TERMINAL GUIDANCE SYSTEM FOR SOFT LUNAR LANDING

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ABSTRACT

A terminal guidance system for landing on the moon with a velocity of about 20 ft/sec is described and its operation analyzed. The system design employs state of the art hardware and emphasizes realistic performance and accuracy requirements upon sensing, computation, propulsion and attitude control sub-systems. The guidance rules and mechanization block diagrams are presented in the paper. The dynamic response of both the longitudinal and lateral guidance channels to initial errors is derived and the implications to the propulsion, attitude control, and sensor subsystems discussed. An error analysis of the system shows that bias errors of the sensors are the most important contributors to the landing velocity error.

INTRODUCTION

A primary objective of early lunar landing missions is to gather scientific data about the lunar surface and environment by soft landing an unmanned, instrumented spacecraft which can telemeter information back to Earth. A landing velocity of about 20 fps permits a payload of sensitive scientific instruments to be landed. Relatively simple calculations suffice to show that a landing velocity of this small a magnitude cannot practically be achieved without a terminal guidance system, utilizing on-board measurements of velocity and altitude with respect to the lunar surface. Hord (1)[1] has considered relative motion during the terminal phase of an intercept mission, with emphasis on proportional navigation systems. Proportional navigation techniques, though useful

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[1]Numbers in parentheses indicate References at end of paper.

for advanced lunar landing missions requiring landing site selection, are probably not required for early missions. Brady and Green (2) and Green (3) have considered the application of logarithmic guidance to lunar landing missions. The purpose of the present paper is to describe and analyze a simplified terminal guidance system which is capable of achieving soft landing without site selection and which may be used with a landing vehicle having body-mounted sensors and propulsion units.

MISSION AND SPACECRAFT DESCRIPTION

A launch vehicle boosts the spacecraft into a transit trajectory to the moon. A midcourse correction, using radio tracking data from Earth-based facilities, will control the landing to a predesignated area within reasonable tolerances. A cold gas attitude control system maintains attitude during coast and orients the spacecraft in the proper thrust direction for a midcourse correction, which is controlled both in thrust and attitude by three liquid propellant vernier engines.

Approaching the moon with a relative velocity in excess of 8000 fps, the spacecraft is oriented by ground command to the proper direction for application of retrothrust. Basic attitude reference is provided by sensors which track the sun and a star, or the sun and Earth. At some altitude, such as 40 nautical miles above the lunar surface as determined by a radar altimeter, the main solid propellant engine is ignited providing retrothrust to slow the vehicle to a relative velocity of about 660 fps. Burnout of the solid fuel engine occurs at a 6 nautical miles altitude, shortly after which the engine is jettisoned and the three vernier engines provide the remainder of the necessary retrothrust velocity increment in order to achieve a soft landing.

During the vernier engine phase, body-mounted radar sensors furnish the necessary altitude and velocity information for continuous control of body maneuvering and thrust. The vernier engines are shut off at an altitude of about 30 ft and the spacecraft falls to the surface, landing at a nominal speed of about 20 fps.

Fig. 1 shows a schematic representation of the spacecraft. A three-legged landing gear absorbs the residual energy at landing to limit the shock experienced by the spacecraft body and the scientific instruments. Stability during landing, for the expected tolerances in both the vertical and horizontal

components of the landing velocity, as well as the expected
variations in the surface slope, is provided by the wide
spread of the landing legs.

TERMINAL DECELERATION PHASE

The terminal deceleration phase of the landing operation be-
gins at an altitude of about 50 nautical miles above the lunar
surface. The vehicle will have been oriented through ground
command such that its thrust axis is very nearly parallel to
the approach velocity vector. First, the vernier engines are
reignited to achieve attitude control by means of a differen-
tially throttling control system. Shortly thereafter, at a
nominal altitude of about 40 nautical miles, the main engine
is fired.

During the main engine phase, no guidance with respect to
altitude-velocity profile is incorporated. This results from
the consideration that accurate measurements by terminal sen-
sors might be difficult during this phase because of the high
altitude, the effects of the large flame, and the vibration
environment. Instead, the system is designed to be able to
accommodate the probable tolerances in main engine burnout
altitude and velocity due to the aggregate of all error con-
tributing sources, such as total main engine impulse, thrust
level, uncertainties in altitude and velocity at initiation
of burning etc. The dispersions in staging altitude and
velocity are represented by a 99% probability ellipse shown in
Fig. 2.

After staging, the vernier engine thrust level is reduced to
that which gives a net thrust acceleration of one lunar-g.
The spacecraft, whose thrust direction has been aligned to the
vertical during the short period between main engine burnout
and staging, descends at constant velocity. The terminal
sensors, namely an altimeter and three Doppler radars (located
symmetrically about the vehicle thrust axis at a certain
squint angle), begin to obtain altitude and velocity informa-
tion in the body coordinate reference system.

For vertical descent, a constant deceleration trajectory is
chosen in the h (altitude) - $\dot{z}$ (downward velocity) phase
plane as the nominal. The end point (h_f, $\dot{z}_f$) on the tra-
jectory is chosen to be at 100 ft and 10 fps while the nomi-
nal thrust acceleration for the trajectory is 2.7 lunar-g's.
When the sensor data indicate that the spacecraft has de-
scended from the actual staging point A' to a point B' on
the nominal trajectory (as shown in Fig. 2), longitudinal or

vertical guidance begins. This is done by throttling the
total vernier thrust in such a way as to minimize the devia-
tion between the actual and nominal trajectories. A short
time later, lateral or horizontal guidance is initiated.
Lateral guidance takes the form of body maneuvering (i.e.,
thrust axis steering) in response to lateral velocity error in
the body coordinate system. A time-varying gain is incorpo-
rated to optimize the system for maximum effectiveness in
reducing lateral velocity error while minimizing the maximum
angle that the body axis attains from the local vertical.
The latter constraint arises from the consideration that
proper radar sensor operation can only be achieved with rea-
sonably small body angles with respect to the local vertical.

When the spacecraft velocity is reduced to 10 fps (nominally
at a 100-ft altitude) as measured by the Doppler sensors,
lateral guidance terminates and the thrust is reduced to
1 lunar-g level for a constant velocity descent. This point
is hereafter referred to as "guidance termination," since
closed loop commands of body steering and thrust level end at
this time and the guidance system has only an engine shutoff
calculation still to perform. The final 1-g descent phase
absorbs any altitude dispersions existing at the 100-ft level
so as not to contribute directly to landing velocity error.
It furthermore allows smoothing of approximately 7 sec of data
at low altitude and low velocity in order to determine the
engine shutoff with precision. When the calculated landing
velocity is 20 fps, shutoff is effected. Nominally, this
occurs at an altitude of 30 ft and a vertical velocity of
10 fps.

GUIDANCE RULES

Longitudinal (or Vertical) Channel

For guidance in the longitudinal channel, two basic rules of
thrust acceleration control were considered. One rule in-
volves a continuously adjusted acceleration trajectory obeying
the relation

$$a_c = \frac{1}{2} \frac{\dot{z}^2 - \dot{z}_f^2}{h - h_f} + g \qquad [1]$$

where a_c = command acceleration
$\quad g$ = lunar gravity
$\quad \dot{z}$ = instantaneous downward velocity
$\quad \dot{z}_f$ = "target" or offset velocity (= 10 fps)

h = instantaneous altitude

h_f = "target" or offset altitude (= 100 ft)

In other words, when a deviation from the nominal phase plane trajectory is detected, the thrust is controlled to a value which will produce a new constant-acceleration trajectory through the target velocity and altitude.[2]

Another possible rule is to force the spacecraft to cancel any measured deviation by returning to the nominal phase-plane trajectory. The guidance rule in this case is

$$a_c = a_N + K_a (\dot{z} - \dot{z}_d) \qquad [2]$$

where a_N is the nominal acceleration, K_a is a gain factor which may be programmed as a function of time or altitude, and $\dot{z}_d$ is the desired velocity corresponding to the particular altitude h on the nominal trajectory. Thus

$$\dot{z}_d = \sqrt{2(a_N - g)(h - h_f) + \dot{z}_f^2} \qquad [3]$$

If there were no errors, the methods would operate identically. However, in the presence of errors, the second method is preferable. A closer correspondence between the actual and nominal trajectories is obtained and a smaller variation in acceleration results, especially towards the end of the guidance phase.

Due to the fact that the spacecraft attitude is nearly vertical during the greater portion of the solo vernier phase (particularly so near the end of guidance), Eqs. 2 and 3 may be mechanized using slant range R in place of h and velocity along the instantaneous thrust axis v_i in place of $\dot{z}$. The guidance sensors, being mounted fixed in the body frame, yield the required R and v_i data without the need of complex, non-constant matrix transformation.

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[2] In Eq. 1, if the factor 1/2 is replaced by a slightly larger number, the command acceleration is well behaved, in the presence of system errors, decreasing to 1 lunar-g at the end. However, the time required for the maneuver is greater than for the nominally constant acceleration trajectory (with the factor 1/2), thus increasing the gravity loss. Also the large variation in acceleration level can affect the dynamic operation of the lateral channel.

Lateral (or Horizontal) Channel

At the start of the solo vernier phase, a sizable horizontal velocity may be present. Table 1 lists the sources of error and the resulting horizontal velocity at staging. Landing stability considerations dictate that the 3-sigma residual horizontal velocity at impact be no more than 10 fps. Hence, there is a need for lateral steering during the solo vernier phase to remove most of the initial horizontal velocity.

The lateral guidance channel must therefore satisfy the following system requirements:

1) The system must be able to remove at least 100 fps of initial lateral velocity error.

2) The maximum angle θ_{max} of the thrust axis from the vertical during the corrective process must be small enough to allow proper operation of the body-mounted velocity and altitude sensors.

3) The angular acceleration and angular rate must be within the capability of the control system.

4) The attitude of the vehicle at the end of the guidance phase must be very nearly vertical.

5) The landing velocity must be less than 10 fps (3σ) in the horizontal direction.

Two rules were seriously considered for lateral guidance:

Rule A: A linear steering system in which the vehicle thrust axis is directed at a computed angle from the local vertical in the trajectory plane. At the proper time, the angle is decreased at a constant rate to zero such that when the angle goes to zero, the lateral velocity goes to zero also.

Rule B: A proportional rate steering system in which the angle between the velocity vector and the thrust axis is used to generate angular rate commands in pitch and yaw in a manner to gradually reduce the angle to zero (but not to force this angle to be always zero as in an ideal gravity turn system).

The major advantage of Rule A is that it best fulfills the second requirement previously given. However, control of attitude in accordance with this rule requires dead-reckoning attitude using body angular rates. To accomplish this, the required instrumentation and computation are

somewhat complex.

Rule B requires no knowledge of either the lunar vertical or
the vehicle attitude. The combined action of the longitudinal
and lateral channels automatically forces the thrust axis to
approach the lunar gravity vector at the end of guidance.
Because this rule results in a null-seeking system, errors in
initial conditions or those due to transient disturbances,
which might in a dead-reckoning system result in standoff
errors, are largely cancelled out by the time guidance is
completed. The only respect in which Rule B appears inferior
to Rule A is that it results in a slightly larger value of
θ_{max}. However, by proper choice of the gain function relating
angular rate and error angle, a system based on Rule B can
meet all the stated requirements comfortably.

INSTRUMENTATION AND EQUATIONS

Terminal Guidance Instrumentation

The block diagram for the overall terminal guidance instru-
mentation is shown in Fig. 3. The computer accepts slant
range from the altimeter, range rates from the three Doppler
radars and thrust acceleration information from a longitudinal
accelerometer. After proper data processing, the following
fundamental quantities are obtained:

R = slant range along thrust axis to lunar surface
v_i = velocity of spacecraft projected along roll (thrust)
axis
v_j = velocity of spacecraft projected along pitch axis
v_k = velocity of spacecraft projected along yaw axis
a = thrust acceleration

The computer generates the following guidance commands:

a_c = command thrust acceleration
ω_{jc} = command pitch rate
ω_{kc} = command yaw rate

The command acceleration is compared with the measured accel-
eration and the difference is amplified and sent to the
vernier engine valve controls for adjusting the total thrust
level. The angular rate commands are sent directly to the
attitude gyros.

In addition to the guidance commands, the computer also sends
gimbal angle commands for steering a high gain communications
antenna to maintain an Earth pointing despite body maneuvering

during the vernier solo phase.

Guidance Equations

The guidance equations for the various important functions are given in the following subsections.

1 Longitudinal guidance

$$a_c = a_N + K_a(t) \left[v_i - \sqrt{2(a_N - g)(R - h_f) + \dot{z}_f^2} \right] \quad [4]$$

This is in the same form as Eqs. 2 and 3 combined, except with $\dot{z}$ (the vertical velocity) replaced by v_i (the component of velocity along the thrust axis) and h (the altitude) replaced by R (the slant range).

2 Lateral guidance

$$\omega_{jc} = - f_1(t) \frac{v_k}{v_i} = - K' \frac{v_i}{R} \frac{v_k}{v_i} = - K' \frac{v_k}{R} \quad [5]$$

$$\omega_{kc} = f_1(t) \frac{v_j}{v_i} = K' \frac{v_i}{R} \frac{v_j}{v_i} = K' \frac{v_j}{R} \quad [6]$$

The gain function $f_1(t)$ (chosen to be a constant K' times the ratio v_i/R) is nearly an invariant function of altitude due to the fact that the longitudinal guidance channel always forces the spacecraft towards the nominal vertical descent phase trajectory. Roughly, the ratio v_i/R increases as the reciprocal of T, the time-to-go before guidance termination. An upper limit is actually imposed on $f_1(t)$ during the last few seconds of guidance in order to avoid guidance loop instability. Also, the constant K' is made large during the first few seconds of guidance to achieve a rapid swing out of the thrust axis in the proper direction. For the major portion of the lateral guidance interval, however, K' is maintained at a modest value optimized for efficient steering out of lateral velocity combined with minimum body tilt angles.

3 Guidance termination

Guidance termination is defined as the instant when the thrust acceleration is returned to the 1-g level and lateral guidance is terminated by setting $\omega_{jc} = \omega_{kc} = 0$. This occurs when the measured v_i is 10 fps at a nominal altitude of 100 ft.

To minimize the velocity error, guidance termination requires
a timing more accurate than the basic 1-sec computation cycle,
and a simple linear prediction is used to send this signal to
the engines at the proper instant. The time to throttle down
$\Delta\tau_i$ is computed as

$$\Delta\tau_i = \frac{v_i - \dot{z}_f}{a - g} \qquad [7]$$

As soon as $\Delta\tau_i$ becomes less than the computer repetition
period plus the expected engine delay $\Delta\tau_d$, the computer
waits for a time equal to $\Delta\tau_i - \Delta\tau_d$ and then sends the
throttling signal.

4 Shutoff

During the final 1-g descent, an accurate measure of the
vertical velocity is obtained by simple averaging of all
samples of v_i. The desired shutoff range or altitude is then
computed to provide a nominal landing velocity of 20 fps.
Next, the time before cutoff T_{co} is computed and compared
with the sum of the computer repetition period and the
expected engine shutdown delay $\Delta\tau_v$. When T_{co} becomes less
than this sum the computer waits for a time equal to
$T_{co} - \Delta\tau_v$ and sends the shutoff signal. The equations are
as follow

$$\overline{v_i} = \sum_{r=1}^{n} \frac{v_{i_r}}{n} \quad \text{(average velocity)} \qquad [8]$$

$$R_{co} = \frac{20^2 - \overline{v_i}^2}{2g} \quad \text{(shutoff range)} \qquad [9]$$

$$T_{co} = \frac{R - R_{co}}{\overline{v_i}} \quad \text{(time to go before shutoff)} \qquad [10]$$

GUIDANCE LOOP DYNAMICS

The two guidance channels, longitudinal and lateral, are
essentially closed-loop systems whose cross-coupling effect
onto each other is negligible so long as the following two
conditions are satisfied: The vertical descent adheres
closely to the nominal trajectory; and the body tilt angle at
any time during the guidance phase is small so that the

cosine of this angle is nearly unity.

For the spacecraft under consideration, both conditions are in fact fulfilled.

Longitudinal Channel Dynamics

For the case of zero lateral velocity, the relations between acceleration, velocity, and altitude may be obtained from inspection of Eqs. 2 and 3. The block diagram of Fig. 4 shows the closed loop nature of the system. It should be noted that the system is basically nonlinear due to the nonlinear transfer function between the altitude h and the desired velocity $\dot{z}_d$. However, if the assumption is made that the actual trajectory followed does not deviate much from the nominal, it is possible to obtain a linear approximation for the system which involves only a time varying parameter.

For the nominal trajectory

$$\dot{z}_d = \dot{z}_f + (a_N - g)\, T \qquad [11]$$

where T is the time to go before guidance termination. Also

$$\frac{\partial \dot{z}_d}{\partial h} = \frac{a_N - g}{\sqrt{2\,(a_N - g)\,(h - h_f) + \dot{z}_f^2}} = \frac{a_N - g}{\dot{z}_d} \qquad [12]$$

Defining the quantities

$$\Delta \dot{z} = \dot{z} - \dot{z}_{nominal} \qquad [13]$$

$$\Delta h = h - h_{nominal} \qquad [14]$$

$$\Delta a = a - a_{nominal} \qquad [15]$$

$$\Delta \dot{z}_d = \dot{z}_d(h) - \dot{z}_{nominal} \qquad [16]$$

where the terms in the right-hand members of Eqs. 13 through 16 correspond to the same instant of time, it may be shown that the block diagram of Fig. 4 is reducible to that of Fig. 5 in which only the perturbations are involved.

In practice, the time to go T decreases very nearly in a one to one correspondence with real time

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$$(dT/dt) \simeq -1 \qquad [17]$$

The differential equation describing the dynamic behavior of the system of Fig. 5 is, with the utilization of Eq. 17

$$\frac{d\epsilon_V}{dT} = \epsilon_V \left[K_a - \frac{a_N - g}{\dot{z}_f + (a_N - g) T} \right] \qquad [18]$$

Eq. 18, K_a need not be a constant but may be any function of time. The special case of constant K_a results in the following solution for the velocity error ϵ_V

$$\frac{\epsilon_V (T)}{\epsilon_V (T_o)} = \frac{\dot{z}_f + (a_N - g) T_o}{\dot{z}_f + (a_N - g) T} \exp K_a (T - T_o) \qquad [19]$$

At guidance termination T equals zero and the ratio of the initial to the final velocity error is

$$\frac{\epsilon_V (T_o)}{\epsilon_V (0)} = \frac{\dot{z}_f}{\dot{z}_f + (a_N - g) T_o} \exp (K_a T_o) \qquad [20]$$

This ratio is plotted against T_o for several values of K_a in Fig. 6. For large values of T_o, the reduction of initial velocity error is generally quite effective. However, as T_o decreases, the velocity error may actually increase towards the end. The latter is an inherent characteristic of the geometry, not a fault of the system. Fortunately, there is a better criterion for judging the system performance than ϵ_V. Since guidance termination is based on velocity information, concern is primarily with the altitude dispersion at this point. It may be shown that the altitude error between the actual and nominal trajectory corresponding to some value of $\dot{z}$ is

$$\delta h \cong - \frac{\partial h}{\partial \dot{z}_d} \Delta \dot{z} + \Delta h$$

$$= - \frac{\partial h}{\partial \dot{z}_d} (\Delta \dot{z} - \Delta \dot{z}_d)$$

$$= - \frac{\dot{z}_d}{a_N - g} \epsilon_V \qquad [21]$$

Eqs. 20 and 21 may be combined to give

$$\delta h (0) = \delta h (T_o) \exp (- K_a T_o) \qquad [22]$$

Hence, the altitude error diminishes rapidly in an exponential manner. As an example, suppose guidance begins at $T_o = 50$ sec and an initial altitude error of 1000 ft (equivalent to 20 fps in velocity) is assumed. Then, even with a K_a of only 0.1, the final altitude error is but 6.7 ft. In the actual system, K_a is increased from 0.1 initially to about 0.3 at the end of guidance, and the residual altitude error due to initial offset becomes quite negligible. Analog simulation results have also verified the validity of the conclusions of this simple, linearized approach.

Lateral Channel Dynamics

The geometry of the lateral channel is shown in Fig. 7. For small values of β and θ, the closed-loop representation of Fig. 8 applies. Assuming the control system (autopilot) response is essentially instantaneous, the differential equation characterizing the guidance loop is

$$\frac{\dot{z}}{a_N f_1(t)} \ddot{x} + \frac{\dot{z}}{a_N} \ddot{x} + \dot{x} = 0 \qquad [23]$$

The form chosen for $f_1(t)$ is

$$f_1(t) \cong K' \frac{v_i}{R} \qquad [24]$$

As long as the error angle η is small, the ratio v_i/R is very nearly $\dot{z}/h$. In terms of the time to go T, Eq. 23 may be rewritten as

$$\frac{1}{K' a_n} \left[h_f + \dot{z}_f T + \frac{1}{2} (a_N - g) T^2 \right] \frac{d^2 \dot{x}}{dT^2}$$

$$- \frac{1}{a_N} \left[\dot{z}_f + (a_N - g) T \right] \frac{d\dot{x}}{dT} + \dot{x} = 0 \qquad [25]$$

Using the approximate expression

$$\dot{z} \cong \dot{z}_f + (a_N - g) T \qquad [26]$$

to change the independent variable in Eq. 25, one may obtain

$$\frac{a_N - g}{2K' a_N} \left[\dot{z}^2 + 2h_f (a_N - g) - \dot{z}_f^2 \right] \frac{d^2 \dot{x}}{d\dot{z}^2}$$

$$- \frac{a_N - g}{a_N} \dot{z} \frac{d\dot{x}}{d\dot{z}} + \dot{x} = 0 \qquad [27]$$

Except for the last few seconds of guidance

$$\dot{z}^2 >> 2h_f (a_N - g) - \dot{z}_f^2 \qquad [28]$$

Thus, Eq. 27 may be approximated by the homogeneous linear differential equation

$$\frac{a_N - g}{2K' a_N} \dot{z}^2 \frac{d^2 \dot{x}}{d\dot{z}^2} - \frac{a_N - g}{a_N} \dot{z} \frac{d\dot{x}}{d\dot{z}} + \dot{x} = 0 \qquad [29]$$

The solution of Eq. 29 is

$$\dot{x} = c_1 \dot{z}^{m_1} + c_2 \dot{z}^{m_2} \qquad [30]$$

where m_1 and m_2 are the roots of the auxiliary equation

$$\frac{a_N - g}{2K' a_N} m^2 - \frac{a_N - g}{a_N} \left(\frac{1}{2K'} + 1 \right) m + 1 = 0 \qquad [31]$$

Clearly, in order to effect a reduction of the lateral velocity $\dot{x}$ as $\dot{z}$ decreases, the real part of m_1 and m_2 must be positive. Actually, there is no advantage in making m_1 and m_2 complex at any event. The gain K' actually chosen is such as to make $m_1 = m_2$. Any increase in K' tends to increase one and decrease the other, resulting in less effective velocity reduction. For the equal root case

$$\frac{\dot{x}}{\dot{x}_o} = \left[\left(1 + m \ln \frac{\dot{z}_o}{\dot{z}}\right) - \frac{a_N}{a_N - g}\frac{\theta_o}{\beta_o}\ln\frac{\dot{z}_o}{\dot{z}}\right]\left(\frac{\dot{z}}{\dot{z}_o}\right)^m \qquad [32]$$

From Eq. 32 it is apparent that the lateral velocity reduction is also strongly dependent on the initial conditions. For the parameters involved for the spacecraft, $K' = 2.06$ is the optimum choice resulting in $m_1 = m_2 = m = 2.56$. Thus

$$\frac{\dot{x}}{\dot{x}_o} = \left[1 + \left(2.56 - 1.59\frac{\theta_o}{\beta_o}\right)\ln\frac{\dot{z}_o}{\dot{z}}\right]\left(\frac{\dot{z}}{\dot{z}_o}\right)^{2.56} \qquad [33]$$

Several response curves are shown in Fig. 9. Curve A is for $\theta_o = 0$, corresponding to the case where the optimum gain K' is used initially as well. Curve C is for $\theta_o = \beta_o$. This may be achieved by using a very large gain initially for a short time, forcing the thrust line into coincidence with the velocity vector and then switching over to the optimum gain. Curve D is for the case where the logarithmic term disappears from Eq. 33. This can be achieved only through an under-damped response in θ/β and by switching the gain to the proper optimum value when θ has overshot past β with the exact ratio as indicated. Curve E is one for which the co-efficient of the logarithmic term is negative. Although not shown, the velocity error will become negative at some small value of $\dot{z}$, indicating over-correction.

The actual K' program places the response somewhere between Curves A and C. Curve B is representative. A large value of K' of about 15 is used initially for three seconds, after which the lower value of 2.06 is used. The reduction in lateral velocity is more than adequate for the initial magnitudes anticipated. The approximate analysis is accurate down to a $\dot{z}$ of about 80 fps corresponding to a time to go of 8 sec. If $\dot{z}_o$ and $\dot{x}_o$ are assumed to be 600 and 100 fps, respectively, the residual $\dot{x}$ will be about 4.5 fps at this time. Further correction during the last few seconds results in a final $\dot{x}$ of less than 0.5 fps, as verified through analog simulation studies.

The body attitude response curves are found through the following relation

$$\theta \cong -\frac{\ddot{x}}{a_N}$$

$$= -\frac{1}{a_N}\frac{d}{dt}\left\{\dot{x}_o\left[\left(1 + m\ln\frac{\dot{z}_o}{\dot{z}}\right) - \frac{a_N}{a_N - g}\frac{\theta_o}{\beta_o}\ln\frac{\dot{z}_o}{\dot{z}}\right]\left(\frac{\dot{z}}{\dot{z}_o}\right)^{m-1}\right\}$$

$$= \left\{\theta_o + \left[\frac{a_N - g}{a_N}m^2\beta_o - m\theta_o\right]\ln\frac{\dot{z}_o}{\dot{z}}\right\}\left(\frac{\dot{z}}{\dot{z}_o}\right)^{m-1} \qquad [34]$$

Fig. 10 is a plot of θ vs $\dot{z}/\dot{z}_o$ for several values of θ_o.
Again it is seen that the choice of θ_o somewhere between 0
and β_o has a beneficial effect of reducing the maximum value
of θ.

CONCLUSIONS

Analysis and tests of the landing dynamics of the spacecraft
showed that stability is critically dependent upon keeping the
two components of touchdown velocity within prescribed bounds.
Conservative allowances of $\pm$ 5 and 10 fps were then made for
the vertical and horizontal velocity errors, respectively. A
detailed error analysis, based on anticipated tolerances in
the actual instrumentation, revealed the following 3-sigma
landing velocity errors

    Vertical velocity error (3σ)   3 fps

    Horizontal velocity error (3σ)   10.3 fps

The vertical velocity error is due primarily to an assumed
error of $\pm$ 10 ft (3σ) in the altimeter reading at vernier
engine shutoff. The horizontal velocity error is almost com-
pletely due to an assumed random bias error of 3 fps (3σ) in
each Doppler radar beam.

The principal errors in landing velocity are, therefore, not
inherently a result of any deficiency or over-simplification
of the guidance philosophy but are rather due to sensor limita-
tions. Therefore, if sensor accuracy should be better than
assumed, an even lower velocity landing could be achieved,
possibly less than 10 fps.

REFERENCES

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Source	Horizontal Velocity (3σ) at Staging, fps
1 Radio tracking error	42
2 Attitude error (0.61°)	83
3 Main engine velocity increment uncertainty (for a 30° initial approach angle)	41
R.S.S.	102 (3σ)

Table 1 Horizontal velocity error at staging

Communications
Antenna

Radar
Altimeter
Antenna

Landing
Leg
(3 places)

Vernier
Engine
(3 places)

Doppler
Radar
Antenna
(3 places)

Solid Propellant
Main Engine
(Jettisoned after
Burnout)

Bottom
View of
Space-
craft
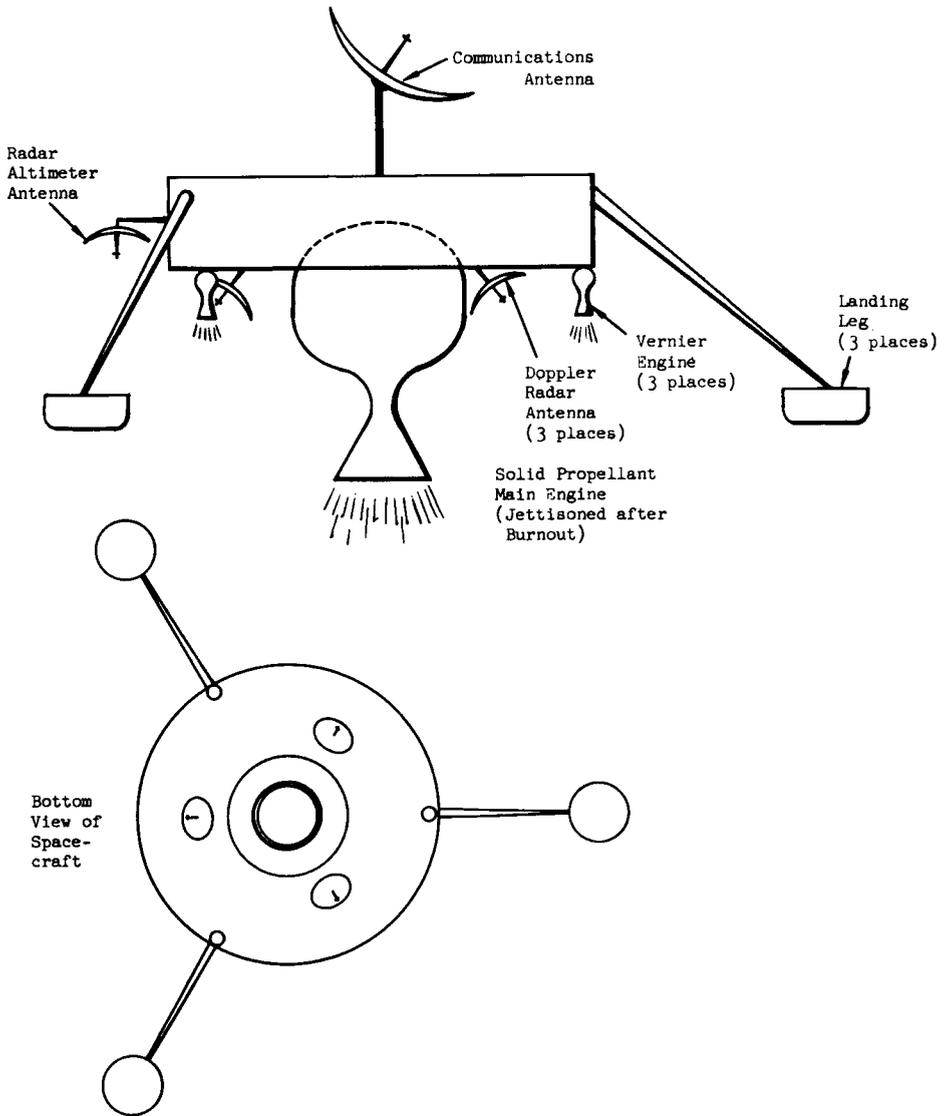
Fig. 1 Schematic representation of spacecraft

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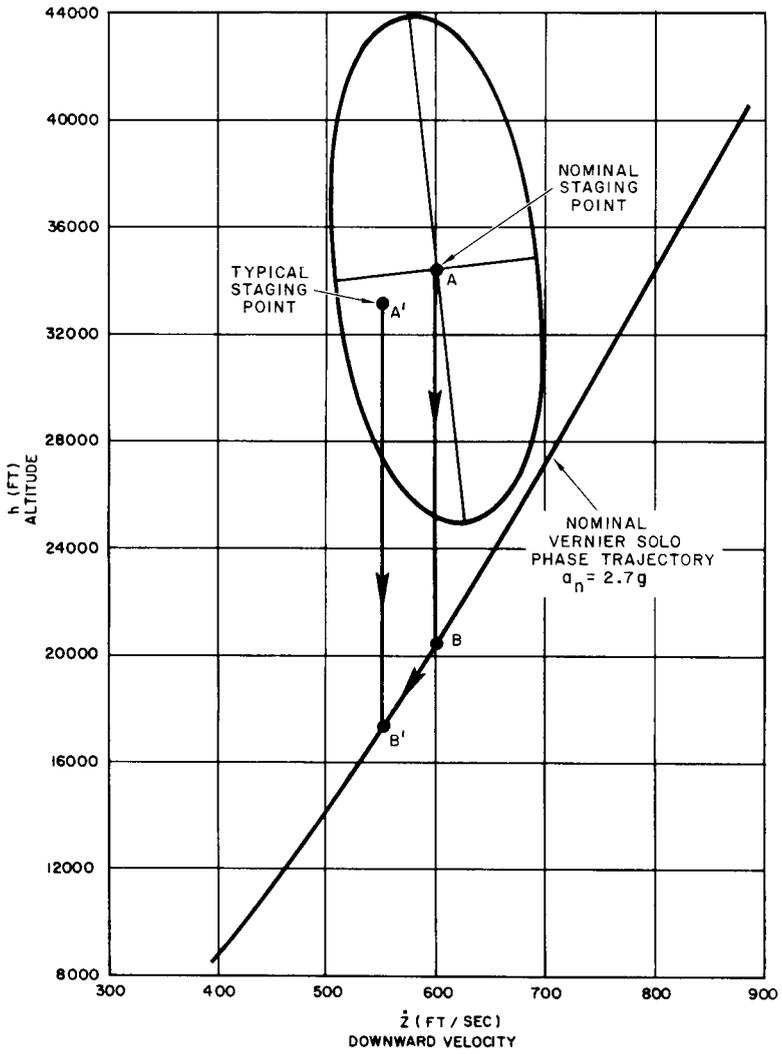
Fig. 2 Ninety-nine % probability ellipse
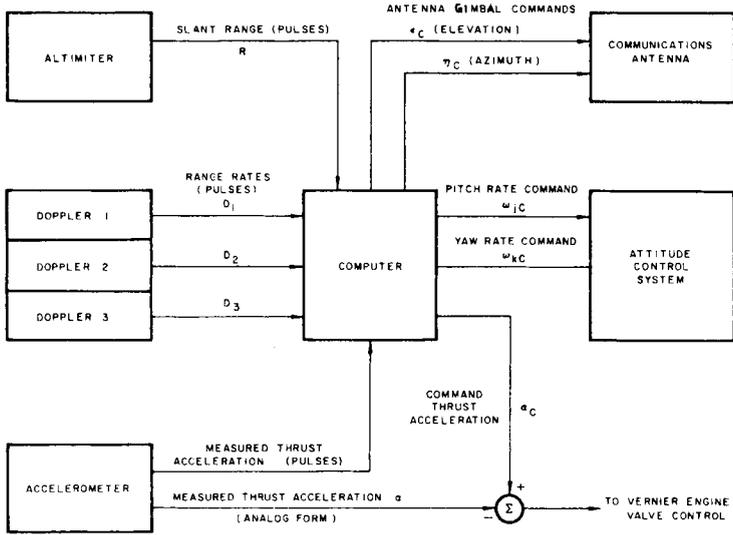
Fig. 3   Terminal guidance instrumentation
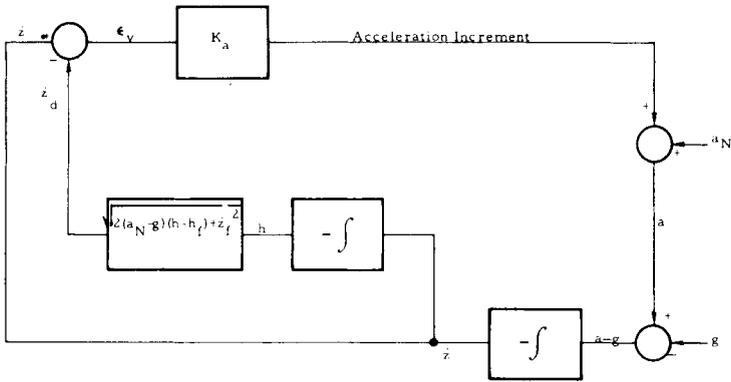
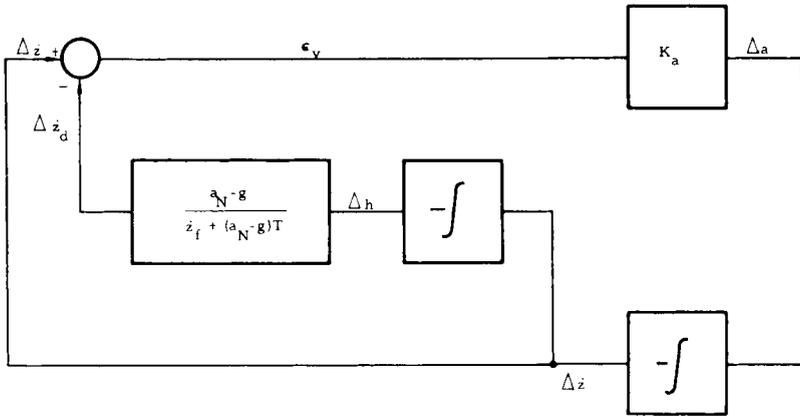Fig. 4   Closed-loop representation of longitudinal channel

Δz +

ϵ_v

K_a

Δa

$\Delta \dot{z}_d$

$$\frac{a_N - g}{\dot{z}_f + (a_N - g)T}$$

Δh

$-\int$

$-\int$

$\Delta \dot{z}$

Fig. 5 Loop representation of perturbations (vernier solo)

$\dot{z}_f = 10$ ft/sec
$a_N - g = 9$ ft/sec^2
(g = lunar g)

$$\frac{\epsilon_v(T_o)}{\epsilon_v(o)} = \frac{\dot{z}_f e^{K_a T_o}}{\dot{z}_f + (a_N - g)T_o}$$

$K_a = 0.3$

$K_a = 0.2$

$K_a = 0.1$

$\epsilon_v(T_o)/\epsilon_v(o)$

T_o (sec)
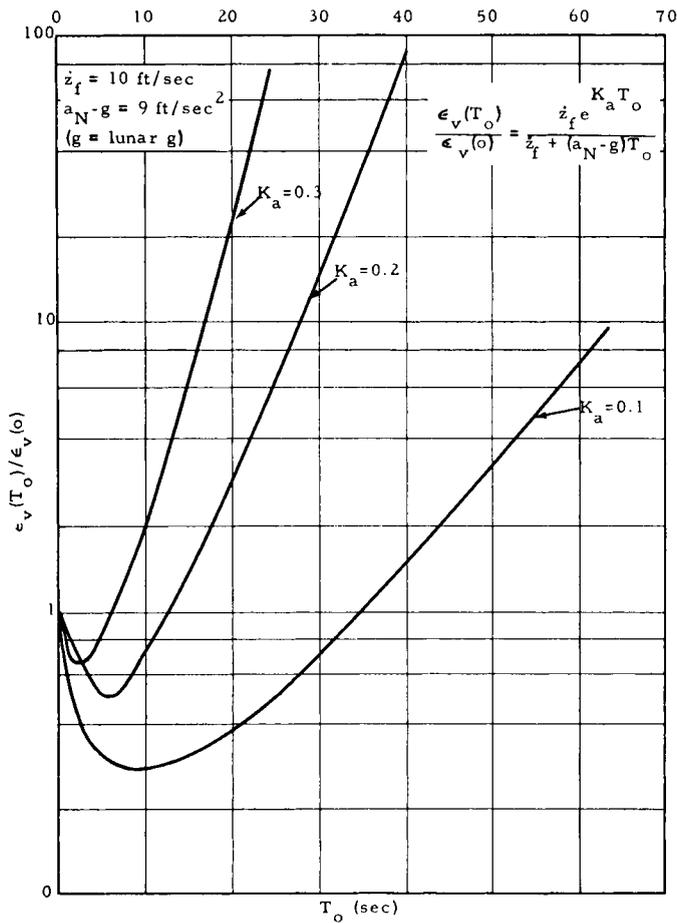
Fig. 6 Velocity error reduction ratio for constant K_a

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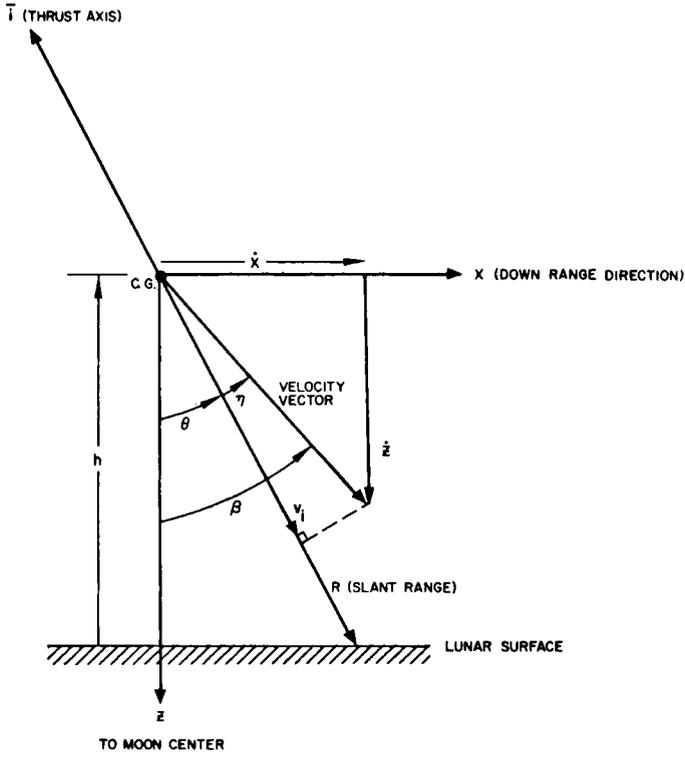
Fig. 7   Lateral channel geometry
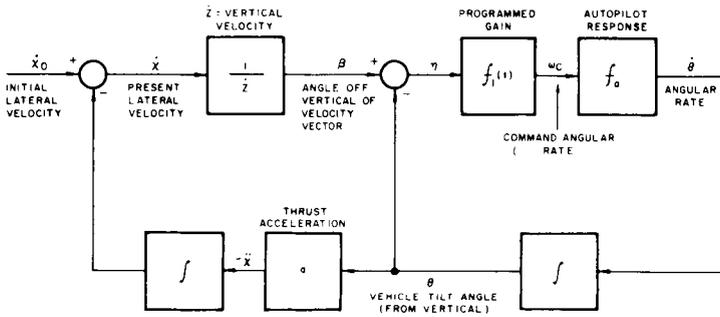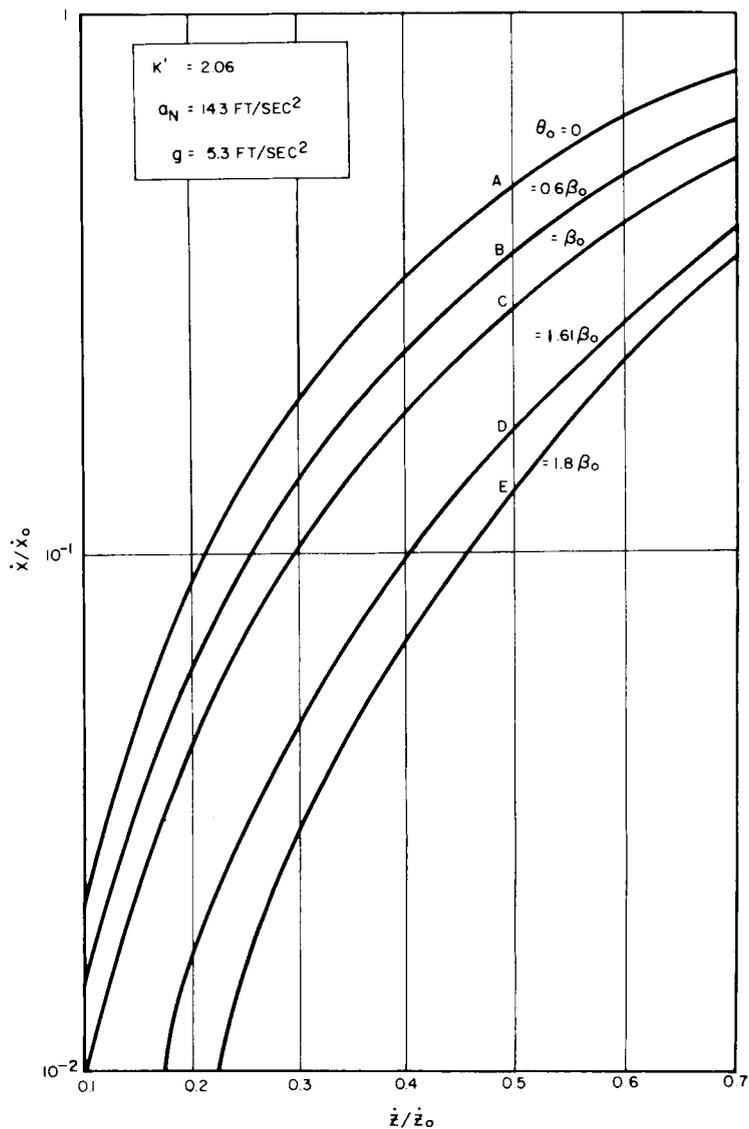

Fig. 8   Lateral channel dynamic representation

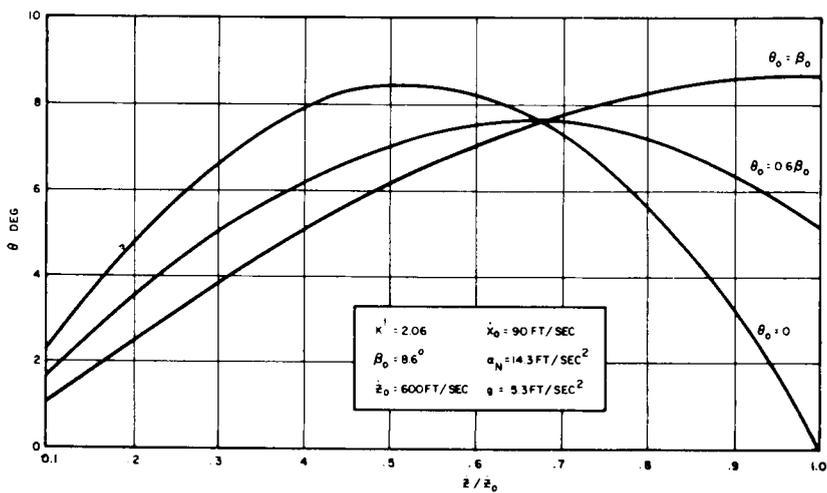Fig. 9   Lateral velocity reduction ratio vs. vertical velocity ratio

Fig. 10   Body attitude vs. vertical velocity