AUTOMATIC RE-ENTRY GUIDANCE AT ESCAPE VELOCITY

Paul C. Dow Jr.,¹ Donald P. Fields,² and Frank H. Scammell³

Avco Corp., Wilmington, Mass.

ABSTRACT

Two methods are considered in detail of guiding a spacecraft to a safe point landing after re-entering Earth's atmosphere at escape velocity, for example, after return from a lunar mission. In order to ease the task of midcourse guidance the range of acceptable re-entry conditions is maximized within certain constraints; these are that peak deceleration loads be less than a specified value (arbitrarily set at 10 g) and that after re-entry the vehicle does not "skip out", that is, the trajectory remains below some specified altitude (arbitrarily set at 300,000 ft). Within these constraints it is demonstrated that both guidance systems are capable of achieving high impact accuracy at ranges of 1000 to 2500 miles from re-entry and ±200 miles cross-range, with re-entry flight path angles between -5.6° and -7.8° at 400,000 feet altitude, corresponding to a corridor width of 31 miles.

The first guidance scheme uses an apparent target which differs from the actual target only in altitude. A proportional steering signal is obtained by computing the vector cross product of the unit velocity vector and the vector along the line of sight from the vehicle to the apparent target. The vector form of the computation was chosen to simplify the design of the digital computer. The proper motion of the apparent target toward the actual target is crucial and it is shown that this can be a single function of range to go for all re-entry conditions.

¹Presented at ARS Guidance and Control Conference, Stanford California, Aug. 7-9, 1961.
²Chief, Guidance Design and Integration Section.
³Staff Engineer.
⁴Staff Engineer.
The second scheme predicts the impact point by explicitly solving the trajectory equations. The difference between the predicted impact point and the target is resolved appropriately and used as the steering command. This process is repeated continuously to impact.

Both systems temporarily delay control over range early in re-entry to insure safe passage through the "survival phase", i.e., that phase when avoidance of excessive decelerations and altitudes is critical. A trajectory is flown in which angle of attack is programmed as a function of time, depending on re-entry angle, until the guidance system takes over control. In the predictive scheme the predictor modifies the programmed trajectory if necessary on the basis of predicted loads and altitudes.

INTRODUCTION

Recent interest in circumlunar flight and in return trajectories following launch from the moon have resulted in extensive investigations of guidance, control, and heating problems associated with re-entry into Earth's atmosphere following such missions. Philips (1) discusses the design problems of re-entering manned space vehicles and points out the effect of lift and drag modulation. The subject of lift modulation is pursued further by Lees et al. (2). A comprehensive analysis of corridor and guidance requirements is given by Chapman (3), and this work is further expanded on by Galman (4) and Teague (5) who consider dispersion, maneuverability, and guidance accuracy. Attention to the problem of lateral maneuverability is given by Slye (6). Methods of controlling a re-entry vehicle to achieve the desired trajectories are considered by Cheatham et al. (7) for a re-entering satellite, and by Wingrove and Coate (8) for a pilot operated prediction system. A simplified approach to the range prediction problem for such a system is given by Rosenbaum (9) which makes use of final value techniques for re-entry at less than satellite velocity. The purpose of this paper is to present two different guidance concepts which provide accurate control over the landing point and at the same time obey the requirements of specified constraints and afford considerable flexibility over the downrange and crossrange distances from re-entry to landing. For definiteness in most of the analysis a vehicle is selected which has

---

1Numbers in parentheses indicate References at end of paper.
a maximum lift-drag ratio of 0.5 and a ballistic parameter \(W/C_{pA}\) of 100 psf.

This paper incorporates concepts for guidance through the "survival-phase" of re-entry and range control guidance from the termination of the survival phase to touchdown. The term "survival-phase" refers to those portions of the bounding trajectories where there is danger of exceeding the design constraints. This phase occurs in the early super-circular velocity portions of flight. It will also be shown that with the proper L/D-flight path angle combination during the survival phase range control can be simplified, since the trajectory parameters at the initiation of range-control are similar over approximately 80% of the allowable re-entry corridor.

Following a discussion of the survival phase, the two guidance techniques are described. Both systems are completely self-contained and do not depend for their operation on external sources of information other than the desired landing point. The difference between them is in the method of computing the error signals used to develop the required commands to the vehicle autopilot. The prediction guidance system uses a high speed computer to solve the trajectory equations explicitly and to determine the predicted impact point, whereas the apparent target system controls the vehicle velocity vector so that it always points at an apparent target which differs from the actual target only in altitude.

Since this paper was originally prepared, other concepts of guidance during the survival phase have been presented (10, 11).

SURVIVAL PHASE

During the survival phase it is a matter of prime importance that neither excessive loads nor skip trajectories result; therefore, range control was not attempted during this phase. For this analysis, a drag polar was employed such that a \(W/C_{pA}\) of 100 psf was obtained at an L/D of 0 and a \(W/C_{pA}\) of 80 psf at an L/D of 0.5. The initial altitude and velocity were selected as 400,000 ft and 36,800 fps respectively. During the entire re-entry phase the resultant aerodynamic load was constrained to be less than 10 g and the maximum attainable altitude after pullup, \(\gamma = 0^\circ\), was constrained to be less than 300,000 ft. The 10 g load constraint is obvious for manned flights. From a guidance and control point of view, the 300,000 ft maximum altitude constraint
after pullup results from:

1) Loss of aerodynamic control above 300,000 ft.
2) Necessity of adding a reaction control system for vehicle orientation and stability.
3) Unnecessarily tight tolerances on platform accuracies, knowledge of initial conditions, the actual values of W/CpA and L/D, and the density profile so that the vehicle can be accurately guided to the required exiting conditions.

Small errors in exiting conditions can result in large range deviations during the resulting near vacuum flight which may greatly complicate the subsequent re-entry maneuver. Nevertheless, if greater range capabilities are desired, the above constraints must be relaxed.

Normal Re-entry Corridor

The re-entry angles defining the ballistic overshoot boundary, the 10 g ballistic undershoot boundary and the 10 g lifting undershoot boundary (constant L/D of 0.5) are \(-5.63^\circ\), \(-6.21^\circ\), and \(-7.82^\circ\), respectively. Maximum aerodynamic loads less than 10 g may be obtained at re-entry angles as steep as \(-8.9^\circ\) with proper L/D modulation but re-entries at angles greater than \(-7.82^\circ\) should not be considered except under abort conditions since the range overlap would be greatly diminished. A range of 2750 miles can easily be obtained from the 10 g lifting undershoot boundary using a constant L/D of 0.5 as opposed to only 1300 miles from the 10 g lifting boundary using a modulated lift. For re-entries along the 10 g lifting undershoot boundary using a constant L/D of 0.5, range control is initiated at an altitude of 175,300 ft and a velocity of 31,360 fps as opposed to an altitude of 158,500 ft and a velocity of 27,900 fps for re-entries along the 10 g lifting undershoot boundary using a modulated L/D.

Allowable Variation In Pullup Parameters

The maximum loads obtained during the survival phase, as a function of initial re-entry angle and constant L/D, are presented in Fig. 1. This figure also shows the permissible range of L/D variation without exceeding a particular load for all re-entry angles.

A considerable variation in the altitude and velocity at the end of the survival phase (or initiation of range control) can occur; i.e., for a re-entry of \(-6.75^\circ\) the pullup altitude can vary between 193,500 and 156,500 ft whereas the pullup velocity can vary between 33,200 and 23,600 fps for L/D's
ranging between 0.5 and 0.1. For these conditions the maximum load varies between 6 and 10 g. The variation in pullup altitude and velocity as a function of L/D and re-entry angles are shown in Figs. 2 and 3, respectively.

A wide variation in range from re-entry to pullup is also experienced with the range increasing as the re-entry angle is decreased for a constant L/D and as L/D is decreased for a constant re-entry angle. For a re-entry angle of -6.75° the range increases from 440 to 595 miles as L/D decreases from 0.5 to 0.1. The variation in range from re-entry to the end of the survival phase as a function of L/D and re-entry angle is presented in Fig. 4.

Another parameter is the heating rate, which is as yet unconstrained but which may be eventually required to play an important part in the guidance scheme, should it prove necessary to obtain minimum heating trajectories for a given range. The heating rate increases with both an increase in re-entry angle for a constant L/D and a decrease in L/D for a constant re-entry angle. At a re-entry angle of -6.75° the maximum laminar heating rate that occurs during the survival phase varies between 750 Btu/ft²/sec and 960 Btu/ft²/sec, whereas the maximum turbulent heating rates are 330 Btu/ft²/sec and 550 Btu/ft²/sec as the L/D is decreased from 0.5 to 0.1. Therefore, unless flying along the 10 g lifting undershoot boundary where an L/D of 0.5 is required, tradeoffs in L/D are available depending upon whether it is desirable to minimize the load, maximize or minimize the range, or minimize the heating rates or the total integrated heating. The variation in maximum heating rates that occur during the survival phase are presented in Fig. 5. These heating rates are based upon a 1-ft nose radius. As may be expected, similar trends are experienced for the total integrated heating and this information is presented in Fig. 6.

Re-entries Requiring Negative Lift

A range of re-entry angles that has not yet been discussed is defined by those angles shallower than the ballistic overshoot boundary and steeper than the lifting overshoot boundary. Although re-entries in this region might only be considered during an abort mode of operation or under abnormal density variations, guidance through this regime poses no serious difficulties. In this instance, however, negative lift must be applied to avoid skipping. If this negative lift is removed too late peak loads up to approximately 60 g may result, and if it is removed too early skipping will still result.
One method of solving this energy management problem is to compare the instantaneous altitude and flight path with one or more reference "altitude-flight path angle" histories. These reference trajectories may be the ballistic overshoot, the ballistic undershoot and a nominal trajectory between these two ballistic boundaries. If the negative lift is removed when the instantaneous altitude-flight path angle history is identical to any one of the three reference histories, the resulting trajectory will closely approximate that reference trajectory.

This reference technique is only applied during the survival phase until range control is initiated. However, the reference trajectory can be selected as a function of range to go since the ballistic undershoot and ballistic overshoot trajectories approximately define the range overlap. The time difference between crossing the ballistic overshoot and undershoot reference trajectories is approximately 20 sec and is independent of re-entry angle.

Survival Phase Guidance Scheme

A guidance scheme can now be developed for use during the survival phase up to a point where range control can be initiated safely. In order to simplify the energy management problem once range control has been initiated and also to minimize the number of guided trajectories that are required to define the range overlap, an attempt has been made to obtain trajectory parameters at the end of the survival phase that are approximately equal regardless of the initial re-entry angle.

A nominal trajectory is selected which is midway between the ballistic overshoot and ballistic undershoot boundaries. Limitation of the nominal re-entry corridor to the ballistic corridor allows safe passage through the survival phase if either or both the pilot and the automatic guidance system are inoperative. Under normal operating conditions range control can be initiated near the termination of the survival phase.

For re-entry angles shallower than the nominal trajectory the maximum negative lift is employed from re-entry, until, as has been previously shown, the altitude or flight path angle indicates a return to the nominal trajectory. The nominal ballistic trajectory is now flown to an altitude of 182,000 ft where range control is initiated. An altitude of 182,000 ft was selected for initiation of range control, since this is approximately the lowest altitude at which a range of 2750 miles can be realized and the highest altitude at which
GUIDANCE AND CONTROL

a range of 1000 miles can be realized without excessive loads. The ballistic portion of the flight before the initiation of range control can be utilized to roll the vehicle through 180° if it is anticipated that positive L/D will initially be required during the range control mode of operation. For re-entry angles steeper than the nominal trajectory, the value of L/D to be employed is selected as a function of the re-entry angle such that pullup is achieved at 182,000 ft. This obviously can only be accomplished up to the re-entry angle which requires an L/D of 0.5 to pullup at 182,000 ft; steeper angles will result in a somewhat lower pull-up altitude.

This procedure results in trajectory parameters at the beginning of range control which are not sensitive to initial re-entry conditions, and hence permits the guidance techniques for range control to begin with fairly standard conditions.

THE PREDICTION GUIDANCE CONCEPT

The central piece of equipment for the first guidance system which will be described is the prediction computer, or predictor. In a typical configuration it would operate in conjunction with an inertial platform, a general purpose digital computer, and a flight control system. Prior to initiating control, the predictor takes position and velocity information from the general purpose computer and solves the vehicle trajectory in advance. The impact point thus computed is compared with the desired target, and an error signal is developed which is used to furnish commands to the autopilot. Using the new commands the predictor again computes the predicted impact point, and a new error signal is developed to modify the previous command. Each prediction takes only a few seconds, depending on range and accuracy required, and the process is repeated continuously throughout the flight. During the solution of a predicted trajectory the maximum deceleration load and maximum altitude are noted; if these exceed allowable values the commands to the autopilot are limited to keep the trajectory within the desired bounds. A significant modification to this general outline is made to provide for energy management. Energy management is here defined as the ability to command the vehicle to fly a trajectory such that the vehicle will approach the end of its flight with the target in the center of its maneuver capability. That is, the trajectory will be such as will give the maximum ability to maneuver in any direction to correct for unpredictable winds, errors in guidance, etc. This energy management function is performed by overcontrolling early in the trajectory so that
the full maneuver capability will be available later. A block diagram of the system is shown in Figure 7.

**Range Control**

In the first computation, the predictor uses values of $L/D$ and $\phi$ equal to zero to make the first prediction. This results in a predicted impact point, and an error signal is determined. The error signal is used to generate new values of $L/D$ and $\phi$, which are used in the next prediction. The second prediction again runs to impact and the new error signal modifies the previous values of $L/D$ and $\phi$, until after several predictions the solution converges on the proper values required in order to hit the target. If, during any one of these predictions, the altitude exceeds 300,000 ft, the solution stops, the upper limit on $(L/D) \cos \phi$ is decreased and the prediction is repeated with the decreased value. This is repeated until a solution is obtained which does not exceed 300,000 ft. Conversely, if the trajectory does not reach 300,000 ft the limit is raised. This causes the upper limit to oscillate about the desired value. Similarly, if the maximum load exceeds 10 g the lower limit on $(L/D) \cos \phi$ is increased, and it is decreased if the load is less than 10 g.

Before being transmitted to the autopilot, the predicted commands for range control are "optimized". Fig. 8 shows the command sent to the autopilot as a function of the value determined by the predictor. The effect of this optimization is to overcontrol the vehicle so that it will ultimately be capable of maximum range increase as well as range decrease. In other words, the trajectory is adjusted to locate the center of the maneuver footprint on the target. In the example shown in Fig. 8, the vehicle will finally follow a trajectory corresponding to $(L/D) \cos \phi$ equal to 0.2; this corresponds to approximately the center of the maneuver capability for a vehicle having a maximum $L/D = 0.5$. Optimization therefore, provides an energy management function by insuring that the vehicle will have the maximum capability to maneuver in order to compensate for unpredictable effects, such as errors in the range prediction, unforecast winds, guidance errors, nonstandard atmospheric density, etc.

**Crossrange Control**

It is possible to determine the crossrange error signal by use of the crossrange component of the predicted miss distance. However, in order to use the energy management advantages of
GUIDANCE AND CONTROL

Optimized control it is desirable to approach the target with a bank angle of zero. This is accomplished by turning the velocity vector toward the target as early as possible. The error signal in this case is the horizontal component of the angle between the vehicle velocity vector and the line of sight to the target. It is generated by taking the vertical component of the vector cross product of the unit velocity vector and the line of sight vector to the target. This component is used to command \((L/D) \sin \phi\), the horizontal component of \(L/D\). It will be seen later that this is the same as the method used for lateral control in the apparent target scheme.

**Control Law**

It has been pointed out that the commands to the autopilot are angle of attack and bank angle. At the end of a prediction cycle an error in range is determined

\[
\epsilon_x = R_{xtg} - \Delta x \tag{1}
\]

where \(\epsilon_x\) = predicted range error
\(R_{xtg}\) = range to go to the target from vehicle present position
\(\Delta x\) = range of predicted trajectory

This error is divided by the range to go to maintain a convenient gain throughout the flight and multiplied by a gain constant. It is then added to the vertical component of \(L/D\) used in the prediction just completed

\[
\left(\frac{L}{D} \cos \phi\right)_i = \left(\frac{L}{D} \cos \phi\right)_{i-1} + \frac{K \epsilon_x}{R_{xtg}} \tag{2}
\]

where \(\left(\frac{L}{D} \cos \phi\right)_i\) = vertical component of \(L/D\) used in the \(i\)th prediction
\(K\) = gain constant

The next prediction is made with the new value of \((L/D) \cos \phi\), and at the same time this value is optimized as shown in Fig. 8.

The computer obtains the vector product

\[
\vec{I}_v \times \vec{R}
\]

where \(\vec{I}_v\) = unit velocity vector
\(\vec{R}\) = vector from vehicle to target (range vector)

and uses the vertical component as the error signal to command the horizontal component of \(L/D\)
GUIDANCE AND CONTROL

\[ \frac{L}{D} \sin \phi = -\left( I_v \times \bar{R} \right)_{\text{vert}} \]  

The two components of L/D are then combined to give L/D and \( \phi \) separately

\[ \frac{L}{D} = \left[ \left( \frac{L}{D} \cos \phi \right)^2 + \left( \frac{L}{D} \sin \phi \right)^2 \right]^{1/2} \]  
\[ \phi = \arctan \left( \frac{\frac{L}{D} \sin \phi}{\frac{L}{D} \cos \phi} \right) \]

These are then used as the command inputs to the autopilot.

It will be noted that reference to the horizontal and vertical components of L/D as \( (L/D) \sin \phi \) and \( (L/D) \cos \phi \), respectively, implies that small flight path angles have been assumed. This assumption is valid since very little control over range is possible after the flight path angle becomes large.

Lift, Drag, And Density Correction

In order to make an accurate prediction of the trajectory, it is apparent that the parameters which enter into the trajectory equations must also be known accurately. The velocity and position information will be provided by the inertial navigator and will be sufficiently accurate. In the absence of some other means of determining it, density must be obtained from altitude. Since considerable uncertainty exists in the present knowledge of density at high altitudes, this method of obtaining density can cause large miss distances, as described in the section on errors. It is possible to compensate for this effect, however, and also to compensate for uncertainties in \( W/\sigma A \) by making use of the acceleration and velocity data which is available from the inertial platform and computer.

\[ \frac{D}{m} = \frac{1}{2} V_R^2 \left( \frac{C_D A}{m} \right) \rho_0 \exp \left( -\frac{g}{h} \right) \]  
\[ \frac{L}{m} = \left( \frac{D}{m} \right) \left( \frac{L}{D} \right) \]

where
\[ D = \text{drag} \]
\[ L = \text{lift} \]
\[ m = \text{vehicle mass} \]
\[ V_R = \text{velocity relative to air mass} \]
\[ \rho_0 = \text{sea level density} \]
\[ g = \text{density scale height} \]
\[ \phi = \text{altitude} \]
\[ \frac{C_D A}{m} = \text{ballistic coefficient} \]

From the inertial guidance system the relative velocity vector \( V_R \) and the acceleration vector \( \bar{A} \) are available. Their dot and
vector products yield the lift and drag, respectively
\[
\left( \frac{D}{m} \right)_{\text{actual}} = \frac{\vec{V}_r \cdot \vec{A}}{|V_r|} \quad [8]
\]
\[
\left( \frac{L}{m} \right)_{\text{actual}} = \frac{\vec{V}_r \times \vec{A}}{|V_r|} \quad [9]
\]
At the beginning of a prediction, the actual (measured) drag is divided by the value computed at that instant from Eq. 6. The quotient is used as a correction factor to correct the drag used in the prediction
\[
C = \left( \frac{\left( \frac{D}{m} \right)_{\text{actual}}}{\frac{D}{m}} \right) \frac{L}{m} \frac{\rho}{\rho_0} \exp(-\beta h) \quad [10]
\]
The correction factor C now multiplies the value of \( \frac{D}{m} \) and \( \frac{L}{m} \) used in the prediction equations. It is seen that this not only eliminates errors in uncertainty of \( \frac{C_p A}{m} \) and \( \rho \), but it also removes dependence on knowledge of altitude, since a constant percentage error in density results from a constant magnitude error in altitude. This is true even though the altitude error changes during the trajectory due to, for example, platform drift. This occurs because the prediction takes place during a very short time during which the platform errors are essentially constant. It should be pointed out, however, that this technique will not correct for errors in the slope of the density curve or random variations in density which are unpredictable. The effectiveness of this technique is described in the error analysis.

The computation of \( \frac{L}{D} \) is accomplished by dividing the actual lift (obtained by the vector cross product, Eq. 9) by the actual drag to give the actual \( \frac{L}{D} \) ratio. This is transmitted to the autopilot and is compared with commanded \( \frac{L}{D} \) to provide the error signal.

Effect of Earth's Rotation

The predictor will determine the time of flight to impact and this will be relayed to the digital computer for computation of future target position. This will be used to determine miss distance in comparison with predicted impact point. The only other correction required by Earth's rotation is the use of velocity relative to the air mass to compute aerodynamic forces.

Maneuver Capability

In order to determine the range capability of the prediction guidance system a variety of trajectories were simulated at
initial conditions varying from the ballistic overshoot boundary to the lifting undershoot boundary. These simulations began at 300,000 ft to avoid the analog computer difficulties of handling the wide range of density which would be required otherwise. However, all reference to re-entry angles in this section refers to the value at 400,000 ft in order to be consistent with usage elsewhere in the study. The corresponding angle at 300,000 ft for the various corridor boundaries is, of course, different. It may be noted that although the target is on the ground the trajectory ends at 50,000 ft. This is to simulate the opening of the parachute at that altitude. The vehicle in effect is aimed at 50,000 ft over the target. An appropriate offset in aiming point would be used to account for the residual horizontal component of velocity at that point.

Figs. 9 and 10 show typical trajectories with targets displaced from the orbital plane and re-entry from the lifting undershoot boundary and the ballistic overshoot boundary. In these examples cross range maneuvering did not begin until the minimum flight path angle was reached, when range control is initiated.

Figure 11 shows the maximum maneuver capability for re-entry on each of the corridor boundaries. The shaded area, which is the overlap between these two maneuver diagrams, is the locus of possible impact points for re-entry at any flight path angle between the boundaries. For comparison the capabilities with a W/C_{PA} of 50 and 100 are shown. The vehicle used in this study has a W/C_{PA} = 100.

Fig. 12 shows the maneuver capability of the vehicle when it is 100 and 200 miles from the target. This gives an indication of the maximum inertial guidance errors which can be tolerated at those ranges if a terminal radio guidance system is employed. It will be noted that optimized control, described earlier, has tended to center the target within the maneuver capability, giving a more nearly equal uprange and downrange capability than would be possible on a ballistic trajectory. On the other hand the total maneuver capability has been reduced, as can be seen by comparison with the terminal maneuver capabilities using the apparent target scheme. That is, the optimized control has permitted greater control over errors early in the trajectory at the expense of large maneuver capability at the end.

Effects Of Errors On Prediction System

The effect of a number of errors was considered with the
prediction system to determine their influence on performance. These were errors in range prediction, altitude errors, errors in flight path angle, density errors, and frequency of predictions.

1) Range Prediction Errors
The range computed by the predictor was intentionally increased (or decreased) by a constant percentage before being sent to the controller. That is, the controller used a predicted range which was 100 miles in error when it was 1000 miles from the target and in which the error was 10 miles when the vehicle was 100 miles from the target, in the case of a 10% error in range.

Fig. 13 shows the effect of a 10% prediction error for ranges from 1000 to 2500 miles, with and without optimized commands. The dramatic decrease in miss distance resulting from the use of the optimized commands is apparent. The explanation is simply that the optimized control technique causes the vehicle to overcontrol in such a way that it has a margin of maneuver capability available with which to overcome unpredictable effects such as the predictor range error.

2) Altitude Error
In the prediction guidance system an altitude error from the inertial navigator can have two effects. It can cause the predictor to use the wrong value of density in the trajectory solution, since the density value is obtained by a knowledge of altitude. This is discussed further in the section on density errors, and the method for correcting density errors was described earlier. The second effect is to cause the predictor to end the predicted trajectory at the wrong altitude, thus in effect aiming for the target at the proper range but at the wrong altitude. This in turn causes the vehicle to arrive over the target but at the wrong altitude, so that when it reaches the proper altitude it is no longer over the target. This effect is shown on Fig. 14, in which the nominal target altitude is 50,000 ft. These are the miss distances which would result if the parachute opens at 50,000 ft, say by a pressure sensing device followed by vertical descent. If, on the other hand, the parachute is opened when the inertial navigator indicates 50,000 ft, the error shown here will be zero. The guidance errors of the inertial navigator will, of course, still exist, and the parachute will be deployed at the wrong altitude.

3) Flight Path Angle Error
The effect of an error in flight path angle furnished
to the predictor was investigated. An error of 0.1° was found to have no effect on accuracy but did cause an error of 10,000 ft in setting the upper altitude limit on those trajectories which go to the maximum altitude. The effect of an error in flight path angle on maximum loads at pullup was discussed earlier.

4) Density Errors
The effect of an error in density causes the predictor to use the wrong value of density in the solution of the trajectory equations. Density errors can arise by use of the wrong altitude or if the actual density is different from that corresponding to the standard atmosphere, since the predictor uses an exponential approximation to the standard atmosphere. An error in altitude of a constant magnitude causes a constant percentage error in density. Fig. 15 shows a trajectory re-entering on the lifting undershoot boundary with a standard density. Next it shows the trajectory in which the density has been reduced by a factor of two above 100,000 ft in the actual atmosphere (but not in the predictor). It is seen that the vehicle overshoots the target by 250 miles due to an erroneous prediction of the trajectory. In the section Lift, Drag, and Density Correction the method of correcting for density errors is described. When this method is used here, the third trajectory in Figure 15 results, showing that the density correction eliminates the miss distance error. Similar results were obtained when the density was increased by a factor of two and also when the magnitude of the density error was reduced as a function of altitude from a factor of two at re-entry to zero at the ground.

5) Frequency of Predictions
Simulation has shown that a prediction time of 10 sec is more than adequate for range control, and that reduction of the prediction time will not result in any significant improvement in accuracy. To avoid excessive oscillation of the load and altitude limits and to insure rapid convergence on the proper vehicle commands, a prediction time of between 1 and 4 sec is desirable. Prediction times of this magnitude are possible using a digital differential analyzer with errors in predicted range of less than 1%.

THE APPARENT TARGET GUIDANCE CONCEPT
The apparent target guidance scheme combines features of the tail chase and the nominal trajectory schemes to provide range control for a space vehicle re-entering Earth's atmosphere at supercircular velocities. As in the tail chase, the apparent target scheme points the vehicle velocity vector at
a target, in this case at an apparent target differing from the actual target in altitude only. The difference between the actual target and the apparent target allows the trajectory to be shaped for energy management purposes in a manner similar to the nominal trajectory scheme. A proportional steering signal is generated by digitally computing the vector cross product of the unit vehicle velocity vector and the line of sight vector from the vehicle to the apparent target. The cross product form of steering signal calculation was chosen to simplify the digital computation required.

The Apparent Target Altitude Table

An apparent target location is generated in the digital computer by adding to the location of the actual target an apparent target altitude. This altitude is obtained from a table stored in the computer as a function of slant range from the vehicle to the actual target. The shaping of this altitude table as a function of slant range allows the vehicle's energy to be controlled so that landings can be accomplished for a wide variation of target locations with respect to the re-entry point. Shaping of the table also allows the incorporation of a vertical descent on the target during the terminal phase of the re-entry maneuver. This may be desirable for parachute deployment or for the reduction of the effect of altitude errors in the computer. Shaping of the table also allows a reduction in the magnitude of the deceleration peak experienced during the range control portion of the re-entry.

The apparent target altitude table shown in Fig. 16 has been used for all of the trajectories discussed in this section. No attempt to optimize the table has been made, but satisfactory performance for all target locations using the same table for all conditions has been achieved. It should be noted that no range control is contemplated during the survival phase. Range control and the use of the apparent altitude table begin at the termination of the survival phase. The complete separation of survival phase and range control insures the safe re-entry of the vehicle, although the chosen landing site may not be reached, even in the event of a failure of the range control portion of the guidance scheme. It is possible to extend somewhat the maneuvering capability of the vehicle by performing some range control functions during the survival phase. This is not considered to be desirable at the present time, since more than adequate maneuvering capabilities have been achieved without making this compromise.
Guidance Signals

The essential function of the guidance computer during the range control phase is to provide guidance signals that will cause the vehicle to fly so that the vehicle velocity vector with respect to the actual target is pointed at the apparent target. The block diagram presented in Fig. 17 shows, in simplified form, how this function is accomplished. The inertial platform provides acceleration information which is combined with a computation of gravity. The integration of this total acceleration produces vehicle position and velocity information with respect to an inertial reference. Initial target location (on the surface of Earth) and a knowledge of Earth's rotational rate are used to calculate actual target position and velocity with respect to the inertial reference. The vehicle velocity vector \( \vec{V}_v \) is subtracted from the target velocity vector \( \vec{V}_T \) to obtain the velocity of the vehicle with respect to the target \( \vec{V}_{v-T} \). A similar operation provides the position vector \( \vec{R}_{TG0} \) of the vehicle with respect to the target. The magnitude of \( \vec{R}_{TG0} \) is used in a table lookup routine with linear interpolation to provide the apparent target altitude \( \text{ALT} \). The apparent target position vector is generated by adding the apparent target altitude to the actual target position vector. The vector from the vehicle to the apparent target \( \vec{R}_{ls} \) is then computed and used to generate the cross product guidance signal. Fig. 18 illustrates the vector relationships just described.

The guidance signal is computed by vectorially crossing the unit velocity vector of the vehicle with respect to the target with the vector from the vehicle to the apparent target. The magnitude of the resultant vector is equal to the product of the range to the apparent target \( \vec{R}_{ls} \) and the sine of the angle between \( \vec{V}_{v-T} \) and \( \vec{R}_{ls} \). Thus, at a given range the guidance signal is approximately proportional to the angular deviation of the velocity vector from its desired orientation. When the vehicle is at long ranges from the apparent target, small deviations of the velocity vector from the line of sight are more significant than they are at the shorter ranges. This effect is compensated for by allowing the guidance loop gain to vary by the magnitude of \( \vec{R}_{ls} \).

The vector form for the generation of guidance signals was chosen for its suitability to digital computation. Addition, subtraction, multiplication, division, the square root operation, and table lookup are the only operations required for the generation of the guidance signal. The minimum number of calculations required and the elimination of trigonometric
operations should simplify the design, reduce the complexity and increase the reliability of the computer.

The resolution of the guidance signal into the appropriate steering command is accomplished using a transformation matrix whose elements are obtained from the platform gimbal angle pickoffs. The guidance signal vector is transformed into the vehicle coordinate frame and is then resolved into two components (one along the vehicle pitch axis, the other along the vehicle yaw axis). The third component is neglected since the roll axis of the vehicle will be relatively well aligned with the vehicle velocity vector. The angle of attack command is proportional to the component lying along the vehicle pitch axis, whereas the roll command is obtained by taking the arctangent of the pitch component divided by the yaw component.

Maneuvering Capability

The feasibility of the apparent target guidance scheme was investigated using a digital program written for the IBM-704. The program is capable of simulating guided re-entry into the atmosphere of a rotating, oblate Earth. For the purposes of this study, however, most trajectories were run on a spherical, nonrotating Earth. A drag polar, a first-order lag in angle of attack response, and a limit on maximum angle of attack were used to approximate the vehicle characteristics. All of the trajectories were run using the conditions at the end of the survival phase as initial conditions.

The region of attainable landing sites for all re-entry conditions is presented in Fig. 19. This figure is presented to indicate that a satisfactory maneuvering capability can be obtained. It is anticipated that an increase in the maneuver capability will result as the apparent target altitude table is refined. In all cases, maximum loads were within the specified limits and miss distances were negligible. Typical trajectories are shown in Figs. 20-23.

System Errors

The effect on the guidance scheme of altitude and density errors was investigated. Density errors did not appear to affect significantly the performance of the guidance scheme. Trajectories were run with the density doubled and reduced by a factor of 10 at 400,000 ft. These initial density errors were decreased linearly with altitude blending into the standard atmosphere at 100,000 ft. Although a significant change in the altitude at the initiation of range control occurred, system performance was not affected since the vehicle still had
approximately the same energy at the beginning of the range control phase.

The effect of altitude errors is more serious, however. The exact effect of altitude errors was not simulated but an approximate simulation was performed as follows. A trajectory was run with no errors in altitude information. The acceleration time history from this run was used to calculate the error in the computer's knowledge of altitude as a function of time. This altitude error time history was then used in the trajectory program to modify the altitude used in the computation of the steering signal.

Two cases were run for the lifting undershoot boundary. In the first case the error caused the steering signal computation to be made at an altitude lower than the vehicle's actual altitude. This caused the vehicle to fly higher than it would with the correct altitude information. The vehicle overshot the target by 5.0 miles and hit a maximum load of 16.5 g. The second case involved an altitude error of opposite sign and resulted in the vehicle falling short of the target by 22.9 miles. The maximum load was only 5.5 g, however.

Terminal Maneuvering Capability

Position and altitude errors resulting from initial condition errors and platform drift build up in an approximately exponential fashion as a function of time. The vehicle, however, travels a large fraction of the total range in about half the total time of flight. The last 300 to 100 miles takes a relatively long time, since the vehicle has been considerably slowed down by its passage through the denser parts of the atmosphere. The errors in altitude and position, therefore, build up disproportionately fast over the last 10% of the total range.

A ground based tracking and telemetry facility could provide updated position and velocity information to the vehicle which, if its maneuvering capability were sufficient, could then substantially reduce the errors that had accumulated. With this possibility in mind, the maneuvering capability of the vehicle was determined at 100 and 300 miles slant range from the target.

Lifting undershoot trajectories were used to determine the maneuver capability, since these result in the minimum maneuverability at a specified range from the target. Fig. 24
presents a typical example of the results and shows the maneuver capability remaining at ranges to go of 100 and 300 miles. It will be noted that substantial maneuvering capability is available even for only a 100 mile range to go.

Platform Error Analysis

An inertial platform error analysis program was written for the Philco 2000. The program integrates the actual three dimensional vehicle trajectory based on a set of initial conditions in position and velocity and a time history of three inertial components of acceleration. Position and velocity errors and initial platform misalignments are combined with the actual initial condition information to provide initial conditions for the computed trajectory. The acceleration time history modified by the instrument errors is then used to simultaneously compute the vehicle trajectory as seen by the computer. The two trajectories are compared at some specified time (for the cases presented, the time corresponds to passing through 50,000 ft altitude on the actual trajectory).

The types of errors that can be handled by the program include initial condition errors in three components of position and velocity, platform misalignments about three axes, and instrument errors such as gyro fixed drift, mass unbalance along either spin or input axes, or anisoeastic drift and accelerometer bias, scale factor, and anisoeastic errors. The initial condition errors can be handled either singly or in combination, thus allowing them to be considered as either dependent or independent. The instrument errors are considered to be independent of other instrument errors and therefore cannot be run in combination, but they can be run in combination with initial condition errors if such a dependency exists.

Six initial condition errors and six instrument errors were investigated (see Table 1). The range error was assumed to have a dependent velocity error since separate calculations aboard the vehicle provide knowledge of the vehicle velocity with respect to the local vertical. Therefore, an uncertainty in position (and a resultant uncertainty in the local vertical) produces a dependent uncertainty in velocity. A similar effect in track (normal to the range direction) produces a track velocity error. The six independent initial condition errors and the six instrument errors were run and the results are presented in Table 2. The resultant errors are those attained at 50,000 ft altitude (corresponding to
parachute deployment) on the nominal 2500 mile trajectory. The root sum square values for these errors are presented at the bottom of Table 2.

REFERENCES


8 Wingrove, R. C., and Coate, R. E., "Piloted simulator tests of a guidance system which can continuously predict landing point of a low L/D vehicle during atmosphere re-entry," NASA TN 787, March 1961.


GUIDANCE AND CONTROL


Table 1  Initial conditions for platform error analysis

<table>
<thead>
<tr>
<th>Run no.</th>
<th>Error source</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gyro fixed drift</td>
<td>3.3 mru</td>
</tr>
<tr>
<td>2</td>
<td>Gyro mass unbalance drift</td>
<td>-6.6 mru/g</td>
</tr>
<tr>
<td></td>
<td>along input axis</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gyro mass unbalance drift</td>
<td>6.6 mru/g</td>
</tr>
<tr>
<td></td>
<td>along spin axis</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gyro anisoelasticity</td>
<td>1.33 mru/g^2</td>
</tr>
<tr>
<td>5</td>
<td>Accelerometer bias</td>
<td>1.0 x 10^{-4}g</td>
</tr>
<tr>
<td>6</td>
<td>Accelerometer scale factor</td>
<td>1.0 x 10^{-4} g/g</td>
</tr>
<tr>
<td>7</td>
<td>Altitude error</td>
<td>-608.0 ft</td>
</tr>
<tr>
<td>8</td>
<td>Range error</td>
<td>-36181.2 ft</td>
</tr>
<tr>
<td></td>
<td>Altitude velocity error</td>
<td>63.6 fps</td>
</tr>
<tr>
<td></td>
<td>Range velocity error</td>
<td>8.8 fps</td>
</tr>
<tr>
<td>9</td>
<td>Track error</td>
<td>-36181.2 ft</td>
</tr>
<tr>
<td></td>
<td>Track velocity error</td>
<td>8.7 fps</td>
</tr>
<tr>
<td>10</td>
<td>Altitude velocity error</td>
<td>-12.8 fps</td>
</tr>
<tr>
<td>11</td>
<td>Range velocity error</td>
<td>-12.8 fps</td>
</tr>
<tr>
<td>12</td>
<td>Track velocity error</td>
<td>-12.8 fps</td>
</tr>
</tbody>
</table>

291


<table>
<thead>
<tr>
<th>Run no.</th>
<th>Alt., ft</th>
<th>Range, ft</th>
<th>Track, ft</th>
<th>Alt. velocity, fps</th>
<th>Range velocity, fps</th>
<th>Track velocity, fps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-749</td>
<td>35</td>
<td>13</td>
<td>-4.2</td>
<td>-0.8</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-263</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2142</td>
<td>-243</td>
<td>692</td>
<td>-9.8</td>
<td>-1.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>487</td>
<td>0</td>
<td>0</td>
<td>-0.1</td>
</tr>
<tr>
<td>5</td>
<td>135</td>
<td>1317</td>
<td>-1092</td>
<td>0.4</td>
<td>-3.1</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>295</td>
<td>-1192</td>
<td>-51</td>
<td>0.1</td>
<td>3.1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>-1033</td>
<td>-550</td>
<td>-51</td>
<td>-1.7</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>27388</td>
<td>40763</td>
<td>-51</td>
<td>85.7</td>
<td>3.2</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>-35</td>
<td>-141</td>
<td>14771</td>
<td>0.3</td>
<td>-0.2</td>
<td>-1.4</td>
</tr>
<tr>
<td>10</td>
<td>-10737</td>
<td>-7084</td>
<td>-51</td>
<td>-21.9</td>
<td>7.6</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>-4764</td>
<td>7532</td>
<td>-51</td>
<td>-8.9</td>
<td>-5.8</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>8516</td>
<td>0</td>
<td>0</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Root sum square values

29907  42096  17109  28.3  11.1  1.6
Fig. 1 Load vs. re-entry angle

$V_0 = 36,800\text{ FPS}$
$h_0 = 400,000\text{ FT}$

$\text{L/D} = 0, 0.1, 0.2, 0.3, 0.4, 0.5$

RE-ENTRY ANGLE, $\gamma$ (degrees)

LOADS (lbs)

13 12 10 8 6 4 2 0.5 0.2 0.1 0.08 0.06 0.04 0.02 0 0.02 0.04 0.06 0.08 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Fig. 2 Pullup altitude vs. re-entry angle

V_0 = 36,800 FPS
h_o = 400,000 FT
Fig. 3 Pullup velocity vs. re-entry angle
Fig. 4 Range from re-entry to pullup vs. re-entry angle
Fig. 5 Maximum heating rates vs. re-entry angle
Fig. 6 Total integrated heating at pullup vs. re-entry angle
Fig. 7 Block diagram of prediction guidance system
Fig. 8 Prediction guidance system - autopilot commands for optimized control

Fig. 9 Prediction guided trajectories from 10 g lifting undershoot boundary
Fig. 10  Prediction guided trajectories from overshoot boundary

Fig. 11  Maneuver capability using prediction guidance scheme
Fig. 12 Terminal maneuver capability using prediction guidance scheme

Fig. 13 Envelope of maximum miss distance from 10% error in range prediction
Fig. 14 Prediction guidance miss distance resulting from altitude error
Fig. 15 Effect of halving density above 100,000 ft

Fig. 16 Apparent target altitude vs. range to go
Fig. 17 Block diagram of apparent target guidance scheme

Fig. 18 Vectors used in apparent target guidance scheme
Fig. 19 Maneuver capability using apparent target guidance scheme

Fig. 20 Apparent target guidance scheme - altitude vs. range
Fig. 21  Apparent target guidance scheme - total integrated turbulent heating

Fig. 22  Apparent target guidance scheme - total integrated laminar heating
Fig. 23 Apparent target guidance scheme - load vs. time

Fig. 24 Terminal maneuver capabilities using apparent target guidance scheme