DAMPING AN INERTIAL NAVIGATION SYSTEM

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ABSTRACT

When an inertial navigation system designed for a freely maneuverable vehicle is excited by the random drift of its gyros, error oscillations develop which tend to build up with time. The system equations can be modified so that the root-mean-square values of the errors approach limiting values; however, such a modification invariably introduces an undesired response of the system to vehicle motion. It can be shown that the performance of an inertial navigation system may be expressed so that the responses of the vertical errors, and of the azimuth error, to gyro drift display a strong analogy to the closed loop response function of a simple servomechanism. By pursuing this analogy, using standard techniques of servomechanism theory, the ultimate performance limitations on the damped inertial navigation system can be displayed and specific damping equalizers designed.

INTRODUCTION

The errors of an inertial navigation system arise from the various component imperfections, from the theoretical approximations that have been built into the system, and from inaccuracies in the setting of initial conditions. The most serious source of navigation system errors is gyro drift.

Analysis shows that constant gyro drift, or an impulse of gyro drift, will cause sustained oscillations. To the accu-

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racy that constant or other predictable gyro drift can be measured, it can effectively be compensated. There remains a random, in general noncompensable, component of gyro drift that will act on the system like a succession of small impulses. The effect of the random drift is cumulative, and the resulting error oscillations will tend to grow with time. It can be shown that the root-mean-square (rms) value of the error oscillations increases as the square root of time.

If a single impulse of gyro drift produced a damped error response instead of producing a sustained sinusoidal response, the effect of a train of impulses would not tend to accumulate, since the effect of each member of the train would disappear with time. It is seen, therefore, that from the standpoint of the effect of gyro drift on a system, it would be highly desirable that the response to a single impulse be a damped function; this situation that would be realized if the poles of the system response functions had negative real parts instead of being pure imaginary quantities.

Therefore, the possibility might be considered of modifying the navigation system equations so that the poles of the system response functions all acquire negative real parts. It will develop that such a modification is possible. In this paper, the methods by which the modified set of system equations can be obtained, and the advantages and disadvantages of using such a set of equations, will be explored.

It is immediately apparent that if the design of an inertial navigation system is modified in any respect that requires a change in the ideal specific force equations, the system will develop errors that depend on vehicle acceleration. For an aircraft, the effect of the high accelerations that may occur could be so severe that a damped system would not be useful. For slow speed vehicles, however, the errors introduced in a damped system by changes of motion are small, and it is profitable to consider the creation of a favorable balance between such errors and the reduction in the errors excited by random gyro drift.

If externally derived velocity information is available, a compensating modification is possible. It is known that velocity information, properly injected into a damped inertial navigation system, can return the system to a state in which errors are not excited by vehicle accelerations and in which the desired damping effect on gyro drift-excited errors is still achieved. A serious objection to such a procedure is that the accuracy of the external velocity information must rival that produced by the navigation system itself. There is, therefore, implied as an element in such a hybrid system, a completely redundant system whose only function is to provide compensation for the dynamic effects of damping. Use of such a system may be considered questionable. In the following discussion, however, the method of introducing externally measured velocity into an inertial navigation system will be considered.

THE EFFECT OF A RANDOM INPUT ON A LINEAR OSCILLATOR

The case of a linear oscillator excited by a random input function is considered first. This example will prove directly applicable to the navigation system problem.

The system to be examined has one input and one output and can be represented as a transfer function as shown in Fig. 1. It is supposed that the switch shown in Fig. 1 is closed at time $t = 0$, and that the input to the system is a random function having a zero mean and a constant rms value. The output of the system will be an oscillatory random function, and the peak amplitudes will tend to grow with time. Since the output is zero, up to the instant the switch is closed, and grows with time thereafter, it is not a member of a stationary random process and it is not possible to determine an rms value by averaging with respect to time. An rms value can be defined, however, by considering an ensemble of linear oscillators, all energized at $t = 0$ by stationary random input functions having the same statistics. The rms value is defined by taking an average over the entire ensemble at a given time.

Using the properties of the input process, and of the oscillator, it is possible to calculate the rms value of the output process, as a function of time, by using the equation

$$\overline{e^2(t)} = \int_0^t \int_0^t \phi(x - y) f(t - x) f(t - y) \, dx \, dy \qquad [1]$$

where $\overline{e^2(t)}$ is the mean-square error, $\phi(\tau)$ is the autocorrelation function of the input process, and f(t) is the response of the system to a unit impulse (1).[2] Eq. 1 holds for general systems and is not limited to the oscillator under discussion.

For the linear oscillator shown in Fig. 1, the response to a unit impulse function is simply

$$f(t) = \sin \omega t \qquad [2]$$

Valid use of Eq. 1 to predict an average error now requires that the function $\phi(\tau)$ be the autocorrelation function of gyro drift and that it be suitably obtained from experimental data. For the immediate purpose of the discussion, which is to observe the general effect of damping on the rms errors, and to compare the effectiveness of various damping modes, it will be assumed that the system input is white noise. This is equivalent to assuming that the input process has a flat average spectrum over the system pass band. The assumption introduces an analytical simplification, and will not affect the design of the equalizers to be developed later in the paper, since they are not designed with an explicit dependence on the statistics of gyro noise.

Thus

$$\phi(\tau) = \pi P \delta(\tau) \qquad [3]$$

the autocorrelation function of white noise. The function $\delta(\tau)$ is the unit impulse function, and P is the average power per unit bandwidth. The autocorrelation function is the Fourier transform of the power spectral density, and the factor π in Eq. 3 arises from the convention employed in writing the Fourier integral theorem as a set of direct and inverse transforms.

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[2]Numbers in parentheses indicate References at end of paper.

With Eq. 3, the mean-square value of the output becomes

$$\overline{e^2(t)} = \pi P \int_0^t \int_0^t \delta(x - y) f(t - x) f(t - y)\, dx\, dy$$

$$= \pi P \int_0^t f^2(t - y)\, dy \qquad [4]$$

Substituting Eq. 2 in Eq. 4, the rms value is obtained as

$$\sqrt{\overline{e^2(t)}} = \sqrt{\frac{\pi P}{2\omega}\left(\omega t + \frac{1}{2\omega} \sin 2\omega t\right)} \qquad [5]$$

A plot of the function

$$\sqrt{\omega t + (1/2\omega) \sin 2\omega t}$$

is shown in Fig. 2. It is apparent from Eq. 5 and from the figure that $\sqrt{\overline{e^2(t)}}$ increases as the square root of time.

The rms output of a damped linear oscillator having a transfer function

$$F(p) = \frac{\omega}{p^2 + 2\zeta p\omega + \omega^2} \qquad [6]$$

is now considered. The impulse response is the damped function

$$f(t) = \frac{\omega}{\beta} e^{-\alpha t} \sin \beta t \qquad [7]$$

where

$$\alpha = \zeta\omega$$

and

$$\beta = \omega\sqrt{1-\zeta^2}$$

If $\sqrt{\overline{e^2(t)}}$ is calculated for the damped system, as before, it is found that a steady value is approached. If

$$\alpha/\beta = n$$

then the mean-square value is

$$\overline{\epsilon^2(t)} = \frac{\pi P}{4\omega\sqrt{1-\zeta^2}}\left[\left(n+\frac{1}{n}\right)\left(1-e^{-2\zeta\omega t}\right)\right.$$
$$\left. + e^{-2\zeta\omega t}\left(n\cos 2\sqrt{1-\zeta^2}\,\omega t - \sin 2\sqrt{1-\zeta^2}\,\omega t\right) - n\right] \quad [8]$$

A plot of $\sqrt{\overline{\epsilon^2(t)}}$ (neglecting the factor $\sqrt{\pi P/2\omega}$ as before) is shown in Fig. 3 for various values of the damping constant ζ. It is noted that after a time equal to three natural periods of the oscillator, a damping constant equal to 0.1 will reduce the average error to about half its value for an undamped system. A damping constant of 0.5 reduces the average error to about one quarter of the value for an undamped system.

Fig. 4 shows a similar set of average errors for the system whose transfer function is

$$F(p) = \frac{p}{p^2 + 2\zeta\omega p + \omega^2} \quad [9]$$

The general behavior of the curves is similar to that obtained for the system of Fig. 1, and the final values are the same for the same values of the damping constant.

SIMPLIFIED INERTIAL NAVIGATION SYSTEM

A highly simplified model of an inertial navigation system is now considered. The system is assumed to be constrained to move only over a meridian. It has a single axis, normal to the plane of the meridian, and is designed to indicate the vertical and compute latitude. The case of a three-axis system will be treated in a later section; it should be noted that the behavior of the single-axis system described here is the same as the behavior of a three-axis system about its north pointing axis when the navigation system is located at the equator.

A block diagram of the system showing the signal flow and the effect of an error in vertical indication as a feedback signal affecting the accelerometer output is shown in Fig. 5. The establishment of the block diagram of Fig. 5 is treated in Ref. 2 (pp. 9-12). It is noted that the figure contains a block representing an as yet undetermined transfer function H(p). Writing the vertical error β as a function of the north velocity ω_n and the gyro drift U, the result is

$$\beta = \frac{p\omega_n}{g/R}(1 - H)\frac{\frac{g/R}{p^2}}{1 + H\frac{g/R}{p^2}} + \frac{pU}{g/R}\frac{\frac{g/R}{p^2}}{1 + H\frac{g/R}{p^2}} \qquad [10]$$

For an unmodified system the transfer function H(p) is equal to 1. In this case the first term on the right-hand side of the equation disappears, and the error angle β becomes

$$\beta = U\frac{p}{p^2 + g/R} \qquad [11]$$

which is analogous to the case of the linear oscillator considered in the previous section.

By suitably selecting the function H, the response of the system to gyro drift can be damped. If H is different from 1, however, the nonvanishing factor 1 - H in the first term shows that vehicle motion will contribute a component to the error β.

It is desired to make the function

$$\frac{\frac{g/R}{p^2}}{1 + H\,\frac{g/R}{p^2}}$$

be equivalent to the response function of a damped oscillator. If this function is written in the form

$$\frac{1}{H}\;\frac{H\,\frac{g/R}{p^2}}{1 + H\,\frac{g/R}{p^2}}$$

it is apparent, since the second factor has the appearance of a single loop servomechanism response function

$$Y/(1 + Y)$$

that conventional servomechanism synthesis techniques may be employed to develop an equalizer transfer function H. The function

$$G = \frac{g/R}{p^2}$$

may be conveniently represented graphically by making the substitution

$$p = i\omega$$

expressing the result in the form

$$G(i\omega) = |G(\omega)|\, e^{i\,\theta(\omega)}$$

and then plotting the gain function

$$M = 20 \log |G(\omega)|$$

and the phase shift function

$$\theta = \theta(\omega)$$

against the logarithm of ω (see Ref. 3 or any text on servomechanism theory).

The gain of $G(i\omega)$ will be plotted as a line having a slope of 12 db per octave, crossing the zero db line at the angular frequency

$$\omega_s = \sqrt{g/R}$$

The phase shift will be a constant negative angle of 180° over the entire frequency range.

Elementary servomechanism theory indicates that this is a situation in which the system is at the boundary of stable behavior, and that a disturbance will cause the system to have sustained oscillations.

The response can be improved by selecting the design of the cascaded equalizer H so that the gain of the function

$$H(i\omega)\, G(i\omega)$$

crosses the zero db line before a condition of 180° phase shift is reached.

It is apparent that the function H need be different from 1 only in the neighborhood of the frequency $\sqrt{g/R}$. The desirability of maintaining the term $1 - H$ as small as possible shows that the function H should be different from 1 over as narrow a frequency band as possible.

It is supposed that H is required to be a lead function

$$H = \frac{\omega_1}{\omega_o} \frac{p + \omega_o}{p + \omega_1} \qquad \omega_o < \omega_1 \qquad [12]$$

having a value of one at low frequencies. Fig. 6 illustrates:

1) the undamped navigation system response curves; 2) the frequency response of H for an appropriate selection of the constants ω_o and ω_1; and 3) the frequency response of the combination HG.

The phase shift curve is -180° at low frequencies and then becomes less than 180° in magnitude in the region where the gain curve crosses the zero db axis. According to fundamental servomechanism theory, the resulting closed loop system should be stable.

Fig. 7 shows the response function plotted as a curve on a gain vs. phase shift diagram. A diagram of this type is sometimes called a Nichols chart. The contour lines of the chart are lines of constant gain and phase of the closed loop response function. It is apparent from the figure that the modification provided by the function H allows the characteristic to bypass the critical (0 db, -180°) point on the gain-phase diagram.

The closed loop response curves representing the functions

$$HG/(1 + HG)$$

and

$$G/(1 + HG)$$

are shown in Fig. 8. The resultant closed loop response function has been adjusted to peak at 1 db.

Another possible equalizer transfer function H can be realized by simply bypassing the first integrator of the navigation system with a direct connection. For this case, the transfer function can be written

$$H = 2\zeta\sqrt{R/g}\, p + 1 \qquad [13]$$

Fig. 9 shows the gain and phase shift characteristics of this function. The necessary positive phase shift is obtained but unnecessary gain and positive phase shift is continued into the high frequency region. As might be suspected, this condition produces an undesired response to sudden vehicle motions.

Fig. 10 shows the gain and phase shift characteristics of several equalizer transfer functions. The functions corresponding to curves A and B have already been considered. The transfer function illustrated by curve C has a gain of zero db at high frequencies and at low frequencies and, according to the criterion that 1 - H should be as small as possible, would be expected to contribute less error because of vehicle motion.

Fig. 11 shows the result of a one-knot step in vehicle velocity on the vertical error for all three cases. The closed loop response function was adjusted to peak at 1 db in each case.

As mentioned, the function H that introduces excessive gain and phase shift at high frequencies has a response that is inferior to that resulting from use of the other two functions. Between the two functions shown in curves A and C of Fig. 10, there seems little choice. It may be concluded that the small gain contributed by the H function first considered has negligible effect because of the attenuation of the closed loop system transfer function that multiplies (1 - H) in Eq. 10.

THE AZIMUTH ERROR

To determine the behavior of the azimuth error of a damped inertial navigation system, it is assumed that the navigation system to be considered is the one illustrated in Fig. 12. This is a very general system whose behavior, in the undamped case, is discussed in Ref. 2 and 4. In the undamped case, the blocks shown dotted are not included. For the present, the blocks represent unknown transfer functions which, it is hoped, will produce the desired damped behavior of the system. To avoid introducing unnecessary complexity in this description, the equations will be developed for the simplified case of a system in a vehicle near the equator. The corresponding equations for a vehicle at any latitude can be developed by a similar but more detailed analysis. For simplicity, the 84-min oscillations will be considered undamped, and concern will be with only the more important

24 hr oscillations.

It can readily be shown that vertical errors about north and east, α and β, satisfy the following two equations

$$\begin{aligned} \delta \dot{\omega}_n &= (g/R)\ \beta \\ \delta \dot{\omega}_e &= -\ (g/R)\ \alpha \end{aligned} \qquad [14]$$

where $\delta\omega_n$ and $\delta\omega_e$ are the errors in north and east angular velocity computation. Also, if ω_x, ω_y, and ω_z are the angular rate components of a geographic coordinate system with origin at the vehicle, and if ω'_x, ω'_y, and ω'_z are the corresponding computed rates that are used to torque the three gyros, then the vertical errors and the azimuth error γ satisfy

$$\begin{aligned} \dot{\alpha} &= \omega'_x - \omega_x - \gamma\omega_y + \beta\omega_z + U_x \\ \dot{\beta} &= \omega'_y - \omega_y - \alpha\omega_z + \gamma\omega_x + U_y \\ \dot{\gamma} &= \omega'_z - \omega_z - \beta\omega_x + \alpha\omega_y + U_z \end{aligned} \qquad [15]$$

where U_x, U_y, and U_z are the drift rates of the x, y, and z gyros.

The last equation of this set is considered. For general vehicle motion, the term ω_z is given by

$$\omega_z = -\ \Omega \sin \lambda - \omega_e \tan \lambda \qquad [16]$$

where λ is the latitude.

The torquing signal ω'_z would, in the undamped case, be

$$\omega''_z = -\ \Omega \sin \lambda' - \omega'_e \tan \lambda' \qquad [17]$$

In the damped case, this expression is modified and the z gyro is torqued with a term

$$\omega_z' = -Y_2 (\Omega \sin \lambda') - \omega_e' \tan \lambda' \qquad [18]$$

The function Y_2 corresponds to one of the blocks indicated in Fig. 12. It should be noted that the second term of this expression is not modified. A positive or negative ω_e will change the resonant angular frequency of the system from the value Ω to some neighboring value, but it is assumed that the vehicle is slowly moving and that the resonant angular frequency is effectively dependent only on the first term.

If the differences are formed between computed and actual rates around z, and the computed angle λ' is replaced by $(\lambda + \delta\lambda)$ and the computed rate ω_e' by $(\omega_e + \delta\omega_e)$, then

$$\omega_z' - \omega_z = -Y_2\, \Omega \cos \lambda\, \delta\lambda + (1 - Y_2)\, \Omega \sin \lambda - \omega_e \sec^2 \lambda\, \delta\lambda - \delta\omega_e \tan \lambda \qquad [19]$$

Near the equator, the entire expression for $\dot{\gamma}$ becomes

$$\dot{\gamma} = -Y_2\, \Omega\, \delta\lambda - \delta\omega_e \lambda - \beta\, \Omega - a\omega_n + U_z + (1 - Y_2)\, \Omega\, \lambda \qquad [20]$$

The terms $\delta\omega_e\lambda$ and $a\omega_n$ may be neglected in comparison with the other terms, and the resulting equation is

$$\dot{\gamma} = -Y_2\, \Omega\, \delta\lambda - \beta\, \Omega + (1 - Y_2)\, \Omega\, \lambda + U_z \qquad [21]$$

If a similar procedure is applied to the β equation, another equation of the same type may be developed. The three equations

$$\delta\dot{\omega}_n = \frac{g}{R}\beta$$

$$\dot{\beta} = -Y_1\, \delta\omega_n + \Omega\, \gamma + U_y + (1 - Y_1)\, \omega_n \qquad [22]$$

$$\dot{\gamma} = -Y_2\, \Omega\, \delta\lambda - \beta\Omega + U_z + (1 - Y_2)\, \Omega\lambda$$

and the definition

$$\delta \dot{\lambda} = \delta \omega_n \qquad [23]$$

are seen to describe the behavior of the variables $\delta \omega_n$, λ, β, and γ, and the equations for $\delta \dot{\omega}_e$ and $\dot{a}$ may be handled independently.

If this set of equations is solved for γ in terms of the vehicle velocity ω_n, and the gyro drift rates U_x, U_y, and U_z, one obtains

$$\gamma = \frac{1}{\Omega^2} \left[- \frac{Y_3 \frac{\Omega^2}{p^2}}{1 + Y_3 \frac{\Omega^2}{p^2}} \Omega U_y + \frac{\frac{\Omega^2}{p^2}}{1 + Y_3 \frac{\Omega^2}{p^2}} p U_z + \frac{\frac{\Omega^2}{p^2}}{1 + Y_3 \frac{\Omega^2}{p^2}} \left(\frac{R}{g} p^2 + 1\right)(1 - Y_3) \Omega \omega_n \right] \qquad [24]$$

where

$$Y_3 = \frac{(R/g) p^2 + Y_2}{(R/g) p^2 + Y_1} \qquad [25]$$

Eq. 24 is directly analogous to the equation that was used to describe the vertical error in the single-axis case considered previously. Each disturbing influence is modified by a transfer function that is in the form of a closed loop response function, and can obviously be handled by the servo techniques that were applied earlier. The presence may be noted of the factor $1 - Y_3$ in the last term, which shows that when Y_3 is equal to 1, no error will result from vehicle motion.

When a satisfactory form has been found for Y_3, it is necessary to break it up into components Y_1 and Y_2 which are mechanized individually by the navigation system. The selection of Y_1 and Y_2 has been made on an arbitrary basis by letting Y_1 equal 1 and solving for Y_2. Since the way that Y_3 is broken up will not affect γ, the criterion to be used is the magnitude of the effect on the vertical errors. It can be shown that the arbitrary selection of Y_1 as 1 will give satisfactory results.

The synthesis procedure for Y_3 is similar to the one used to find the H function. Rather than repeat the discussion of the synthesis procedure, some of the results will be given for various values of an effective damping constant ζ. Experimentation shows that the shape of the characteristic Y_3 has little effect in changing the response to motion provided that the same effective damping constant is used and the closed loop response functions

$$\frac{Y_3\,(\Omega^2/p^2)}{1 + Y_3\,(\Omega^2/p^2)}$$

and

$$\frac{(\Omega^2/p^2)}{1 + Y_3\,(\Omega^2/p^2)}$$

have unity gain at low frequencies. An effective value of ζ corresponds to peak values of the gains of the two closed loop response functions. For example, $\zeta = 0.5$ corresponds to a peak gain of 1 db for the second of the two functions listed. The insensitivity of the system to modification by a Y_3 that has a high gain at frequencies above Ω, seems to result from the presence of an integration in the last term of Eq. 24 as compared with the last term Eq. 10. This condition differs from the situation that occurred in the case of vertical damping, where it was apparent that some advantage could be derived by careful shaping of the modifying transfer function.

If a vertical error equalizer function H is included Eq. 24 is unchanged. However Y_3 becomes

$$Y_3 = \frac{(R/g)\, p^2 + Y_2 H}{(R/g)\, p^2 + Y_1 H} \qquad [26]$$

If H is appropriately selected, it can be shown that the response of the system to motion along a meridian differs little from the case in which H is neglected.

Figs. 13, 14, and 15 show the response in azimuth error, under this assumption, for a step function input and various values of the damping constant.

If external velocity information is available, it may be introduced as shown in Fig. 16. If the external velocities $\overline{\omega}_n$ and $\overline{\omega}_e$ are precisely measured, the connections shown will eliminate the component of error resulting from vehicle motion. Except for the external inputs and the vertical error equalizers H, the diagram can be shown to be equivalent to that of Fig. 12.

The operation of a damped five-gimbal system can be given in completely analogous form with that of the three-gimbal system that has been considered.

An undamped five-gimbal system can be arranged which does not require torquing of the system gyros. The input axes of the gyros define an orthogonal system which is rotated so that it always maintains almost the same orientation with respect to the figure of the earth, but no gyro is rotated about its input axis. Torquing signals must be provided for damping, however, and Fig. 17 shows the damping signal resolution for a system in which one gyro input axis is parallel to the polar axis, another input axis points east, and the third axis points toward the polar axis, and is parallel to the equatorial plane. The diagram is equivalent to that of Fig. 16.

REFERENCES

1 Laning, J. H., Jr. and Battin, R. H., Random Processes in Automatic Control (McGraw-Hill Book Co., Inc., New York, 1956) Chap. 6.

2 Broxmeyer, C., "Analysis of an Inertial Navigation System," Rep. R-241, M.I.T. Instrumentation Laboratory, Cambridge, 1959.

3 Brown, G. S. and Campbell, D. P., Principles of Servomechanisms (John Wiley and Sons, Inc., New York, 1948) Chap. 8.

4 Broxmeyer, C., "Foucault Pendulum Effect in a Schuler-Tuned System," Journal of the Aero/Space Sciences, May, 1960.

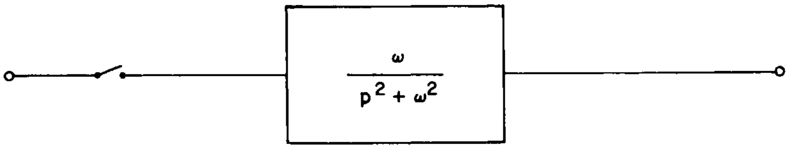

Fig. 1 A linear oscillator

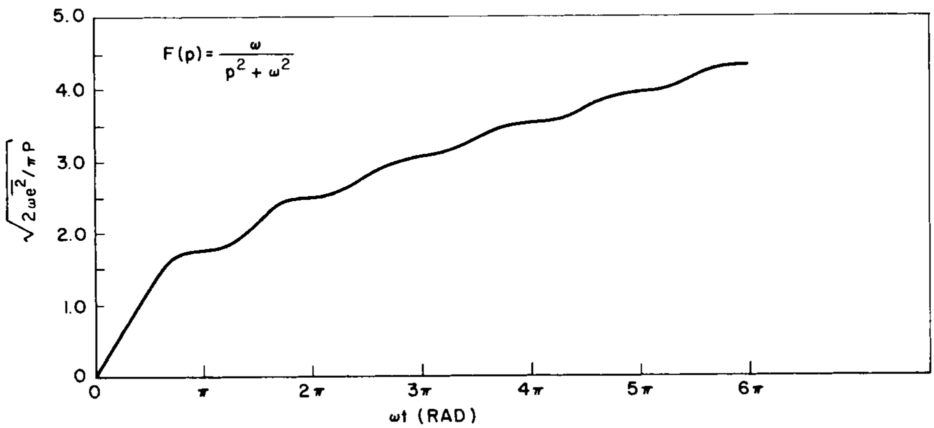

Fig. 2 RMS output of undamped linear oscillator excited by white noise

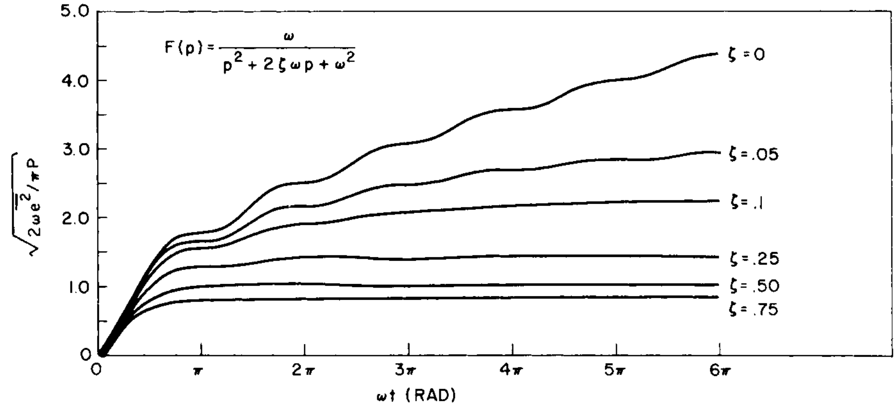

Fig. 3 RMS outputs of damped linear oscillators excited by white noise

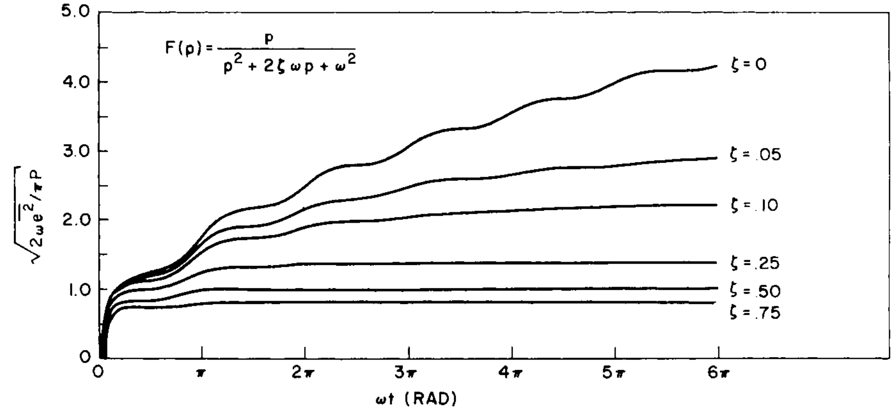

Fig. 4 RMS outputs of damped linear oscillators excited by white noise

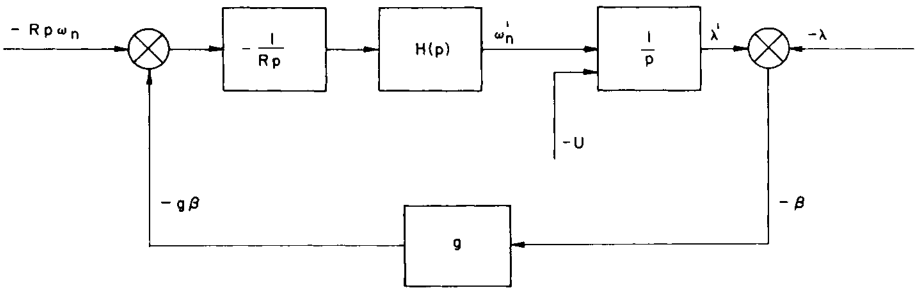

Fig. 5 Single loop system

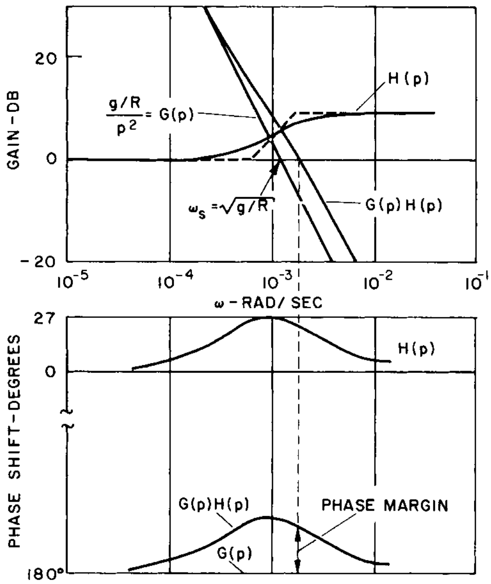

Fig. 6 Gain and phase shift vs. log angular frequency (open loop)

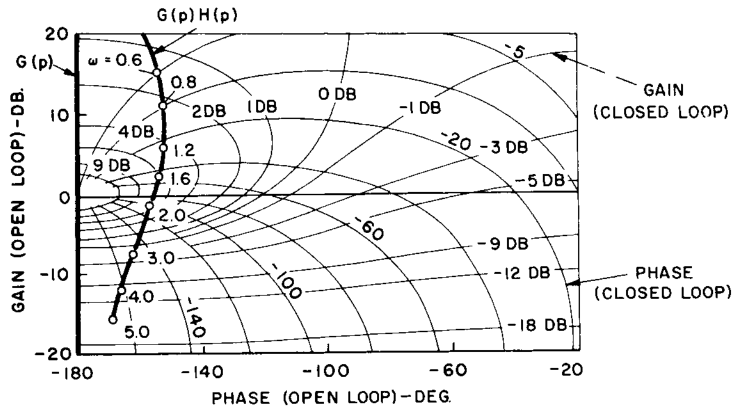

Fig. 7 Gain vs. phase diagram

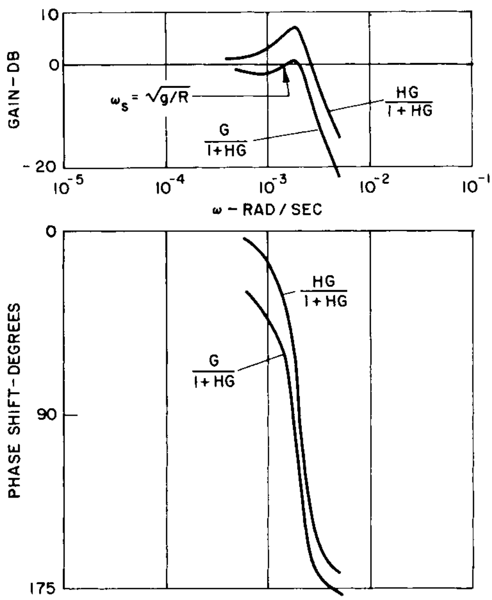

Fig. 8 Gain and phase shift vs. log angular frequency (closed loop)

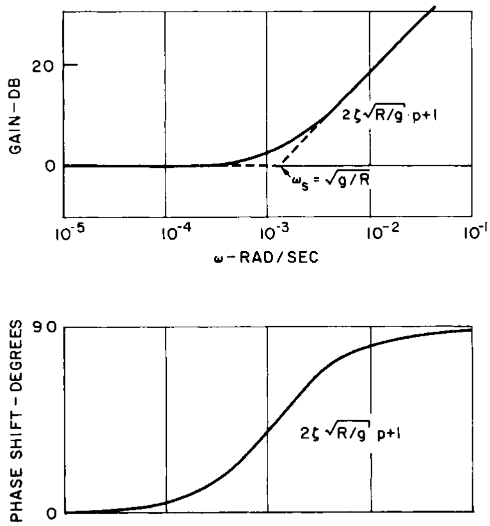

Fig. 9 Gain and phase shift vs. log angular frequency of simple lead equalizer

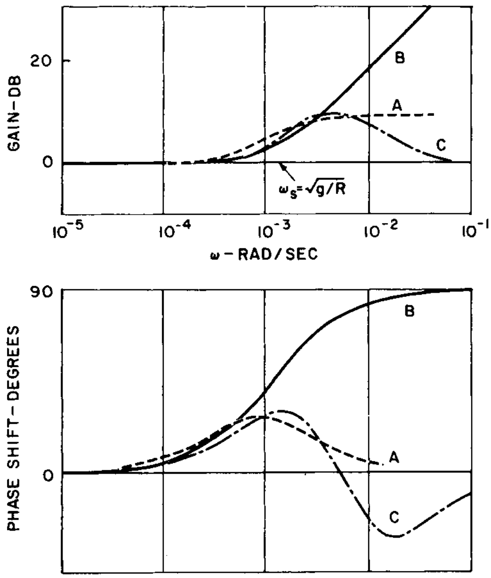

Fig. 10 Gain and phase shift vs. log angular frequency for three typical equalizers

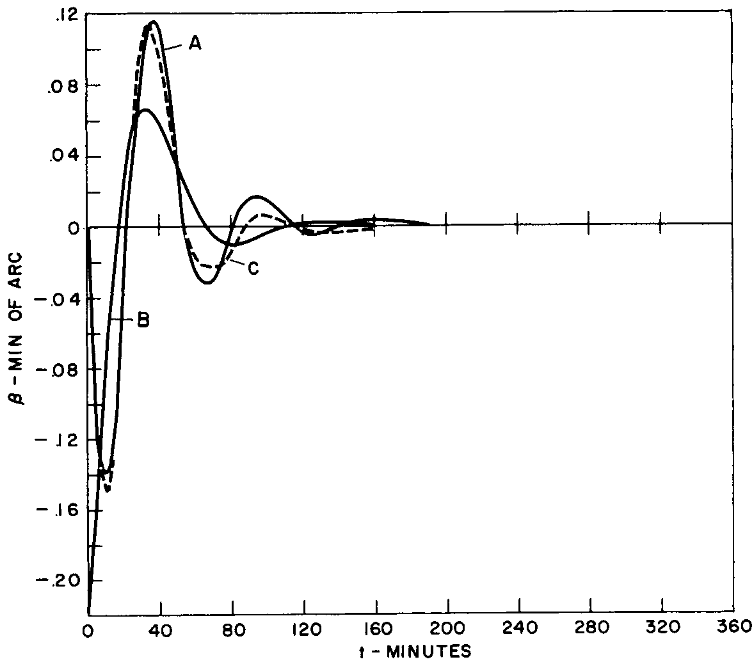

Fig. 11 Verticle error for one knot step input

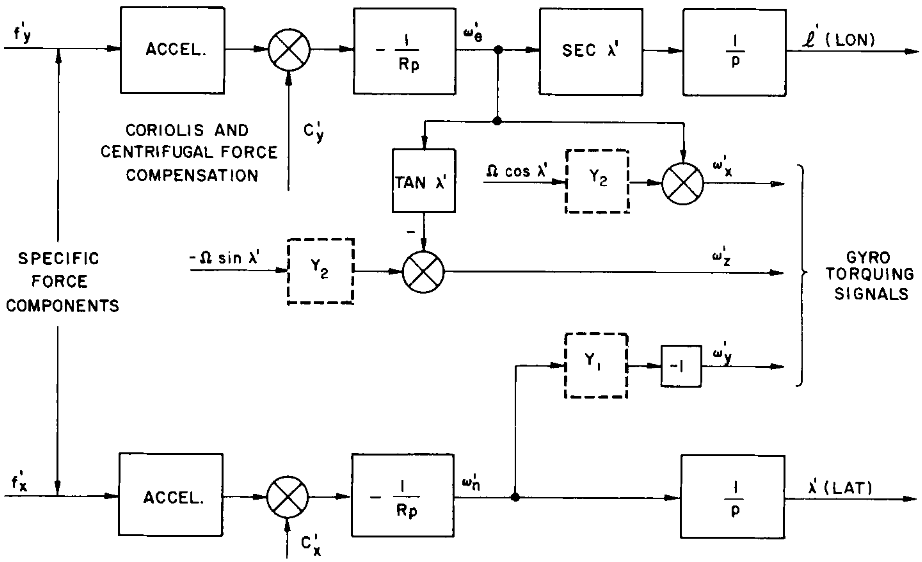

Fig. 12 Damped three-axis inertial navigation system (three gimbal)

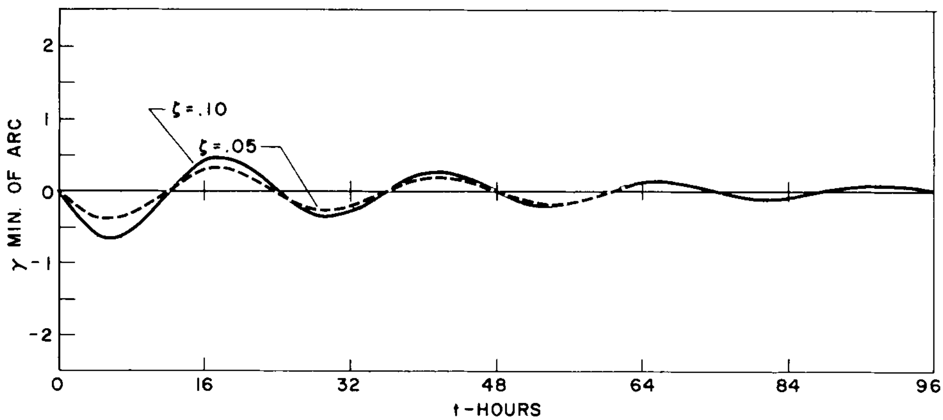

Fig. 13 Azimuth error for one knot step input

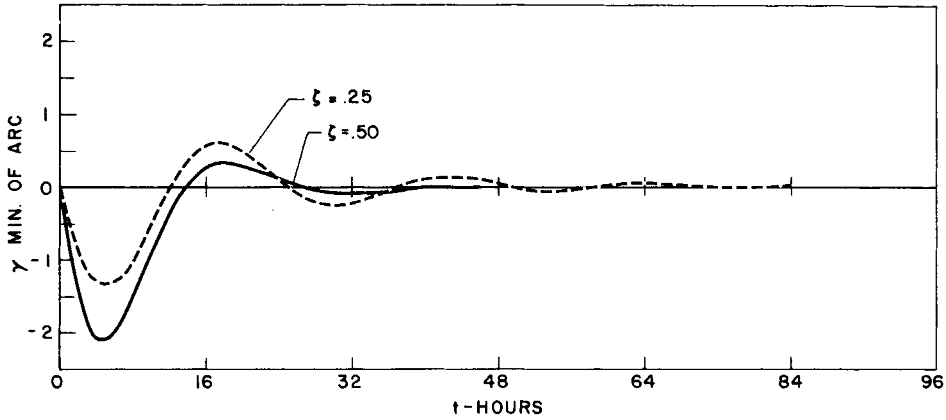

Fig. 14 Azimuth error for one knot step input

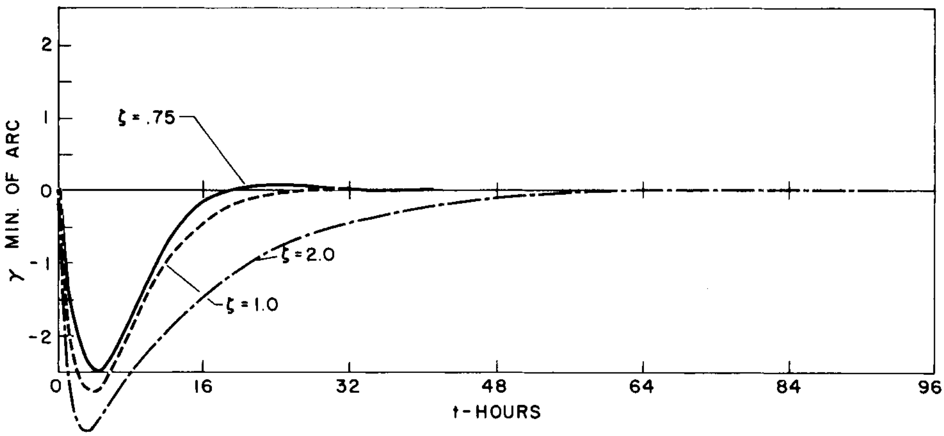

Fig. 15 Azimuth error for one knot step input

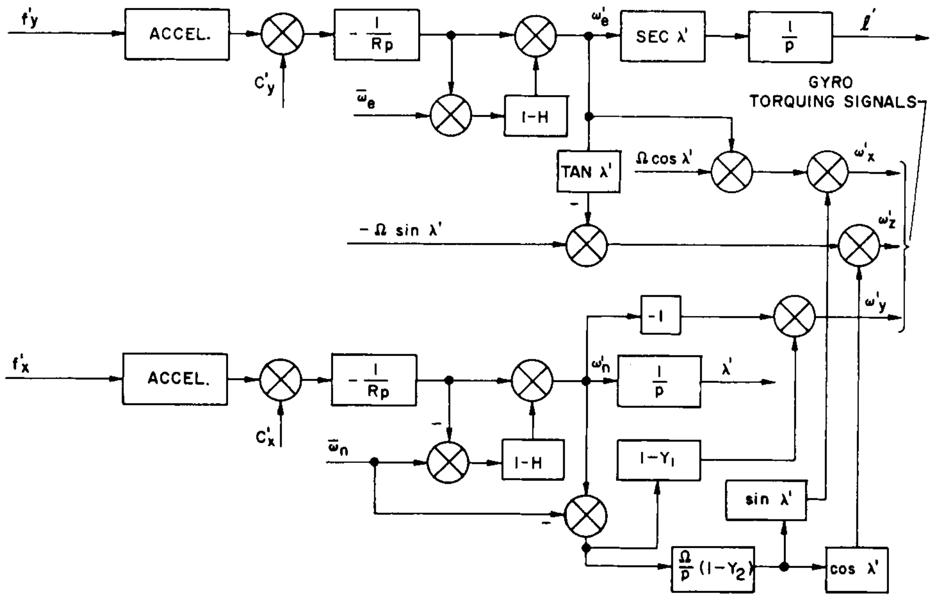

Fig. 16 Damped three-axis inertial navigation system with external velocity compensation

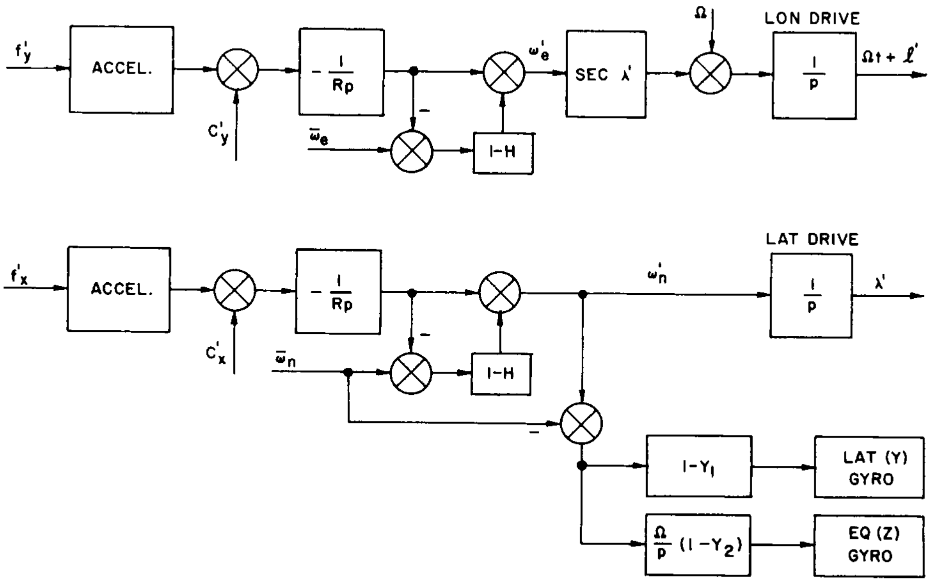

Fig. 17 Damped three-axis inertial navigation system (five gimbal)