

NOISE CONSIDERATION IN DESIGNING A TRAVELING WAVE TUBE
MIXER FOR OPTICAL HETERODYNING

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ABSTRACT

In order to detect the difference frequency between two optical lines, a special electron tube consisting of a photo-sensitive cathode and traveling-wave interaction structure has been investigated. Interference of the two optical signals at the photosurface produces a modulation of the emitted current at the beat frequency which is then coupled to a microwave receiver by means of a helical guide. In this manner, it is possible to obtain a wide bandwidth with a noise figure determined essentially by the photosurface, with a negligible contribution of noise from other parts of the mixer tube.

INTRODUCTION

Measurement of the relative velocity of a vehicle with respect to the sun or stars by means of optical Doppler shifts requires that the frequency of a spectral line emitted by the body be measured precisely. This can be done by determining the difference frequency between this emitted line and the same line generated by a light source on the vehicle. With the use of a non-linear light sensitive device it is possible to generate this difference frequency by the usual heterodyne process. Previous attempts (1)² to make such measurements were not fully conclusive due to the low signal to noise ratio that was attained. In order to verify the feasibility of this technique, the use of a wide band optical mixer tube has been investigated. By including all the available energy at the difference frequency, it was predicted that a substantial increase in signal to noise ratio over previous results could be

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²Numbers in parentheses indicate References at end of paper.

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obtained. The experimental tests were designed to use two Zeeman components of the Hg 5461 A line, which could be expected to have a half-power width of about 1000 Mc. By using this full bandwidth, the desired improvement could be obtained.

Photoelectric emission is the only presently known effect that has a response time short enough to provide a usable output at microwave frequencies. The choice of detector element is therefore limited to this type, and, consequently, the maximum signal to noise ratio that can be obtained will depend primarily on this device. Other sources of noise must be kept insignificant in order to avoid additional reduction of this ratio. After investigating several ways of coupling to the electron beam, it was concluded that a slow wave device, such as a helix, offered the only possible solution to obtain both wide bandwidth and tight coupling. The purpose of this preliminary analysis was to determine if such a device could be built without introducing excessively large additional noise.

ANALYSIS OF INTERACTION STRUCTURE

The solution for the power delivered to the output termination of the slow wave structure follows directly from the normal solution for a Traveling Wave Tube amplifier (2-4). Several conditions exist for the present case which allow certain simplifying assumptions to be made. First, the beam current will be very low, resulting in negligible space charge effects. This allows the use of the simplest solutions for the electromagnetic waves supported by the helix. Secondly, by assuming that it is possible to couple sufficient power from the electron beam to override other sources of noise, it should be possible to operate the tube so that there is very little power gain for signals introduced at the input to the helix (Johnson noise or reflections due to mismatch). Then, it is not necessary to include an attenuator section to suppress oscillations. However, since satisfactory operation is required over a relatively large range of beam currents, the design tends to be complicated.

The major sources of noise are the shot and velocity noises of the electron beam and the Johnson noise of the helix terminations. Additional noise due to such things as secondary emission and heating of the helix can probably be made insignificant by proper design of the electron gun, and, therefore, these effects will not be considered here. Because of the low current density in the beam, there will be no smoothing of the shot noise, and the mean square current

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fluctuations will be given by

$$\overline{i^2} = 2eI_o \Delta f \quad [1]$$

The velocity fluctuations, due to the random energy of the electrons as they are emitted, will be approximately

$$\overline{v^2} = \frac{1}{18} n \frac{(eV_o)}{I_o} \left(\frac{V_m}{V_o} \right)^2 \Delta f \quad [2]$$

The Johnson noise power from the input termination of the helix will be

$$W_j = kT\Delta f \quad [3]$$

The initial amplitudes of the electromagnetic waves supported by the helix will be determined by these three quantities. Since they are all uncorrelated, they can be considered separately when computing the power delivered to the output termination.

Following the usual TWT analysis, the three forward waves are characterized by the propagation constants

$$\Gamma_m = j\beta_e - \beta_e C \delta_m \quad m = 1, 2, 3 \quad [4]$$

where

$$\delta_m = e \exp j \left(\frac{\pi}{2} - \frac{2m\pi}{3} \right) \quad [5]$$

The amplitudes of these three waves are such that they add at the input to the helix to give a value equal to the field due to the Johnson noise, E_{s_1}

$$\left| E_{s_1} \right|^2 = \beta_e^2 K (kT\Delta f) \quad [6]$$

and also satisfy the electronic equations that relate the field quantities to the beam modulation

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$$v_m = - \frac{\eta E_m}{u_o \delta_m \beta_e C} \quad m = 1, 2, 3 \quad [7]$$

$$i_m = j \frac{I_o E_m}{2V_o \delta_m^2 \beta_e C^2} \quad m = 1, 2, 3 \quad [8]$$

At the output of the helix, the combination of these three waves results in an expression for the output power due to each of the noise sources. These are

$$W_1 = G_1 kT\Delta f \quad \text{Johnson noise} \quad [9]$$

$$W_2 = G_2 \frac{(eV_o)}{36C} \left(\frac{V_m}{V_o} \right)^2 \Delta f \quad \text{velocity noise} \quad [10]$$

$$W_3 = G_3 2C(eV_o) \Delta f \quad \text{shot noise} \quad [11]$$

The gain factors G_1 , G_2 , and G_3 are shown in Fig. 1 and are given by

$$G_1 = \frac{1}{9} \{1 + 4 \cosh^2 (\sqrt{3}\pi CN) + 4 \cos (3\pi CN) \cosh (\sqrt{3}\pi CN)\} \quad [12]$$

$$G_2 = \frac{1}{9} \{4 \cosh^2 (\sqrt{3}\pi CN) - 2 - 2 \cos (3\pi CN) \cosh (\sqrt{3}\pi CN) - 2\sqrt{3} \sin (3\pi CN) \sinh (\sqrt{3}\pi CN)\} \quad [13]$$

$$G_3 = \frac{1}{9} \{4 \cosh^2 (\sqrt{3}\pi CN) - 2 - 2 \cos (3\pi CN) \cosh (\sqrt{3}\pi CN) + 2\sqrt{3} \sin (3\pi CN) \sinh (\sqrt{3}\pi CN)\} \quad [14]$$

DISCUSSION

Examination of these curves shows that for the largest shot noise to Johnson noise ratio, the product CN should be about 0.25. However, to obtain the largest shot noise to velocity noise ratio, this product should be as small as possible. The actual value to be used will depend on the values of the other design parameters in Eqs. 9-11 and the noise figure of the following amplifier.

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If the range of some of the design values is considered, it becomes apparent that a value of C greater than 0.001 will be sufficient to insure that the shot noise will be large compared to the Johnson noise. For instance, a beam voltage between 500 and 5000 v gives a value of eV_0 between $8 \cdot 10 \exp(-16)$ and $8 \cdot 10 \exp(-17)$ joule. However, kT is equal to $4 \cdot 10 \exp(-21)$ joule. Thus, if $C = 0.001$, there will be a ratio of between 26 and 16 db, plus the difference in gain factors. The value of CN can be chosen quite low to increase the shot noise to velocity noise ratio before the difference in gain factors, G_1 and G_3 , has an appreciable effect on the shot noise to Johnson noise ratio.

For a numerical estimate, assume that a value of $CN = 0.1$ will be used with a helix 15 cm long. The beam voltage and current will be 2000 v and 3.85μ a. Also, assume an initial normal electron energy of 1 ev maximum. The wavelength is

$$\lambda = 5.932 \cdot 10 \exp(-5) \sqrt{V_0} = 2.66 \text{ mm} \quad [15]$$

Therefore

$$N = 150/2.66 = 56.4 \text{ wavelengths} \quad [16]$$

$$C = 0.1/56.4 = 1.773 \cdot 10 \exp(-3) \quad [17]$$

$$kT = 4.003 \cdot 10 \exp(-21) \text{ joule} \quad [18]$$

$$\frac{eV_0}{36C} \left(\frac{V_m}{V_0} \right)^2 = 1.234 \cdot 10 \exp(-21) \text{ joule} \quad [19]$$

$$2C (eV_0) = 1.135 \cdot 10 \exp(-18) \text{ joule} \quad [20]$$

The relative gains are

$$G_1 = 0 \text{ db} \quad [21]$$

$$G_2 = -14.09 \text{ db} \quad [22]$$

$$G_3 = -4.03 \text{ db} \quad [23]$$

The ratio of shot noise to Johnson noise is then

$$W_3/W_1 = 20.5 \text{ db} \quad [24]$$

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and of shot noise to velocity noise is

$$W_3/W_2 = 39.7 \text{ db} \quad [25]$$

For a receiver noise figure of 8.5 db, the net ratio of shot noise to all other noises is then 11.4 db. The required beam coupling impedance is

$$K = 11.6 \text{ ohms} \quad [26]$$

for which, if a beam to helix diameter ratio of 0.6 is used, the diameter of the helix must be

$$d \approx 2.5 \text{ mm} \quad [27]$$

Over a bandwidth of 1000 Mc, the noise power delivered to the receiver will be

$$W_3 = 4.5 \cdot 10^{-10} \text{ watt} \quad \text{shot noise} \quad [28]$$

$$W_1 = 4.0 \cdot 10^{-12} \text{ watt} \quad \text{Johnson noise} \quad [29]$$

$$W_2 = 4.9 \cdot 10^{-14} \text{ watt} \quad \text{velocity noise} \quad [30]$$

CONCLUSION

This analysis has shown that a Traveling Wave Tube design for the mixer is possible which will result in a signal to noise ratio determined essentially by the photosurface. With a device such as this the detection of the beat frequency between two noncoherent optical lines would be possible with a much greater degree of confidence than for previous experiments.

APPENDIX

The solution for the waves supported by the helix is obtained from the simultaneous solution of the electronic Eq. 8 and the circuit equation

$$E = - \left[\frac{\Gamma_o^2 K}{\Gamma^2 - \Gamma_o^2} + \frac{2jQ\Gamma^2 K}{\beta_e} \right] i \quad [A-1]$$

For negligible space charge effects, $Q = 0$, and if the electron velocity and helix phase velocity are equal, this

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solution gives Eq. 5. In order to satisfy the boundary conditions at the input to the helix, the sum of the individual modulation components, i_m and v_m , must equal the total modulation

$$i = \sum_{m=1}^3 i_m \quad [A-2]$$

$$v = \sum_{m=1}^3 v_m \quad [A-3]$$

Also, the total field at the axis of the helix must equal the sum of the amplitudes of the three electromagnetic waves

$$E = \sum_{m=1}^3 E_m \quad [A-4]$$

These conditions are sufficient to determine the wave amplitudes at the input to the helix

$$E_1 = \frac{1}{3} \left[E_{s_1} + e \exp(-j \pi/6) E_{s_2} + e \exp\left(-j \frac{\pi}{3}\right) E_{s_3} \right] \quad [A-5]$$

$$E_2 = \frac{1}{3} \left[E_{s_1} + e \exp\left(-j \frac{5\pi}{6}\right) E_{s_2} + e \exp\left(j \frac{\pi}{3}\right) E_{s_3} \right] \quad [A-6]$$

$$E_3 = \frac{1}{3} \left[E_{s_1} + e \exp\left(j \frac{\pi}{2}\right) E_{s_2} + e \exp(j \pi) E_{s_3} \right] \quad [A-7]$$

where

$$E_{s_1} = E \quad [A-8]$$

$$E_{s_2} = - \frac{u_o \beta_e C}{\eta} v \quad [A-9]$$

$$E_{s_3} = -j \frac{2V_o \beta_e C^2}{I_o} i \quad [A-10]$$

The total electric field at the output of the helix is then given by

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$$E_L = \left[E_1 e^{-\Gamma_1 L} + E_2 e^{-\Gamma_2 L} + E_3 e^{-\Gamma_3 L} \right] \quad [A-11]$$

Since the noise sources are uncorrelated, the output power due to each can be found by treating them separately. For the Johnson noise of the input termination

$$W_1 = \left| E_{L_1} \right|^2 (\beta_e^2 k)^{-1} \quad [A-12]$$

where

$$E_{L_1} = \frac{1}{3} E_{s_1} \left[e^{-\Gamma_1 L} + e^{-\Gamma_2 L} + e^{-\Gamma_3 L} \right] \quad [A-13]$$

Similar expressions are obtained for the velocity and current modulations of the beam. By squaring and taking magnitudes, Eqs. 9-14 are obtained.

NOMENCLATURE

- C = Pierce's gain factor
- e = $1.6020 \cdot 10^{-19}$ = electronic charge, C
- E = total electric field at input of helix
- E_m = amplitude of individual waves
- E_L = total electric field at output of helix
- E_{L_1} = electric field at output of helix due to Johnson noise
- E_{L_2} = electric field at output of helix due to velocity noise
- E_{L_3} = electric field at output of helix due to shot noise
- E_{S_1} = electric field at input of helix due to Johnson noise
- E_{S_2} = electric field at input of helix due to velocity noise
- E_{S_3} = electric field at input of helix due to shot noise
- Δf = bandwidth
- i = a-c component of beam current
- i_m = component of current modulation
- I_0 = d-c beam current
- j = $\sqrt{-1}$
- k = $1.3803 \cdot 10^{-23}$ = Boltzmann's constant, joule/°K
- K = $C^3 4V_0/I_0$ = beam coupling impedance
- L = length of helix
- N = L/λ = length of helix in wavelengths
- Q = space charge parameter
- T = temperature
- u_0 = average electron velocity
- v = a-c component of electron velocity
- v_m = component of velocity modulation

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V_0 = beam voltage
 V_m = maximum initial energy of electrons (ev)
 W_1 = output power due to Johnson noise
 W_2 = output power due to velocity noise
 W_3 = output power due to shot noise
 W_j = input power due to Johnson noise
 β_e = electron phase constant
 Γ = propagation constant of e-m wave
 Γ_0 = propagation constant of helix
 Γ_m = propagation constant of individual waves
 δ_m = normalized shift of propagation constant
 m = $e/m = 1.759 \cdot 10^{-11}$ = electron charge-mass ratio, C/kg
 λ = $2\pi/\beta_e$ = wavelength

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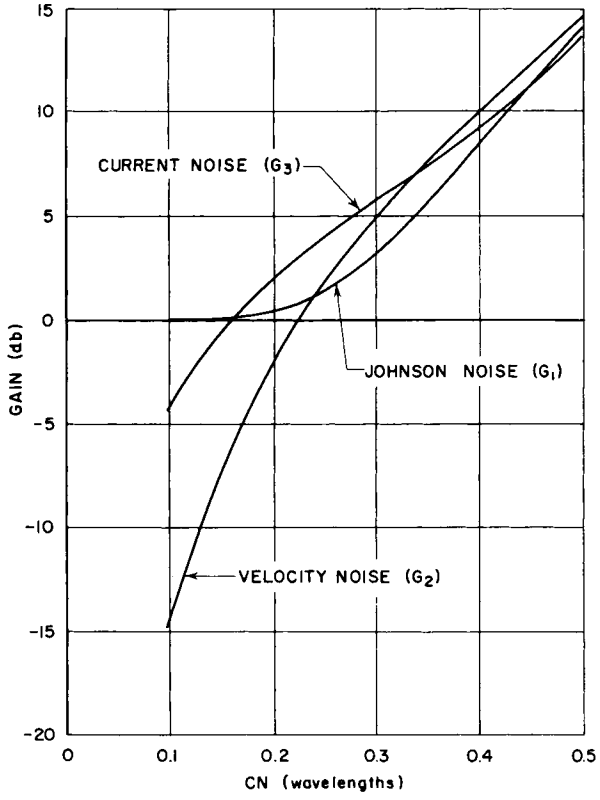


Fig. 1 Gain of traveling wave tube