \approx \frac{\sqrt{2}\,\sigma_R}{b \sin E} \qquad [4]$$

Thus the error is inversely proportional to the length of the equivalent baseline, which is the baseline length as seen from the vehicle.

These errors have been measured (1),[3] and, as should be expected, they are strong functions of the prevailing weather conditions. Typical average results are shown in Fig. 4. The rms angle error is plotted on the vertical scale and the equivalent baseline length (b sinE) on the horizontal. On the left hand portion of the curve, where the baseline is short, the error is independent of the baseline length and is about 100 μ radians. This value is representative of conventional tracking radars. When larger baselines are used the error decreases, and in order to get 1-μ radian accuracy the equivalent baseline must be about 100,000 ft or more.

Required Measurement Accuracy

According to the previous section, very long baselines

[3]Numbers in parantheses indicate References at end of paper.

are required to get precision angle measurements, and the fundamental source of error giving rise to this requirement is the propagation path itself, which causes an error in the measured range difference. In designing the system other contributions to the range difference error must be considered, particularly the errors arising in the equipment. Good guides to the other allowable errors are the propagation errors, since these are more or less irreducible. The measurements errors, since they may be controlled, should not be allowed to contribute significantly to the overall errors. Thus they should be less than the propagation errors, but not so much less as to result in an uneconomical overdesign.

The range difference errors caused by propagation is taken as an upper limit. To determine the value of propagation errors, Fig. 4 may be plotted in a different way. First, the measurement errors are related to the angle errors according to

$$\sigma_p = b \sin E \; \sigma_E \qquad [5]$$

where σ_p is the rms equipment error in the measurement of the range difference. According to Fig. 4, for the small baseline lengths the error caused by propagation is nearly constant. Thus, in this region, the allowable measurement error increases linearly with the baseline length. This is shown on the lower left of Fig. 5, which shows the allowable measurement error for each baseline length.

For large baselines the error in the angle measurement approaches the value given by Eq. 4, which corresponds to the situation when the path errors are uncorrelated. In this case the angle error is inversely proportional to the baseline length, and the allowable measurement error approaches a constant value. This is shown by the dotted horizontal line at the right which, if maintained for very long baselines, would result in angular errors approaching zero. Actually, since the required accuracy is only 1 μ radian, the measurement errors need not be as good as indicated by the dotted curve. The line representing 1 μ radian is shown, and the solid line follows it for baselines longer than 100,000 ft and represents the minimum required measurement accuracy to produce 1-μ radian error.

Implementing the Baseline Measurement

There are a number of ways for making long baseline measurements, some employing multiple ground transmitters and multiple transponders. Only one approach will be considered here, however; this one is most suitable for space programs because a minimum amount of vehicle-borne equipment is required. This system uses a single central station transmitter and a single transponder in the vehicle. The signal transmitter from the vehicle is received at the central station as well as at the outlying stations. As far as the baseline is concerned, the problem may now be reduced to one of measuring the difference between the times of arrival of the signals received at the central station and at the outlying stations.

There appear to be two general approaches to the solution of this problem. In one approach the received signals at one station (or coherent signals derived from them) are brought to another station (usually the outlying station signals are brought to the central station). Here they are compared with signals received at that station from the vehicle, and phase differences are interpreted as time delays or range differences. This is the basis of the baseline systems usually referred to as interferometers.

In the other approach the central station timing is established at the remote station (e.g., by delivering the transmitter signals there). This allows the total path range (central station vehicle outlying station) to be measured at the remote station. This total range (now a number) is then either stored or transmitted by data link to the central station where it is used to compute the angles or other appropriate quantities.

Reflection leads to the conclusion that either of these methods depends on establishing the same time at two remote points separated by a distance equal to the baseline length. The accuracy required is easily obtained by relating the allowable range difference error σ_p with the timing error σ_T through the relation

$$\sigma_T = \sigma_P/C \qquad [6]$$

where c is the speed of light. Using allowable range difference errors as given in Fig. 5, the allowable timing errors are obtained and plotted in Fig. 6.

Timing may be accomplished by transmitting timing signals from one point to another, and the error arising with such a scheme would be equal to the time required for the signals to travel between the points. This error can be calculated approximately or calibrated by some means or other to compensate for the error. However, when the required accuracy is equal to fractions of nanoseconds, the stability of the transmission paths cannot be depended on for this purpose. Clearly a more accurate technique is needed.

One approach that works is to provide a transponder at the receiving end, which, on receiving the timing signal, transmits it back to the sending point. In this way, provided the transmission is reciprocal, twice the propagation time may be accurately measured. This information may be stored and used for later corrections, or it may be used to control the electrical path length and to keep it constant. In any case the transmission time is accurately determined, which, along with the arrival time of the timing signal, allows accurate timing to be maintained at separate points.

The method by which the timing signals are transmitted is not important except from the standpoint of practical considerations. Transmission lines (waveguide or radio frequency coaxial cable) are feasible for short distances, and line of sight radio is useful over somewhat longer distances. Beyond-horizon radio may be used at any distance, but it is probably most suitable for long baselines where the timing errors may be relatively large. A special kind of beyond-horizon link isone using the vehicle as a relay. This could provide excellent timing, but the need for additional vehicle-borne equipment would be a serious disadvantage. An estimated range for baselines for which these approaches are most suitable is indicated in Fig. 6.

Another interesting approach uses atomic clocks. The clocks cannot be used alone, for they must be set initially and must be reset when the accumulated error becomes excessive. Once a clock is set, the period during which its time is sufficiently accurate depends on the stability of the clock and on the required timing accuracy. As an example, for a baseline length of 10^6 ft, the required timing accuracy is one nanosec (Fig. 6). If the stability of the clock is one part in 10^{11} (reasonable for ground-based clocks), the error will reach 1 nanosec within 100 seconds. Thus, in this example, after 100 sec the clock would have to be reset.

Evidently the atomic clocks today are not quite good enough and are only marginally useful, and then only for very long baselines. However, they should receive serious consideration when they are one or two orders of magnitude better or when baselines longer than 10^6 ft are considered. For very long baselines (say 100 miles or more) it may prove difficult to maintain continuous communications for the time synchronization signals. In this case the accurate clocks would be valuable to maintain the timing during the intervals when the communications are out.

GEODETIC SATELLITE TRACKING

One of the more interesting scientific uses to which a precision tracking system may be put is in the determination of Earth's gravitational field. Tracking systems have been proposed for observing geodetic satellites, but the main purpose of such missions is to achieve higher order survey accuracies rather than the probing of Earth's gravitational field. It is true, of course, that whereas the tieing together of continents or the locating of distant points to within a few meters might require a sophisticated gravity model (if the satellite is not simultaneously in view of the cooperating tracking stations), the main emphasis of these proposals has not been on what might be termed the more purely scientific aspects. The variations of the ephemeris parameters, when carefully analyzed, can lead not only to improved values of the coefficients of the zonal harmonics in the expansion of Earth's gravitational field, but also to estimates of the tesseral and sectorial harmonics that would manifest any longitude dependence (i.e., non-rotational symmetry) of Earth's field.

In addition to the geodetic use just outlined, there are a number of other applications for the radio systems described earlier. However, there is not enough space to elaborate on how they are used, what kind of data are obtained how they are interpreted, etc. Some of these applications are:

1) Tracking of rocket vehicles in powered flight to evaluate guidance and other performance; 2) tracking and guidance during the last vernier phase of injection for various space missions; 3) tracking and guidance for satellite rendezvous maneuvers; 4) midcourse tracking for evaluation of corrective thrusts; 5) tracking for special lunar missions (e.g., establishment of a lunar satellite); 6) re-entry tracking and control.

The first example presented will be a discussion of the capability of a precision tracking system to detect the influence on a satellite's orbit of the first longitude dependence term in Earth's gravitational potential. Izsak(2) has recently shown from an analysis of many optical observations of 1959 α 1 (Vanguard 2) and 1959 η (Vanguard 3) that a strong case can be made for the existence of an elliptical shape for Earth's equatorial cross section, with one end of the major axis appearing at $33^\circ.15 \pm 0^\circ.53$ west longitude. He deduces the ellipticity to be $(3.21 \pm 0.29) \times 10^{-5}$, which is equivalent to a 205 m difference between the semimajor and semiminor axis of the ellipse.

From conventional astronomic perturbation theory(3), the rate of change of the six position parameters that locate the satellite with respect to an inertial coordinate system can be expressed by six equations in terms of the disturbing function. Here, the disturbing function is the extra force exerted on the satellite arising from the presence of the $J_{2,2}$ term in the expansion of Earth's gravitational field:

$$U = \frac{\mu}{r}\left[1 - \left(\frac{a_e}{r}\right)^2 \left(\frac{3\sin^2\phi - 1}{2}\right)(J_{2,0} - J_{2,2}\cos 2\lambda)\right] \quad [7]$$

where r = radial distance
a_e = equatorial semimajor axis
ϕ = geocentric latitude
$J_{2,0}$ = first oblateness coefficient
$J_{2,2}$ = coefficient of longitude dependent term
λ = geographic longitude of the major axis of Earth's equatorial cross section
μ = (gravitational constant) times (Earth's mass)

The six equations can be integrated to give expressions for the magnitudes of the ephemeris parameters resulting from the perturbing force. In general, there will be secular, long period and short period contributions to the various terms. The dominant one, due to the equatorial ellipticity, is long period and has a nominal 12-hr period.

Following Izsak, the semidiurnal oscillations in the satellite ephemeris elements are given by terms of the form

$$\delta\omega = -6J_{2,2}\, n a_e^2\ (3-5\cos^2 I)\ \sin 2(\theta_x - \Omega_1)/p^2 8(\theta_1 - \Omega_1) \quad [8]$$

with similar expressions holding for $\delta\Omega$, δI and δM, where (see Fig. 7)

n = mean motion (-2π/period)
ω = argument of perigee
Ω = longitude of the ascending node and its precession rate Ω_1
I = inclination angle
M = mean anomaly
θ_x = mean sidereal time at the intersection of major axis with the equator
θ_1 = Earth's rotation rate
$p = a(1-e^2)$
a = semimajor axis

The effect on the other two independent parameters, the semimajor axis and the eccentricity, is negligible.

The Cartesian inertial coordinates may be expressed in terms of the ephemeris parameters (see Fig. 7) as follows:

$$x = r\left[\cos u \cos\Omega - \sin u \sin\Omega \cos I\right] \quad [9a]$$

$$y = r\left[\cos u \sin\Omega + \sin u \cos\Omega \cos I\right] \quad [9b]$$

$$z = r \quad \sin u \sin I \quad [9c]$$

where f = true anomaly
u = argument of latitude = $\omega + f$

By taking differentials of these expressions and making use of the relations that hold for unperturbed elliptic orbits(4)

$$\delta r = ae(1-e^2)^{-1/2} \sin f\, \delta M + (r/a)\delta a - a \cos f\, \delta e$$

$$\delta f = (a/r)^2 (1-e^2)^{1/2} \delta M + (a/r) \sin f\, (1+r/p)\, \delta e \quad [10]$$

δx, δy, δz can be expressed as functions of the differentials of the six independent orbital parameters.

For satellite 1959 α1 these perturbations were calculated for a time interval on April 17, 1960, when it passed fairly close to Cape Canaveral. A coordinate transformation was applied which expresses the perturbations in the inertial coordinates in terms of local cartesian coordinates (x,y,z), directed east, north, and towards the zenith respectively. Fig. 8 shows the behavior of these three coordinates as functions of the true anaomaly. Since the apogee height of the satellite was about 10^7 ft, it can readily be seen that the detection of a position perturbation amounting to about 2500 ft presents no problem to a tracking system with a 1 ppm accuracy capability. The high accuracy will be advantageous in measuring these perturbations for less favorable geometries and lower elevation angles. Furthermore, the higher the tracking accuracy, the better is the possibility of measuring not only the equatorial ellipticity, but also the effects of higher order longitudinal asymmetries in Earth's gravitational field. For all of these measurements, of course, the very accurate velocity measurements that can also be made would be valuable in delineating all of these perturbations of a satellite's motion.

SYNCHRONOUS SATELLITE TRACKING

The second example deals with the tracking of a synchronous satellite. Whereas in practice long observation times could (and would) be used, the achievement of a precision system in a very short time will be shown here. The accuracy of position measurements (including propagation and survey errors), which can closely approach 1 ppm for a steep elevation angle, will be lessened at smaller elevation angles. Range rate measurements can easily be made to about 0.1 fps. Angular rate capability is a function of data smoothing time, baseline length, and propagation anomalies, as well as the accuracy with which range rate difference can be measured. For the type of system described, an angular rate accuracy of about 2×10^{-8} rad/sec could be attained with only a few seconds of data.

With these sensory capabilities, the errors in measuring the position and velocity components for a synchronous satellite were calculated in terms of coordinates centered at the tracking system (at Cape Canaveral) and pointing north, east, and towards the zenith respectively. To show the effect of different elevation angles from the tracking system to the satellite, the errors were calculated as a function of longitudinal position of the satellite. The resulting position and velocity component errors are shown in Fig 5, 9 and 10.

A simple but tedious transformation permits these errors to be expressed in terms of a local coordinate set at the satellite. However, if this set is defined to be similarly oriented to the first set (viz., north, east, and toward the zenith), the geometry is such that the numerical values are not significantly altered. Consequently this set of values is used in the ensuing calculations.

When the errors are expressed in terms of the local coordinates, it is easy to obtain the most significant quantity for the description of synchronous satellite orbits, namely, the drift rate. This drift results from the satellite's period differing in length from the sidereal day. Ignoring secondary effects, the drift rate is given by

$$\frac{d\lambda}{dt}\ (\text{deg/day}) = 1.08 \times 10^{3}\ (\delta V/V + \delta Z/Z)$$

where V is the speed with respect to the center of Earth, and Z is the height of the satellite.

This drift rate is plotted in Fig. 11 for a satellite situated at 45° W long. as a function of data smoothing time. The exact manner in which the various system errors (including propagation) vary with time for long smoothing times is very difficult to estimate. To simplify matters, an improvement factor varying inversely as the square root of the observation period has been used.

SUMMARY

This paper has presented an outline of the theory and operation, and an application of precision radio tracking systems. It has been shown that by the use of long baseline techniques, accuracies in position and velocity measurements rivaling good optical systems can be obtained.. There are a number of possible applications for such a system, but limitations of space allowed a treatment of only two. The first was an analysis of the ability of such systems to measure the effects in the orbit of a satellite of the fine-grained structure of Earth's gravitational field. It was shown that it is quite within the capability of the type of system discussed to measure the perturbations of the satellite orbit arising from the recently discovered ellipticity of Earth's equatorial cross section. The second example treated the tracking of a synchronous satellite and derived the errors in measured drift rate as a function of observation time.

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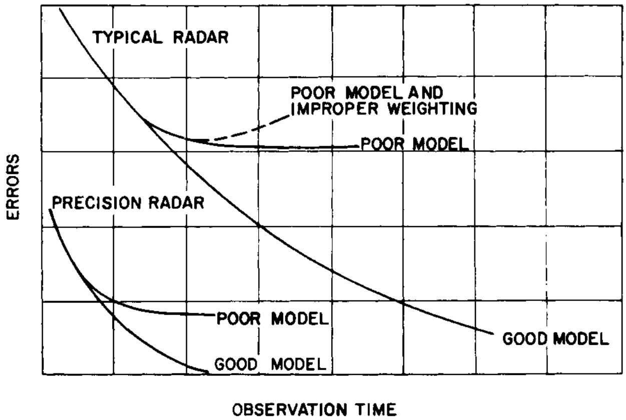

Fig. 1 Effects of smoothing

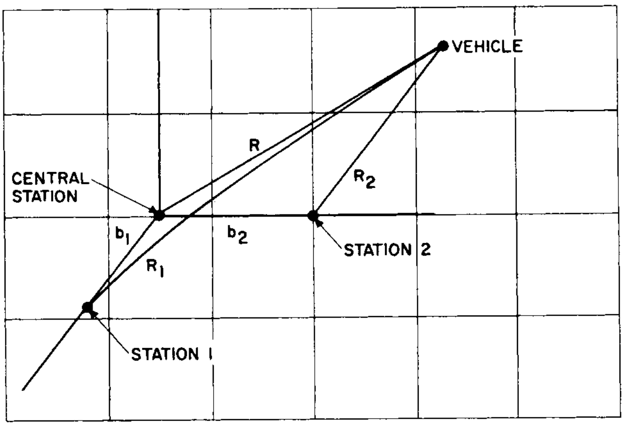

Fig. 2 Typical configuration

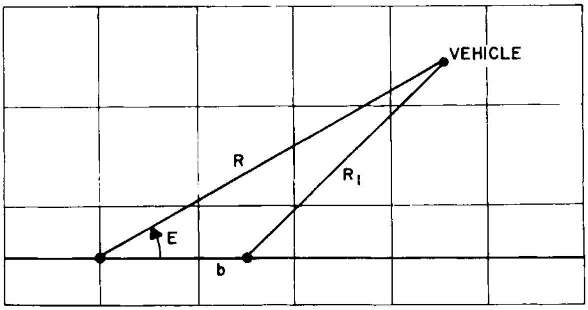

Fig. 3 Simple baseline

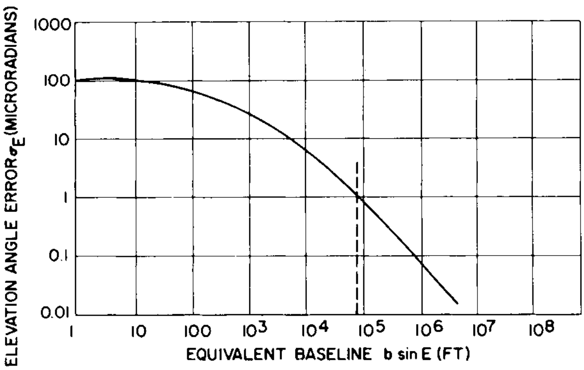

Fig. 4 Angle measurement errors caused by propagation

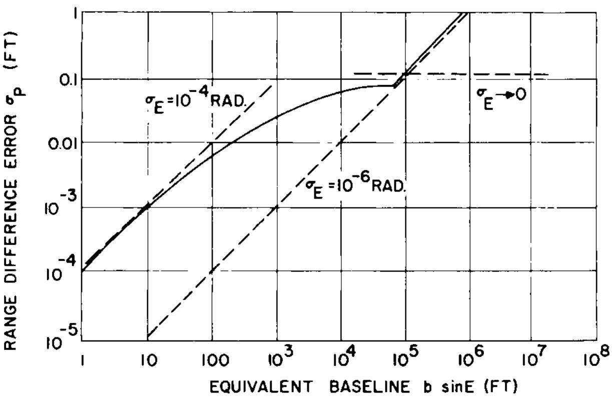

Fig. 5 Allowable range difference errors

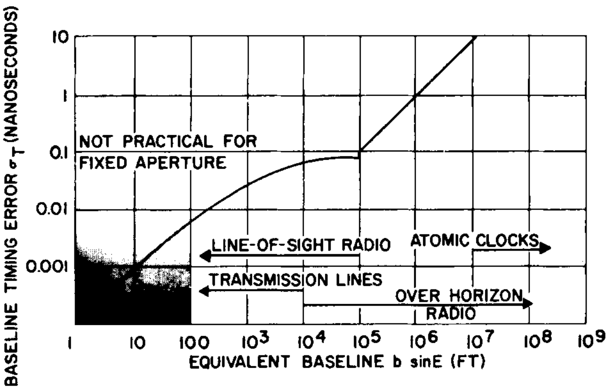

Fig. 6 Allowable timing errors

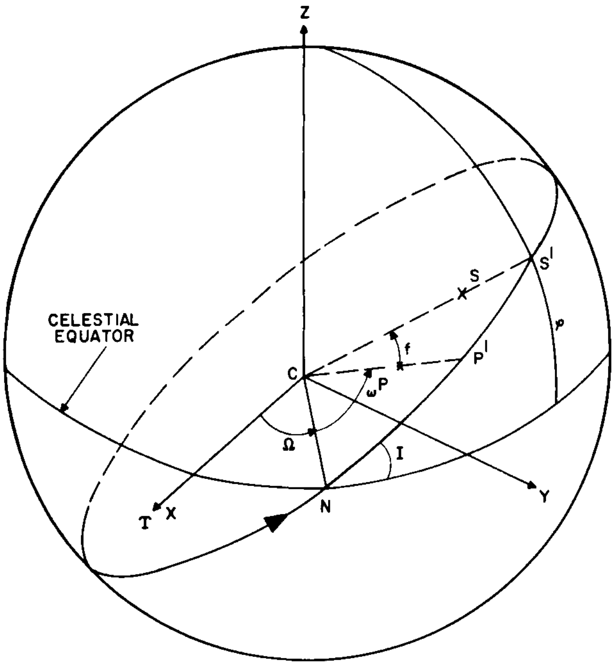

Fig. 7 Satellite motion on the celestial sphere

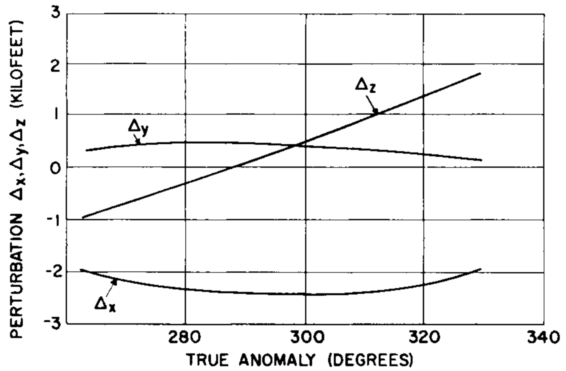

Fig. 8 Position perturbation on satellite 1959 α1

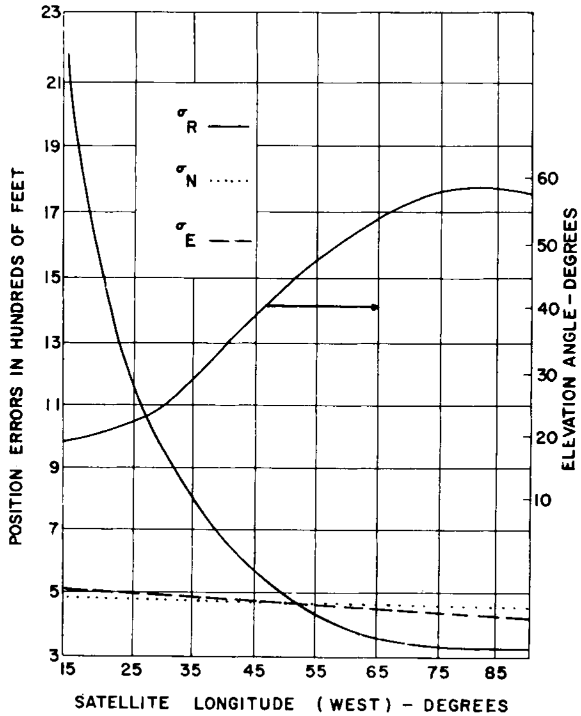

Fig. 9 Position errors - synchronous satellite

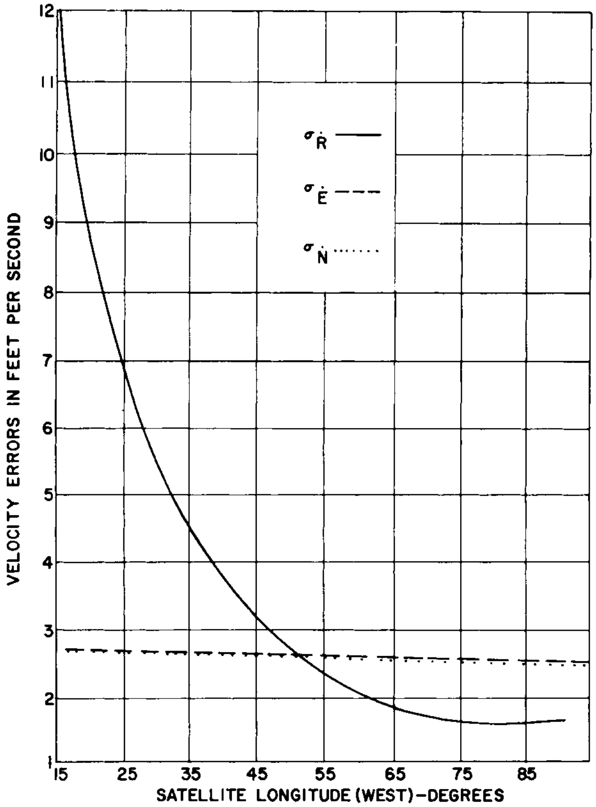

Fig. 10 Velocity errors - synchronous satellite

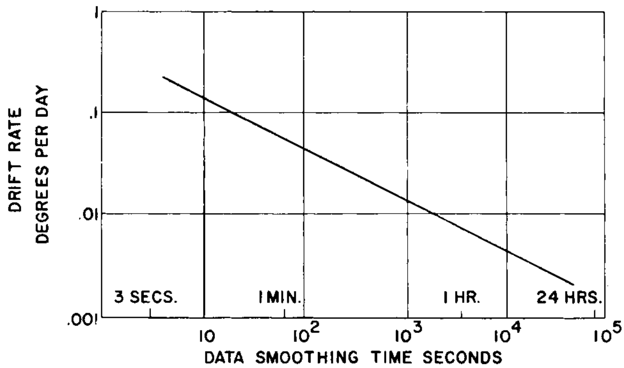

Fig. 11 Apparent drift rate of synchronous satellite as function of data smoothing time