

MULTIPLE INERTIAL SYSTEM OPERATION IN
LONG TERM NAVIGATION

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ABSTRACT

The use of multiple systems in long term inertial navigation is first discussed from the standpoint of reliability. It is then shown that multiple systems are advantageous for other than reliability considerations. In particular, the use of multiple systems in conjunction with platform rotation enables the continuous monitoring and rebiasing of all inertial instruments except the azimuth gyro. In this paper inertial navigation systems are considered which are capable of independent operation and mounted so that platform orientations can be compared. Further, the inertial navigators are mechanized for local level operation (1)³ but the techniques presented here can be applied to other mechanizations (2).

INTRODUCTION

Reliability and Multiple System Operation

In the case of long term operation of an inertial navigator, reliability of the system and its components is of prime importance.⁴ If the mean time to failure of a single system is

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⁴Long term operation implies continuous operation of the inertial navigator for time periods on the order of weeks or months.

less than the required operating period, it may become expedient to have information that can be derived from the operation of two or more inertial navigators (multiple system operation).

In this paper it is shown how the additional information derived from multiple systems can be used to improve the performance of a set of inertial navigators. This additional information is in the form of differences or divergences of the outputs of two or more inertial navigators. The primary differences considered are position differences (from the position counters) and the attitude differences between systems (as given by, say, gimbal angle differences). It is then shown that by use of these differences and operation of the inertial platforms at different heading angles, all constant inertial instrument errors can be computed and corrected for, without the aid of external position, velocity, or azimuth information.

Thus, because of the corrective and monitoring properties of multiple system operation, it is shown that the requirement, in long term navigation, for external reference information can be alleviated and the buildup of errors in the navigation system greatly reduced.

Discussion of Effects Due to Inertial Instrument Errors

In a discussion of the errors associated with long term inertial navigators, several coordinate frames of interest will be used, namely

T = true coordinate frame; a locally level set of axes erected at the actual position of the navigating vehicle

C = computer coordinate frame; a locally level frame erected at the position indicated by the inertial navigator position counters

P = platform coordinate frame; a coordinate frame fixed to the stabilized platform

Each of the frames **C** and **P** will, in general, be misaligned from the true coordinate frame **T** by some vector error angles defined by

$\overline{\delta\theta}$ = vector error angle between the computer and true axes

$\overline{\phi}$ = vector angle between the platform and true axes

$\overline{\psi}$ = vector angle between the platform and computer axes

The error angles $\overline{\delta\theta}$, $\overline{\phi}$, $\overline{\psi}$ are related by the equation

$$\overline{\phi} = \overline{\delta\theta} + \overline{\psi} \quad [1]$$

A complete description of the error propagation in long term inertial navigation will then reside in describing the relationship between the error angles $\overline{\delta\theta}$, $\overline{\phi}$, $\overline{\psi}$ and the inertial instrument errors.

If x , y , z now denote axes in the north, east, and vertical directions, respectively, then for operation in the damped inertial mode⁵ (Eqs A-11, Appendix).

$$\phi_x = \frac{\nabla y}{g} + 2\zeta \frac{\delta V_y}{a\omega_o} \quad [2a]$$

$$\phi_y = -\frac{\nabla x}{g} - 2\zeta \frac{\delta V_x}{a\omega_o} \quad [2b]$$

where

δV_y , δV_x = components of reference velocity errors

⁵Reference velocity for damping in marine applications is usually obtained by resolving ship's speed with respect to the water mass through the platform gimbals angles.

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ϕ_x, ϕ_y = level platform tilt angles

∇_x, ∇_y = north and east accelerometer errors,
respectively

g = local acceleration due to gravity

Eqs. 2 state that in steady state the inertial platform will tilt through the angle ϕ so that the component of force sensed will exactly cancel the accelerometer error ∇ and reference velocity error δV times the damping constant C .

The additional equations giving the position and azimuth error propagation as functions of the gyro drift rate can be derived as follows.

The angular velocity $\overline{\omega}_p$ of the platform with respect to inertial space can be written in terms of the computed angular velocity $\overline{\omega}_c$ and the rate of change of $\overline{\psi}$ as

$$\overline{\omega}_p = \overline{\omega}_c + \overline{\dot{\psi}} \quad [3]$$

The platform angular velocity is also given by the torquing rate $\overline{\omega}_c + \overline{\dot{\psi}} \times \overline{\omega}_c$ (since the platform is misaligned from the computer axis) plus the vector gyro drift rate ϵ :

$$\overline{\omega}_p = \overline{\omega}_c + \overline{\dot{\psi}} \times \overline{\omega}_c + \epsilon \quad [4]$$

Equating Eqs. 3 and 4 gives the $\overline{\dot{\psi}}$ equations

$$\overline{\dot{\psi}} + \overline{\omega}_c \times \overline{\psi} = \overline{\epsilon} \quad [5]$$

For marine applications $\overline{\omega}_c \approx \overline{\Omega}$ = Earth's angular velocity. The transfer function associated with Eq. 5 has poles at $s = \pm j\Omega$ and $s = 0$ (for the polar component); hence, the latitude and azimuth errors for long term marine operations with constant gyro drift rates are oscillatory, with a period of 24 hr (the 84-min or Schuler period errors being damped, using external velocity information). The longitude error

for a constant polar component of gyro drift rate increases linearly with time. If the gyro drift rate is a random function, the root mean square position and azimuth errors build up with the square root of time, since the transfer function for the $\overline{\psi}$ equations has poles on the imaginary axis. In summary, then, it is seen that for long term damped inertial operation:

- 1) The level platform tilts are bounded and related to accelerometer and reference velocity error.
- 2) The long term position and azimuth errors are oscillatory, with a 24-hr periodicity.
- 3) Gyro drift rates cause a square root of time build up of both position and azimuth rms error in long term operation.
- 4) Long term operation dictates the requirement for intermittent position or azimuth fixes for realignment of the inertial system.

In the next section it will be shown how the data available from multiple system operation can be used to reduce the growth of errors in long term system operation.

MULTIPLE SYSTEM OPERATION

Formulation of Error Differences

The various modes of operation of inertial navigation systems can be characterized by the types of information used for damping purposes. Characteristic of these modes is the fact that difference signals are always generated by subtracting some quantity furnished by the system from some external reference quantity, operating on this error signal, and feeding it to various points of the system. A classification of these modes follows.

- 1) Free inertial mode: no error differences used.
- 2) Damped inertial mode (or inertial mode with external references): a) $V_x - V_{rx}$ and $V_y - V_{ry}$, the differences

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between computed and reference velocity, are used to damp the 84-min oscillations; b) $\theta - \theta_c$ and $\Lambda - \Lambda_c$, the differences between computed latitude and longitude and checkpoint latitude and longitude, are used to damp 24-hr oscillations as well as to compute gyro drift rate corrections; and c) $\overline{\psi} - \overline{\psi}_r$, as furnished by a star tracker, can be used to damp 24-hr oscillations and correct for gyro drift rates in a stellar-monitored system.

3) Multiple system operation: a) $\overline{\phi}_1 - \overline{\phi}_2$, the relative platform misalignment of one system with respect to the other, can be used to align one system to another or compute accelerometer bias error corrections by use of platform rotation; and b) $\delta\theta_1 - \delta\theta_2$, the differences in indicated position between two systems, can be used to compute gyro drift rate corrections by use of platform rotation.

In multiple inertial navigator systems, then, the error differences available in the absence of external velocity, position, or azimuth information are

$$\begin{array}{ll}
 \phi_{x1} - \phi_{x2} & \delta\theta_{x1} - \delta\theta_{x2} \\
 \phi_{y1} - \phi_{y2} & \delta\theta_{y1} - \delta\theta_{y2} \quad [6] \\
 \phi_{z1} - \phi_{z2} & \delta\theta_{z1} - \delta\theta_{z2}
 \end{array}$$

where the subscripts 1 and 2 refer to systems number 1 and 2, respectively.

The ϕ angle differences expressed in Eq. 6 can be generated by observing gimbal angle differences between systems (roll, pitch, and azimuth differences). The $\delta\theta$ angle differences for a local level system can be written as

$$\delta\theta_{x1} - \delta\theta_{x2} = \left[\Delta\Lambda_1(t) - \Delta\Lambda_2(t) \right] \cos\theta \quad [7a]$$

$$\delta\theta_{y1} - \delta\theta_{y2} = \left[\Delta\theta_1(t) - \Delta\theta_2(t) \right] \quad [7b]$$

$$\delta\theta_{z1} - \delta\theta_{z2} = -\left[\Delta\Lambda_1(t) - \Delta\Lambda_2(t) \right] \sin\theta \quad [7c]$$

where

θ, Λ = latitude and longitude, respectively

$\Delta\theta, \Delta\Lambda$ errors in indicated latitude and longitude, respectively

The latitude and longitude errors in Eqs. 7 are given by

$$\Delta\theta_1 = \theta_1 - \theta \quad [8a]$$

$$\Delta\theta_2 = \theta_2 - \theta \quad [8b]$$

$$\Delta\Lambda_1 = \Lambda_1 - \Lambda \quad [8c]$$

$$\Delta\Lambda_2 = \Lambda_2 - \Lambda \quad [8d]$$

where θ_1, Λ_1 are indicated latitude and longitude for System 1. Using Eqs. 8 in Eqs. 7 yields

$$\delta\theta_{x1} - \delta\theta_{x2} = [\Lambda_1(t) - \Lambda_2(t)] \cos \theta \quad [9a]$$

$$\delta\theta_{y1} - \delta\theta_{y2} = [\theta_1(t) - \theta_2(t)] \quad [9b]$$

$$\delta\theta_{z1} - \delta\theta_{z2} = -[\Lambda_1(t) - \Lambda_2(t)] \sin \theta \quad [9c]$$

From Eqs. 9 it is seen that the error difference $\overline{\delta\theta_1} - \overline{\delta\theta_2}$ can be formed in terms of the indicated position values only and is not a function of true position $\theta(t), \Lambda(t)$. Hence, all six differences in Eq. 6 can be formed without the aid of any external information.

Use of Error Differences With Platform Rotation For Correction of the Inertial Instruments

1. Correction of Accelerometer Errors in Multiple Systems Using Platform Rotation

Operation of the inertial platform with an arbitrary azimuth orientation about the z axis, as in Fig. 1, must now be considered. In Fig. 1, x_o, y_o, z_o are in the north,

east, and local vertical directions, respectively. The physical platform axes x , y , z are obtained by a rotation about the vertical through the angle α , as shown. To accomplish this rotation with ease, the gimbal order in going from the platform to the ship must be azimuth first (about z) and then pitch or roll.

Fig. 2 depicts the steady state level tilts of two inertial navigators operated with $\alpha_1 = \alpha_2 = 0^\circ$, that is, in north-pointing orientation. From Eqs. 2 the $\bar{\phi}$ differences between systems are given by

$$[\phi_{x1} - \phi_{x2}]_{0^\circ} = \frac{V_{y1} - V_{y2}}{g} \quad [10a]$$

$$[\phi_{y1} - \phi_{y2}]_{0^\circ} = \frac{V_{x1} - V_{x2}}{g} \quad [10b]$$

If platform 2 is now rotated by 180° with respect to platform 1 and allowed to level up, the $\bar{\phi}$ differences will be

$$(\phi_{x1} - \phi_{x2})_{180^\circ} = \left[\frac{V_{y1} + V_{y2}}{g} \right] \quad [11a]$$

$$(\phi_{y1} - \phi_{y2})_{180^\circ} = - \left[\frac{V_{x1} + V_{x2}}{g} \right] \quad [11b]$$

From Eqs. 10 and 11

$$V_{y1} = g \left[\frac{(\phi_{x1} - \phi_{x2})_{180^\circ} + (\phi_{x1} - \phi_{x2})_{0^\circ}}{2} \right] \quad [12]$$

$$V_{y2} = g \left[\frac{(\phi_{x1} - \phi_{x2})_{180^\circ} - (\phi_{x1} - \phi_{x2})_{0^\circ}}{2} \right] \quad [13]$$

$$V_{x1} = -g \left[\frac{(\phi_{y1} - \phi_{y2}) 180 + (\phi_{y1} - \phi_{y2}) 0}{2} \right] \quad [14]$$

$$V_{x2} = -g \left[\frac{(\phi_{y1} - \phi_{y2}) 180 - (\phi_{y1} - \phi_{y2}) 0}{2} \right] \quad [15]$$

The computed accelerometer bias errors (Eqs. 12 and 15) can now be subtracted from the output of the corresponding accelerometer, thus correcting for the level misalignment of both platforms. The implicit assumption that the bias errors do not change during the measurement period should be noted.

2. Correction for Level Gyro Drift Rates in Multiple Systems

The coordinate frame p, y, h shown in Fig. 3, in which x, y, z are local level axes will be considered. The axes p, y, h are defined as follows: p is along Earth's polar axis, y is identical to the east-pointing axis in local level axes, and h completes the triad. From this figure it is seen that the angle between x and p is simply the latitude of the system. Hence

$$\epsilon_x = \epsilon_p \cos \theta + \epsilon_h \sin \theta \quad [16a]$$

$$\epsilon_y = \epsilon_y \quad [16b]$$

$$\epsilon_z = \epsilon_h \cos \theta - \epsilon_p \sin \theta \quad [16c]$$

Similarly

$$\psi_x = \psi_p \cos \theta + \psi_h \sin \theta \quad [17a]$$

$$\psi_y = \psi_y \quad [17b]$$

$$\psi_z = \psi_h \cos \theta - \psi_p \sin \theta \quad [17c]$$

Writing the ψ Eqs. 5 in p, y, h coordinates and integrating, the constant gyro drift rates can be solved for as functions of ψ :

$$\epsilon_p = \frac{1}{t} \left[\psi_p(t) - \psi_p(0) \right] \quad [18a]$$

$$\epsilon_h = \frac{\Omega \sin \Omega t}{2(1 - \cos \Omega t)} \left[\psi_h(t) - \psi_h(0) \right] + \frac{\Omega}{2} \left[\psi_y(t) + \psi_y(0) \right] \quad [18b]$$

$$\epsilon_y = \frac{\Omega \sin \Omega t}{2(1 - \cos \Omega t)} \left[\psi_y(t) + \psi_y(0) \right] - \frac{\Omega}{2} \left[\psi_h(t) + \psi_h(0) \right] \quad [18c]$$

It will now be assumed that there are available two inertial navigators operating in either the free or damped inertial mode. It has been seen how the difference $\bar{\phi}_1 - \bar{\phi}_2$ and $\bar{\delta\theta}_1 - \bar{\delta\theta}_2$ between Systems 1 and 2 can be formed.

Then the $\bar{\psi}$ differences can be formed from Eq. 1:

$$\Delta\bar{\psi} = \bar{\psi}_1 - \bar{\psi}_2 = (\bar{\phi}_1 - \bar{\phi}_2) - (\bar{\delta\theta}_1 - \bar{\delta\theta}_2) \quad [19]$$

Hence, using Eqs. 18 and 19

$$\epsilon_{p1} - \epsilon_{p2} = \frac{1}{t} \left[\Delta\bar{\psi}_p(t) - \Delta\bar{\psi}_p(0) \right]$$

$$\begin{aligned} \epsilon_{h1} - \epsilon_{h2} &= \frac{\Omega \sin \Omega t}{2(1 - \cos \Omega t)} \left[\Delta \psi_h(t) - \Delta \psi_h(0) \right] \\ &+ \frac{\Omega}{2} \left[\Delta \psi_y(t) + \Delta \psi_y(0) \right] \end{aligned} \quad [20b]$$

$$\begin{aligned} \epsilon_{y1} - \epsilon_{y2} &= \frac{\Omega \sin \Omega t}{2(1 - \cos \Omega t)} \left[\Delta \psi_y(t) - \Delta \psi_y(0) \right] \\ &- \frac{\Omega}{2} \left[\Delta \psi_h(t) + \Delta \psi_h(0) \right] \end{aligned} \quad [20c]$$

where the ψ differences in Eqs. 20 are given by

$$\begin{aligned} \Delta \psi_p(t) &= - \left[\Lambda_1(t) - \Lambda_2(t) \right] - \left[\phi_{z1}(t) - \phi_{z2}(t) \right] \sin \theta \\ &+ \left[\phi_{x1}(t) - \phi_{x2}(t) \right] \cos \theta \end{aligned} \quad [21]$$

$$\Delta \psi_y(t) = \left[\phi_{y1}(t) - \phi_{y2}(t) \right] + \left[\theta_1(t) - \theta_2(t) \right] \quad [22]$$

$$\begin{aligned} \Delta \psi_h(t) &= \left[\phi_{x1}(t) - \phi_{x2}(t) \right] \sin \theta \\ &+ \left[\theta_{z1}(t) - \theta_{z2}(t) \right] \cos \theta \end{aligned} \quad [23]$$

If Eqs. 21 and 23 are formed from the differences in system outputs between two inertial navigators with $\alpha_1 = \alpha_2 = 0^\circ$, the differences in gyro drift rate can be computed from Eqs. 16 and 20, as illustrated in Fig. 4.

$$\begin{aligned} \epsilon_{x1} - \epsilon_{x2} &= (\epsilon_{p1} - \epsilon_{p2}) \cos \theta + (\epsilon_{h1} - \epsilon_{h2}) \sin \theta \\ &= (\Delta \epsilon)_{x \ 0^\circ} \end{aligned} \quad [24a]$$

$$\epsilon_{y1} - \epsilon_{y2} = \left(\Delta \epsilon \right)_y 0^\circ \quad [24b]$$

$$\begin{aligned} \epsilon_{z1} - \epsilon_{z2} &= (\epsilon_{h1} - \epsilon_{h2}) \cos \theta - (\epsilon_{p1} - \epsilon_{p2}) \sin \theta \\ &= \left(\Delta \epsilon \right)_z 0^\circ \end{aligned} \quad [24c]$$

If System 2 is now rotated so that the x axis is south ($\alpha_2 = 180^\circ$), the following can be computed observing $\Delta\psi$ at two discrete times:

$$\epsilon_{x1} + \epsilon_{x2} = \left(\Delta \epsilon \right)_x 180^\circ \quad [25a]$$

$$\epsilon_{y1} + \epsilon_{y2} = \left(\Delta \epsilon \right)_y 180^\circ \quad [25b]$$

$$\epsilon_{z1} - \epsilon_{z2} = \left(\Delta \epsilon \right)_z 180^\circ \quad [25c]$$

From Eqs. 24 and 25 level gyro drift rates can now be computed for both systems:

$$\epsilon_{x1} = \left[\frac{(\Delta \epsilon_x)_{0^\circ} + (\Delta \epsilon_x)_{180^\circ}}{2} \right] \quad [26]$$

$$\epsilon_{x2} = \left[\frac{(\Delta \epsilon_x)_{180^\circ} - (\Delta \epsilon_x)_{0^\circ}}{2} \right] \quad [27]$$

$$\epsilon_{y1} = \left[\frac{(\Delta \epsilon_y)_{0^\circ} + (\Delta \epsilon_y)_{180^\circ}}{2} \right] \quad [28]$$

$$\epsilon_{y2} = \left[\frac{(\Delta \epsilon_y)_{180^\circ} - (\Delta \epsilon_y)_{0^\circ}}{2} \right] \quad [29]$$

After the multiple system biasing run is completed, Eqs. 26 and 29 can be applied to the gyros as gyro bias corrections. It is to be noted that neither the accelerometer nor gyro biasing scheme presented requires external position.

SUMMARY

In the previous section it was shown how gyro and accelerometer bias corrections can be generated by the use of data internal to the operation of two or more inertial navigators. By applying these corrections, the effective inertial instrument errors are reduced when the correlation times of the instrument errors are longer than the measurement periods. By these procedures, then, the long term error buildup characteristic of long term navigation systems can be decreased.

Multiple system operation, then, can be used in conjunction with periodic checkpoint information for bounding position, attitude, and azimuth errors to acceptable values for long time periods. This is in addition to the overall gain in reliability afforded by two or more inertial navigators.

APPENDIX: DERIVATION OF STEADY STATE EXPRESSIONS FOR LEVEL PLATFORM TILT ANGLES IN THE DAMPED INERTIAL MODE

The position error vector can be written as

$$\overline{\delta R} = \overline{\delta \theta} \times \overline{R} \quad [A-1]$$

where R denotes the position vector as measured from the center of Earth to the true position, and $\overline{\delta R}$ is the position error in the computed position. Then the position error equation for operation in the damped inertial mode is (where derivatives are taken with respect to the true coordinate system)

$$\begin{aligned} \frac{\ddot{\overline{\delta R}}}{\overline{\delta R}} + C \frac{\dot{\overline{\delta R}}}{\overline{\delta R}} + \omega_o^2 \overline{\delta R} + 2\Omega_z \times \frac{\dot{\overline{\delta R}}}{\overline{\delta R}} \\ = -\omega_o^2 \overline{\psi} \overline{R} + \overline{V} + C \overline{\delta V} \end{aligned} \quad [A-2]$$

where

- ω_o = Schuler angular frequency (4.458 rad/hr)
- Ω = Earth rate (15 deg/hr)
- $\bar{\Omega}_z$ = vertical component of Earth rate ($\bar{\Omega}_z = -\Omega \sin \theta$)
- \bar{V} = accelerometer bias error vector
- C = velocity damping constant ($c = 2\zeta \omega_o$,
where ζ = damping ratio)
- $\bar{\delta V}$ reference velocity error vector

The ψ equations are given by

$$\dot{\bar{\psi}} + \bar{\omega}_c \times \bar{\psi} = \bar{\epsilon} \quad [A-3]$$

where

- $\bar{\epsilon}$ = gyro drift rate
- $\bar{\omega}_c$ = computed angular velocity ($\bar{\omega}_c = \bar{\Omega}$ for slowly moving vehicles)

Eq. A-2 can be rewritten in component form in the frequency domain as

$$\begin{bmatrix} (s^2 + Cs + \omega_o^2) - 2\bar{\Omega}_z s \\ 2\bar{\Omega}_z s (s^2 + Cs + \omega_o^2) \end{bmatrix} \begin{bmatrix} \delta R_x(s) \\ \delta R_y(s) \end{bmatrix} = \begin{bmatrix} Q_x(s) \\ Q_y(s) \end{bmatrix} \quad [A-4]$$

where $Q_x(s)$ and $Q_y(s)$ are the transforms of the x and y components of the driving functions of Eq. A-2.

Eqs. A-4 can be solved for $\delta R_x(s)$ and $\delta R_y(s)$ to give

$$\delta R_x(s) = \frac{(s^2 + Cs + \omega_o^2) Q_x(s) + 2\Omega_z s Q_y(s)}{(s^2 + Cs + \omega_o^2)^2 + 4\Omega_z^2 s^2} \quad [A-5]$$

$$\delta R_y(s) = \frac{(s^2 + Cs + \omega_o^2) Q_y(s) - 2\Omega_z s Q_x(s)}{(s^2 + Cs + \omega_o^2)^2 + 4\Omega_z^2 s^2} \quad [A-6]$$

The poles of the transfer functions in Eqs. A-5 and A-6 are in the left half-plane for $C > 0$. Hence, the position error response for driving functions whose frequency is much less than the Schuler frequency ω_o and for constant driving functions is

$$\delta R_{xSS} = \frac{Q_x(t)}{\omega_o^2} \quad [A-7]$$

$$\delta R_{ySS} = \frac{Q_y(t)}{\omega_o^2} \quad [A-8]$$

or

$$\overline{\delta R} = -\frac{\omega_o^2 \overline{\psi} \times \overline{R} + \overline{V} + C \overline{\delta V}}{\omega_o^2} \quad [A-9]$$

Using the identity $\overline{\psi} = \overline{\phi} - \overline{\delta\theta}$ in Eqs. A-9 and A-1

$$\overline{\phi} \times \overline{R} = \frac{\overline{V}}{\omega_o^2} + C \frac{\overline{\delta V}}{\omega_o^2} \quad [A-10]$$

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Expanding equation A-10 in component form, Eqs. A-11a and A-11b will result

$$\phi_x = \frac{V_y}{g} + 2\delta \frac{\delta V_y}{a\omega_o} \quad [A-11a]$$

$$\phi_y = -\frac{V_x}{g} - 2\delta \frac{\delta V_x}{a\omega_o} \quad [A-11b]$$

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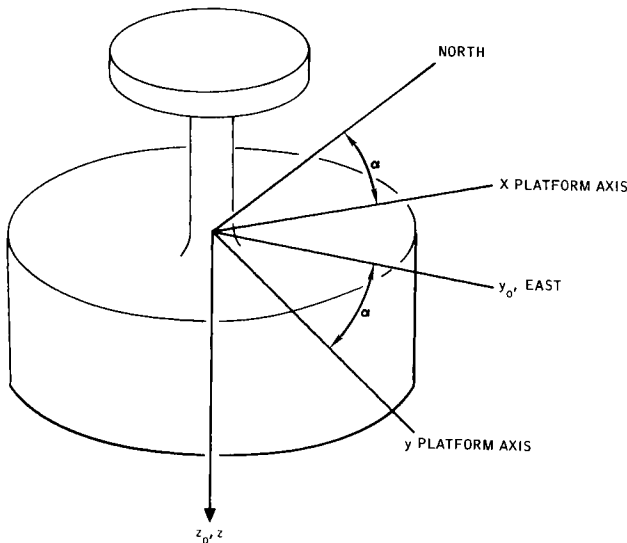


Fig. 1 Operation of an inertial navigation system with arbitrary azimuth angle

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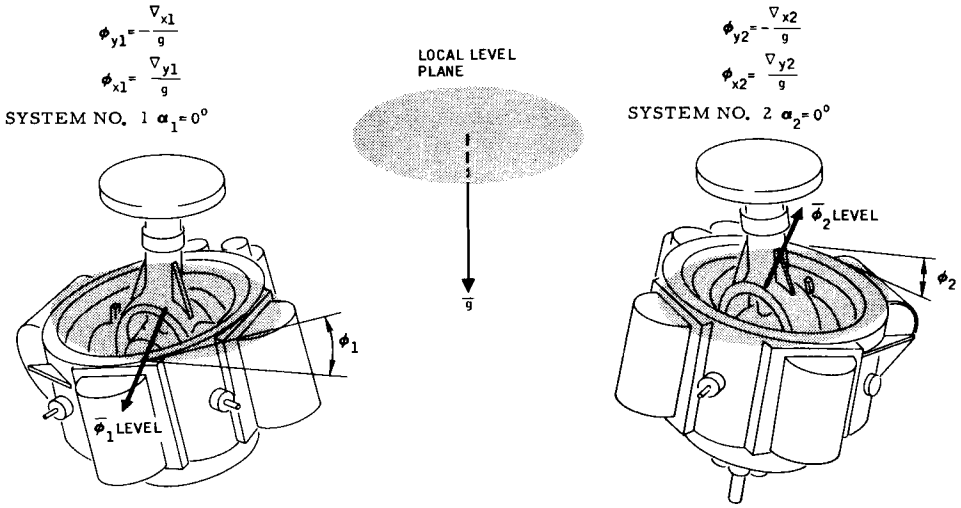


Fig. 2 Platform tilt angles in a multiple system with $\alpha_1 = \alpha_2 = 0^\circ$

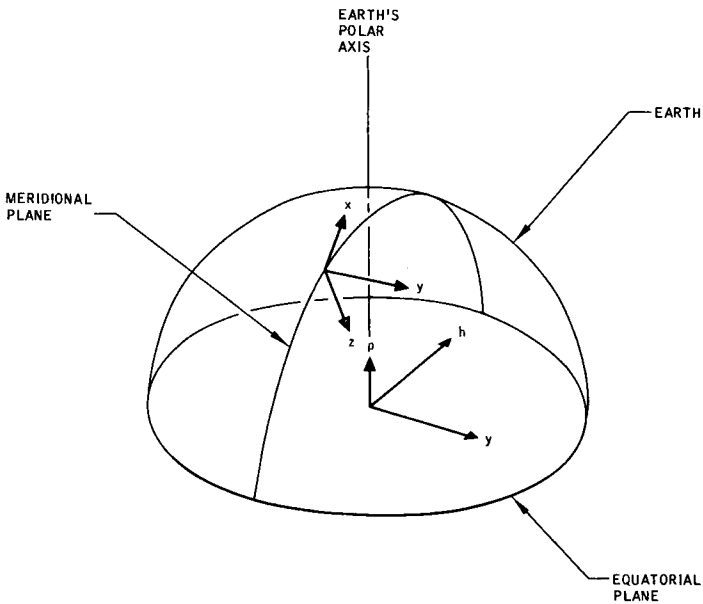


Fig. 3 Platform and Earth-fixed coordinate axes

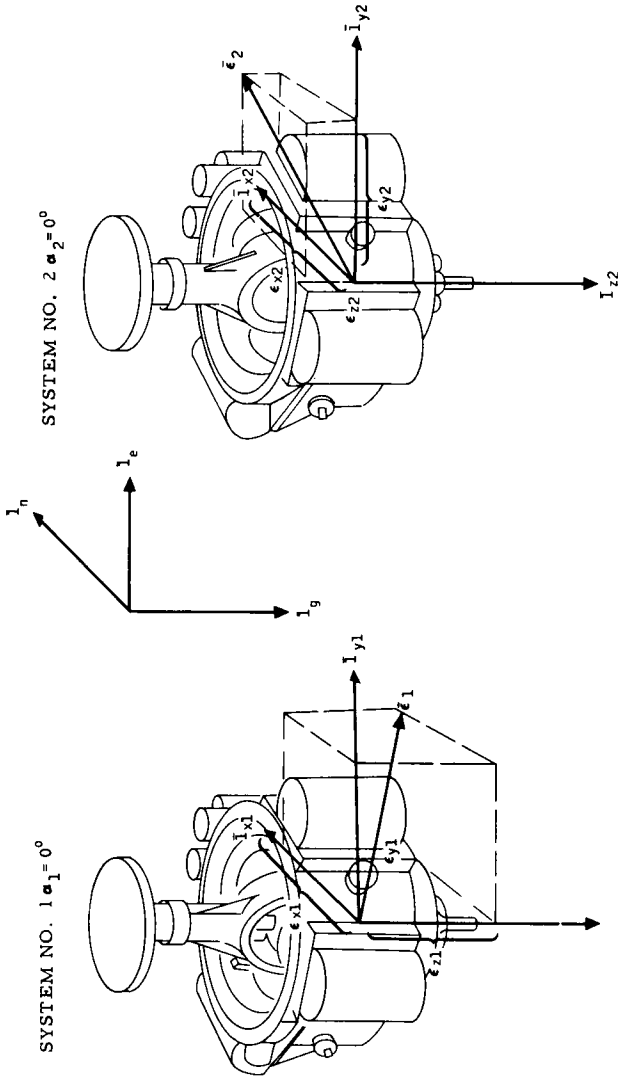


Fig. 4 Gyro drift rate vectors for a two-inertial navigator system with $\alpha_1 = 0^\circ$ and $\alpha_2 = 0^\circ$