

GUIDANCE AND CONTROL

ATTITUDE CONTROL SYSTEM USING LOGICALLY CONTROLLED PULSES

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ABSTRACT

This paper describes the development of a new approach to pulsed jet attitude control. This approach makes use of the minimum impulse capability of the gas jets to design the most efficient possible limit cycle operation and is based on simple logical control of gas thrusts. Reliable and simple on-off solenoid valves could be used, assuring minimum leakage and permitting application to either compressed or liquified gases or hypergolic fuels as the mass impulse element.

It is also shown that by the use of relatively simple switching of the gas reaction thrusts, it is possible to eliminate the conventional rate measuring device or network used for damping. Instead, nonlinear damping is achieved through the switching logic in order to make the system converge into a stable and efficient limit cycle. In addition, the use of controlled pulse lengths can reduce the effect of turn-off delays inherent in the conventional "bang-bang" system, making it possible to reduce the limit cycle amplitude and minimize gas consumption.

INTRODUCTION

Many ways have been proposed to attitude control spacecraft. Some would exert control through the application of internally generated torques due to rotating equipment (1)³ and others would apply external torques to the vehicle. If the space vehicle is a satellite, the Earth's ambient magnetic and gravitational fields could be used to obtain external torques for attitude control. (See Ref. 2 for a discussion of magnetic torquing or Ref. 3 for a discussion of the gravity

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gradient effects upon satellite attitude control.) In present day spacecraft design, the most common source used for external torque is that of mass expulsion, employing either on-off valves ("bang-bang" control, Ref. 4) or valves which have a proportional control over gas flow (5).

This paper describes the development of a new approach to the use of mass expulsion to obtain control torques. The method uses logically controlled torque pulses to obtain efficient use by the control system of the mass which must be stored on the spacecraft. The technique is particularly well suited for the spacecraft attitude control situation where more flexibility is required than simply the ability to follow attitude reference changes. In particular, the case where the torque producing elements of the control system must be capable of handling the high torque requirements needed for controlling the relatively high torque loading which occurs during a period of velocity adjustment from body fixed rocket engines. In addition, if the satellite or spacecraft has a significant duration requirement, a very efficient quiescent period must be designed in order to conserve energy consumed in the control of the vehicle. This is particularly important where gas jet torque producing devices are used, since the mass expelled must be carried as payload.

These conflicting requirements usually lead to the design of control systems with two or more modes of operation; for example, high and low thrust gas jets (6) or gas jets with reaction wheel control systems (7). For the sake of simplicity in logic circuitry and reliability of the control system, it would appear, in many cases, to be ideal to use only one control mode for an entire mission. What appears to be the simplest approach is the on-off controller which applies full control torque whenever the magnitude of a deadband region is exceeded. However, the usual requirements for relatively high torque capability make it difficult, if not impossible, to obtain an efficient limit cycle operation due to non-idealities such as hysteresis, delay, and lags in the controller (8). Thus gas is unnecessarily wasted during the relatively quiescent periods of operation.

Another common solution is one in which the use of a linear controller exerts proportional control over gas flow. Typically, the performance of such systems over long durations is dominated by the non-idealities which result in a "hunting" or limit cycling of the system about the reference command. Both the on-off and proportional control systems require either measured or derived rate information to provide the damping needed for stability. In order to avoid excessive

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noise amplification, a narrow band lead filter can be used to derive rate information; however, the associated lag, combined with any shutoff delay the valve may contain, further degrades the performance of the system. Although it may be entirely possible to use gyros for rate information over very long durations of operation, it seems desirable to examine the feasibility of eliminating the need for rate data altogether.

What is desired, then, is a control system which actually makes use of the non-idealities to achieve minimum gas consumption and, at the same time, assures convergence into a stable and efficient limit cycle. This paper describes the development of a new approach to pulsed jet attitude control which accomplishes this and is based upon simple logical control of gas thrusts in the phase plane.

SYSTEM DESCRIPTION

It has been assumed that the vehicle dynamics in a single control axis may be adequately represented by the following differential equation

$$J\ddot{\theta} = M \quad [1]$$

where the task is, as usual, to find a control torque function M which is in some sense optimum. The other symbols are the moment of inertia J in the control axis, and the attitude acceleration $\ddot{\theta}$ (second derivative of θ with respect to time). The nonlinearities in the torque controller eventually dominate the satellite or spacecraft quiescent control periods and the system will limit cycle in attitude.

As a satellite or spacecraft control system limit cycles in attitude, angular momentum must be added first in one direction and then in the other to maintain the specified tolerance about the reference direction. This may be most easily shown on the phase plane and is depicted in Fig. 1. It may be noticed that during each cycle an angular momentum of $2J\Delta\dot{\theta}$ must be produced. Hence, the thrust impulse per cycle in a control axis can be easily calculated

$$F\Delta t = 2J\Delta\dot{\theta}/l_c \quad [2]$$

and the weight of gas per cycle is

$$w = F\Delta t/I_s = 2J\Delta\dot{\theta}/l_c I_s \quad [3]$$

where w = weight of gas used per cycle

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F = thrust level of gas jet, constant

Δt = thrust on time

I_s = specific impulse

$\Delta \dot{\theta}$ = attitude rate change each half cycle (see Fig. 1)

l_c = jet control thrust lever arm

Thus, for any long period of operation T , the weight of gas consumed in limit cycle operation is given by

$$W = wT/P = 2J\Delta\dot{\theta}T/l_c I_s P \quad [4]$$

where P is the period of the limit cycle and may be quickly approximated by assuming the torque on-time to be very small in comparison with the period P . If the deadband limits are $\pm \theta_o$, P is given by

$$P = 8\theta_o/\Delta\dot{\theta} \quad [5]$$

This expression may be substituted into Eq. 4 to obtain the familiar result

$$W = k (\Delta\dot{\theta})^2 T \quad [6]$$

It is evident that k depends on the design parameters of deadband, control lever arm, and specific impulse

$$k = J/4\theta_o l_c I_s \quad [7]$$

An increase in the value of any one of these design parameters decreases the total weight consumed in the act of controlling the vehicle's attitude. The most critical parameter in the cost of gas weight appears to be $\Delta\dot{\theta}$, or the attitude rate of the limit cycle, because it enters in the second power. It can be seen from Eq. 2 that the key parameter in reducing the attitude rate is the gas jet on-time Δt . Once the minimum control thrust is chosen, limit cycle rate can never be gotten lower than that given by the absolute minimum jet on-time. It would seem desirable to use this fact to design the most efficient of possible limit cycles. Such a design philosophy for this attitude control system permits the use of reliable and simple on-off solenoid valves which assure minimum leakage and allow application to either compressed or liquified gases or hypergolic fuels as the mass impulse element.

Figure 2 is examined to see how the logical solution is

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carried out in the phase plane. Using attitude error levels only (no error rate is needed), the controller meters out the absolute minimum gas pulse when the first error level (switch "+ SW 1" for the example in Fig. 2) has been reached. Thus, the electronic pulse that drives the valve is designed to be the minimum on-time capability of the valve. If the next error level is reached, an identical pulse may be generated, and so on until the vehicle attitude rate changes sign. When the switching lines are approached from a direction of decreasing error they produce no control action. When the trajectory of the vehicle attitude reaches the negative switching lines, an identical control action takes place. For the example represented in Fig. 2, the first negative switching line meters enough of a pulse to change the sign of the rate and "capture" the attitude into the absolute minimum rate limit cycle. Several interesting features of this scheme are apparent. Noise or uncertainty in the precise location of the error levels is somewhat unimportant to the operation of the controller, provided the switches have large enough hysteresis to avoid noise triggering the switching line twice.

Thus, rather than working to reduce the hysteresis, a certain amount is intentionally left in the controller. Valve pure delay and sensor lags do not affect the size of the final limit cycle as the gas jets are not switched off by the error signal, but pulsed a fixed length of time. The final deadband tolerance on attitude accuracy corresponds to the switching lines \pm SW 1 and hence the system "overshoots" while settling out of a transient condition. If this is not acceptable, then the actual operating condition will have to be at a narrower deadband. It is also of interest that the final limit cycle is unsymmetrical about the attitude rate axis, coasting more rapidly in one direction than in the other. The amount of dissymmetry is a function of the initial conditions.

In summary, the basic idea of the control system is to provide control of the attitude of the spacecraft with what amounts to pulse torquing according to error limits and damping the motion in a nonlinear fashion by using phase plane quadrant information only. The knowledge of the quadrant is obtained simply by using a property of the phase plane: An increasing error magnitude defines the first and third quadrants. Thus, only if the error is increasing in magnitude should control action be taken. No measured or derived rate signals are needed. The gas jet minimum pulse is used for an efficient final limit cycle, and the usual system difficulties of noise, hysteresis and delays have no degrading effect on system performance. The practical design aspects of this scheme shall be considered next.

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DESIGN CONSIDERATIONS

Many practical design problems must be solved before this system can be put into application; for example, the design considerations of system settling time, the number and placement of switching lines, the avoidance of two and three pulse limit cycles, the control of torque loading, and the control of other transient conditions.

Pulse Sizing

In order to encourage the system to converge rapidly after a transient, it may be desirable to increase the duration of the pulses subsequent to the first pulse. The phase plane diagram is ideally suited as an analysis tool for this problem consideration. For simplicity at this time, consideration will be directed at two switching lines. Fig. 3 shows a "two-pulse" limit cycle resulting when the second pulse is larger than the first. Whether multipulse limit cycles will occur is a function of the initial conditions. If all initial conditions which can be captured by two switching lines is considered as occurring with equal probability, then, for the case shown in Fig. 4, it can be seen that there are three corridors on each side of the attitude axis for which capture is possible. Two of these corridors will not produce two-pulse limit cycles, but the cross hatched one will; hence, the probability of an undesirable limit cycle may be written as a ratio involving the angular momenta removed by the pulses

$$P(u) = \frac{H_2 - H_1}{H_1 + H_2} \quad H_2 > H_1 \quad [8]$$

The pulse size may be defined from Eq. 2 as the angular momentum change during the pulse on time. Thus, the pulse size for the n^{th} pulse is

$$H_n = J\Delta\dot{\theta}_n \quad [9]$$

Examining Eq. 8, it appears possible to avoid altogether two pulse limit cycles by appropriately sizing the pulses. In fact, it can be seen from Figs. 3 and 4 that if H_2 is equal to or less than H_1 , the pulse at SW 1 will always capture the vehicle into the desired final limit cycle. This line of reasoning may be carried to many pulses. In Fig. 5 a three-pulse limit cycle is shown where the first two pulses are equal and the third pulse is slightly larger than their sum. From this, the pulse sizing which avoids multipulse limit cycles may be found to be

$$H_{n+1} < \sum_{k=1}^n H_k \quad [10]$$

It should be noticed that in the practical selection of the pulse sizing for the first two pulses, the fact that the attitude rates in question are very low indeed may be taken into account. Thus, if it should happen that a pulse put the vehicle phase-plane trajectory very close to the zero rate axis, the time it would take for the attitude to drift out to the next switching line would be so great that aerodynamic, gravity gradient, or solar pressure effects would certainly dominate the behavior. In other words, the vehicle would give the appearance of being "stuck" at that attitude. This occurrence was observed in the laboratory investigation of this device.

Transient Conditions

The occurrence of extremely low attitude rates at an attitude offset (becoming stuck at a fixed attitude error) has been discussed in the preceding paragraph. This leads to the questions: what occurs during a transient, and what are the settling problems with this system?

It appears that as a matter of practical design approach, a simple set of maximum effort switching lines should be used together with a few pulse-torque switching lines in order to control effectively all probable initial conditions and up-setting torque transients. The control torque switching lines and a typical response trajectory are shown in Figs. 6 and 7 for a system of practical design. The response shown represents a sudden torque loading which might occur due to a rocket engine firing time and then recovery from the transient when the engine is turned off. The switching lines are shown for a case in which lead shaping has been used for derived rate. The knee in the switching lines is due to a rate deadband and corresponds to the inability of the system to obtain a meaningful measurement of very low rates. The use of pulse-torque switching lines circumvents any need for low rate measurements.

The transient operation shown in Figs. 6 and 7 also shows a relatively large addition of angular momentum at the negative backup switching lines H_0 . This buildup in angular momentum is due to a combination of non-idealities in a practical design. The dominant effects are hysteresis, sensor lags and jet turn-off delay. The lags and turnoff delays are represented in Fig. 6 by the extension of the trajectory beyond the switching lines. The hysteresis is shown by the difference in the position of the on and off switching lines. The same design philosophy applies to the maximum H_0 that can be tolerated from the backup switches as applies to the pulse sizing. To aid in the examination of this problem, Fig. 8 shows a limit cycle involving

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the backup switches. By inspection, the following relations may be written

$$\text{Stable limit cycle: } \sum_{k=1}^3 H_k + H_{in} = H_o \quad [11]$$

$$\text{Divergent trajectory: } \sum_{k=1}^3 H_k + H_{in} < H_o \quad [12]$$

$$\text{Convergent trajectory: } \sum_{k=1}^3 H_k + H_{in} > H_o \quad [13]$$

In these equations, H_{in} is the angular momentum of the system when the vehicle's attitude trajectory enters the backup switches, and H_o is the angular momentum at control torque removal. Eq. 11 describes a stable limit cycle, Eq. 12 an unstable divergent condition, and Eq. 13 a condition which is converging toward lower rates. The desirable behavior described by Eq. 13 may be used to design the torque pulse switching scheme. The sum of the torque pulses must obey the following equation

$$\sum_{k=1}^n H_k > H_o - H_{in} \quad [14]$$

and the number n may be found by applying Eq. 10 until Eq. 14 is satisfied.

The design of the backup switches follows conventional techniques and is well documented in the literature (9,10). The quantity $H_o - H_{in}$ on the right-hand side of Eq. 14 may be quickly estimated by using the rate diagram method of analysis (10), or by using graphical techniques with the phase plane. Using either technique, the angular momentum H_o may be defined as a function of the angular momentum H_{in} and the indicated difference taken. The region of concern for $H_o - H_{in}$ is from zero initial angular momentum into the switching lines to the momentum values of the stable limit cycle which would exist in the on-off control system without the addition of the pulse torque switching lines. Thus, the next step is to define the maximum angular momentum difference $H_o - H_{in}$, which the torque pulse method must be capable of overcoming in order to produce the desired operation. This design technique may be considered to be simply the addition of pulse torque methods to the "best" on-off design possible in order to greatly reduce the weight of the pneumatic system by obtaining the lowest possible limit cycle rates.

The consideration of system lags dictates the selection of error levels or spacing for the torque pulse switching lines.

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In fact, the response of the sensor and shaping networks (if any) to ramp functions of very low rates, along with the duration and shape of the torque-pulse trajectory, dictate the spacing between pulse switching lines. The lags should be considered because the controller is operating on a measurement of the vehicle's attitude error and must be able to keep up with a change in direction if the desired single pulse final limit cycle is to be reached. Since the rates are very low at the point of final capture this need not pose any serious problem. At the higher rates, the lag does not make much difference in the system's operation, since more than one torque pulse must be used.

The hysteresis of the torque-pulse switching lines must be great enough to avoid an accidental triggering of any one of the switching lines twice in rapid succession due to noise on the error signal. The theory of operation of the controller calls for an increasing error signal to trip the torque pulse output. In the absence of hysteresis, a noisy signal could appear to be an increasing error signal, even if the first triggered pulse had reversed the sign of the error rate. Thus, hysteresis acts as a "reset" feature, and the switching line would not be ready to trigger another pulse unless reset by a decreasing error signal. If quantitative statistical data were available on the noise, the hysteresis could be set at any desirable probability level based on the data (for example, three times the standard deviation). Overlapping hysteresis bands would be perfectly acceptable, provided the bands lie within the deadband region of the controller, thus assuring the reset action.

DEVELOPMENT

In order to examine the expected simplicity and operation of the system in detail, a laboratory version of this controller was designed and built. The system has three pulse-torque switches with adjustable pulse durations for experimental purposes. The block diagram of the system follows quite logically from the sequence of events described on the phase plane and is shown in Fig. 9. An angular reference sensor is shown which is assumed to have a linear voltage output proportional to the angular error from the desired reference direction.

If the pulse-torque switching line system is used without rate information in the actuating signal, so that the switching lines are vertical, even the requirement of linearity reduces to the requirement that the error signal be a monotonic function. In either case, the error detector may be any con-

ventional sensor, such as a gyro or horizon scanner.

Logic and Mechanization

At first glance, it appears necessary to establish switching circuits for each of the plus and minus switching lines shown on the phase plane, or a total of eight for the three-pulse system studied in the laboratory. This may be simplified by using an absolute magnitude circuit to convert bipolar error signals into signals which have a single positive polarity for both positive and negative angular errors. If this is done, a polarity detector must be provided in order to apply the correction torque in the proper direction. The polarity detector must simply generate a discrete signal to one valve for positive angular errors and to the other valve when the angular error is negative.

Thus, it is only necessary to provide a single independent error level detecting circuit for $\pm SW_1$, $\pm SW_2$, $\pm SW_3$ and the \pm backup switches, respectively. These four circuits merely have a discrete logical output when the input exceeds some angular error. As already mentioned, the pulse torque switches SW_1 , SW_2 , and SW_3 must have a certain amount of hysteresis in order to prevent multiple pulse triggering due to a noisy signal. This is easily provided by the inherent characteristics of the conventional voltage level detector. For the backup switches, on the other hand, it is desired to reduce the hysteresis to a minimum. This assures the minimum momentum added by the backup control torque. Even a hysteresis small enough to permit noise uncertainties in switching may be tolerated, since the final limit cycle will not be affected. The hysteresis of the switch may be reduced to any desired level merely by the addition of gain. In the laboratory model, this hysteresis is about 0.8%, whereas the pulse torque switches have a hysteresis of about 10%.

To achieve the desired sequence of thrust pulses, crossing of switching lines SW_1 , SW_2 , and SW_3 must generate a thrust pulse only when the crossings are in the direction of increasing error and not when the error is decreasing. Furthermore, each pulse must be of the proper predetermined length, as previously discussed under the section on pulse sizing. To achieve this, the step function out of the voltage level detectors is differentiated in an RC network to give a positive pulse when the error is increasing and a negative pulse when the error is decreasing. Only the positive pulse triggers the appropriate timing circuit. The timing circuits themselves may be anything which produce discrete outputs for the prescribed thrust times. Fig. 10 shows monostable multivibrators

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although a counter and some associated logic could be used to obtain any accuracy desired.

In order to turn on the valves at the appropriate times and in the correct directions, some simple switching logic is required. The switching signals must be such that a valve is on when a SW_1 , SW_2 , or SW_3 pulse or backup pulse is on and the polarity signal exists for that particular valve. These may be expressed by the logical functions

$$(+ \text{ valve on}) = (P_{SW_1} + P_{SW_2} + P_{SW_3} + P_B) \cdot (- \text{ error}) \quad [15]$$

$$(- \text{ valve on}) = (P_{SW_1} + P_{SW_2} + P_{SW_3} + P_B) \cdot (+ \text{ error}) \quad [16]$$

where $P_{SW_{1,2,3}}$ = on when the pulses corresponding to $\pm SW_{1,2,3}$ are being measured out

P_B = on when the \pm backup switches are on

(- error) = on when angular error is negative

(+ error) = on when angular error is positive

The mechanization of these may be very simply accomplished by diode networks. The output of these gates are low level signals, so switching amplifiers are used to supply sufficient current to positively operate the valve.

Ideally, the valves should be driven by constant current sources. The solenoids of the valves being used have both resistance and a nonlinear inductance. If energized by a voltage, there is a finite delay between the initial application of voltage and the time when the current in the solenoid has risen to a value sufficient to open the valve. Similarly, there is a delay in turning off the valve, since the current takes time to decay to a value small enough to permit the valve to close. It is nevertheless desirable to drive the valve this way because of the greater efficiency and simplicity. With the normal on-off controller, these valve delays add directly to the other system lags in determining the limit cycle. These delays do not appreciably affect the operation of the pulsed system, except when the backup switch becomes necessary. As can be seen from Eq. 2, the most important parameter for the pulse-torque switching scheme is the minimum pulse duration Δt of which the valve is capable. It is obviously

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possible to make the pulse so narrow that the valve is unable to respond to it and either remains closed or only partially opens. Because of the desire to have a somewhat predictable impulse delivered by the valve, the pulse is made just wide enough so that the valve will always open completely under the worst conditions expected in terms of environment and wear. For a typical small solenoid valve considered suitable for this application, the opening and closing times are between 5 and 10 millisecon. The minimum pulse length should be just larger than the worst opening time, or about 10 millisecon.

Laboratory Setup

The closed loop operation of the controller was studied in the laboratory. The instrumentation for the laboratory system is shown in Fig. 10. For this setup a solenoid valve was used to meter the gas. The expansion nozzle was placed at the end of a short length of tubing. The deflection of the tubing was instrumented with a balanced strain gage bridge. The bridge signal was amplified and could be easily scaled to represent any desired torque level in the closed loop studies of system performance. The torque signal contained the valve and tubing delays, along with any electronic lags, and accurately represented the actual torque function that would be applied to a satellite control axis. The vehicle dynamics represented by Eq. 1 could be simulated by a knife-edge or air bearing supported table (11). However, for convenience in scaling the problem and flexibility in applying torque loading disturbances, the dynamics were simulated by electronic integrators, as shown in Fig. 10.

The polarity detector and sign changer were used in the manner shown, instead of having separate valves and instrumentation. Thus, the laboratory setup differed slightly from a flight mechanization in that the polarity detector was not used to gate different valves. It was used to invert the sign of the scaled thrust pulse applied to the vehicle dynamics. External torque disturbances may be simulated by summing d-c voltages with the torque pulse signal at the torque summing point.

With the extremely low rate limit cycles encountered during the test of the system, the drift of the electronic operational amplifiers proved to be a problem. Even with careful balancing of these chopper-stabilized amplifiers, the drift dominated the limit cycle operation of the system just as very low torque loading effects would in a spacecraft system.

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Numerical Example

The results of the experimentation verified that the mechanization was simple and practical, and that the expected performance could be easily obtained in practice. To help indicate the possibilities for a pulse-torque control system, the gas weight consumed in a year's operation in the limit cycle mode may be calculated for a satellite control system using typical numerical values

$$J = 100 \text{ slug-ft}^2$$

$$F = 0.2 \text{ lb}$$

$$l_c = 5 \text{ ft}$$

$$\theta_o = 10^{-2} \text{ radian}$$

$$I_s = 60 \text{ lb-sec/lb}$$

$$\Delta t = 10 \text{ millisecc}$$

Using Eq. 2, it is found that

$$\Delta \dot{\theta} = 10^{-4} \text{ radian/sec} \quad [17]$$

Equation 5 may be used to find the period

$$P = 800 \text{ sec} \quad [18]$$

The weight of gas may be found from Eqs. 6 and 7

$$W = kT (10^{-8}) \quad [19]$$

The parameter k may be evaluated and is

$$k = 8.33 \text{ lb-sec} \quad [20]$$

For one year Eq. 19 becomes

$$W = 2.6 \text{ lb of gas} \quad [21]$$

DIGITAL MECHANIZATION

The possibility is interesting of mechanizing the pulse-torque system with strictly digital hardware. The usual digital control system considered for similar applications follows the philosophy of attempting to approximate a linear system operation by computing with discrete binary quantities.

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This results in some fairly complex equipment which is not needed when the output consists only of timed pulses.

A digital approach toward the pulse-torque hardware becomes attractive if the sensor information is directly available in digital form without requiring an analog to digital converter, or if the remainder of the navigation system is to be digital, allowing the attitude control functions to be assumed by a central computer. The controller then requires a set of gates to detect crossings of each of the switching lines, a register to remember which of the torque pulses have been fired, a counter to meter out the timed pulses, and the associated logic to control the valves. The saving in equipment over the usual digital control system occurs because no arithmetic computations are required.

It should be noticed that the switching lines in such a system are vertical. Thus, the system must be designed such that initial capture and convergence will take place under the worst expected initial conditions. This probably implies that a larger number of pulse switching lines will be required for any reasonable range of control capability than would be the case with the analog pulsed system.

ACKNOWLEDGMENT

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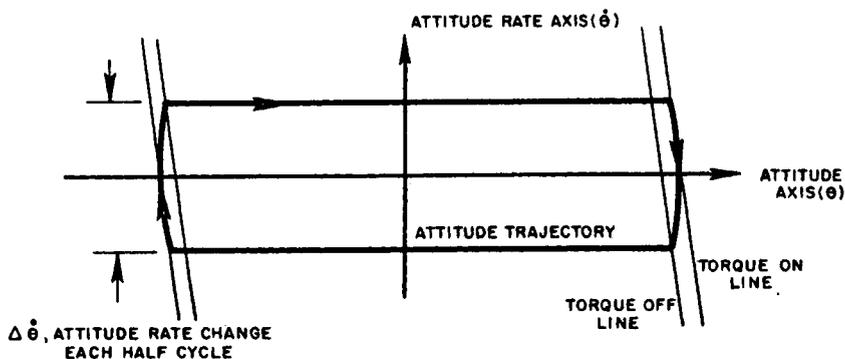


Fig. 1 Phase plane showing attitude limit cycle

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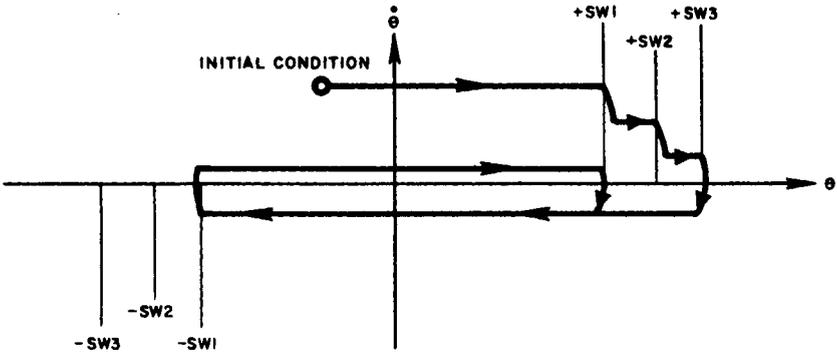


Fig. 2 Phase plane showing multiple switching lines

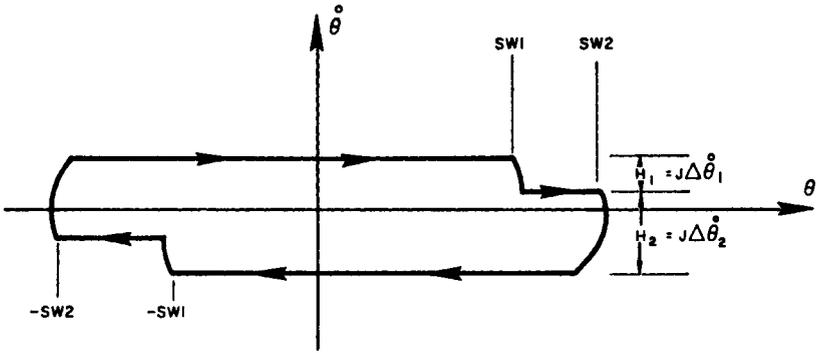


Fig. 3 Two-pulse limit cycle

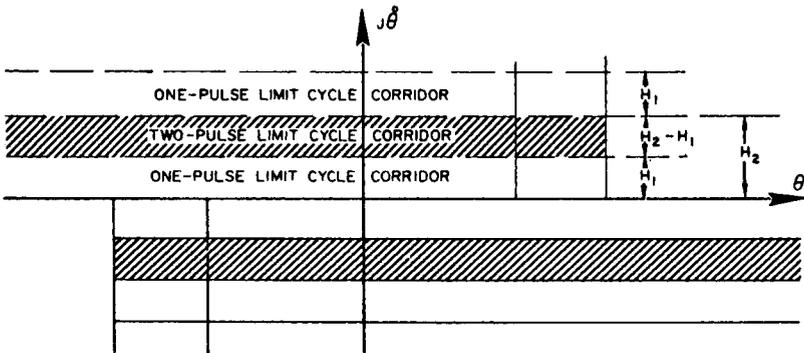


Fig. 4 Phase plane showing the effect of initial conditions on the limit cycle

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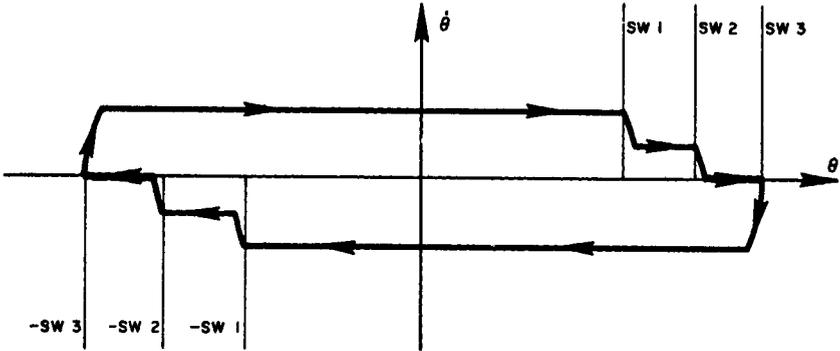


Fig. 5 A three-pulse limit cycle

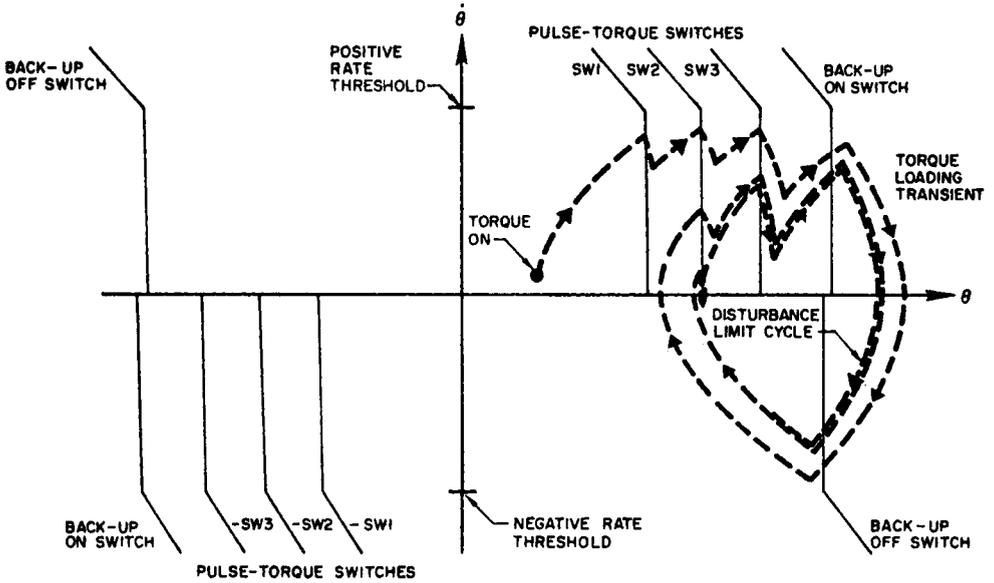


Fig. 6 A typical transient response to high torque disturbance

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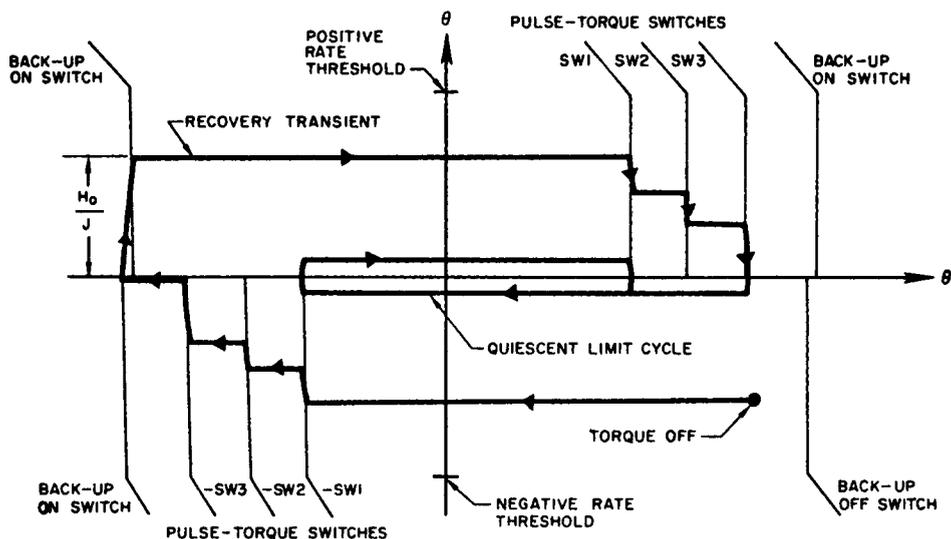


Fig. 7 Recovery of system from transient

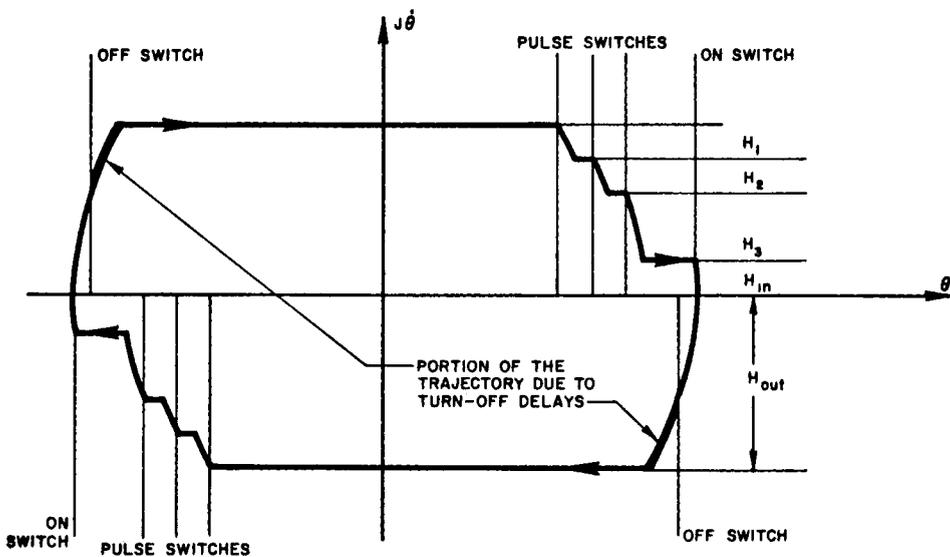


Fig. 8 Phase plane showing limit cycle involving backup switches

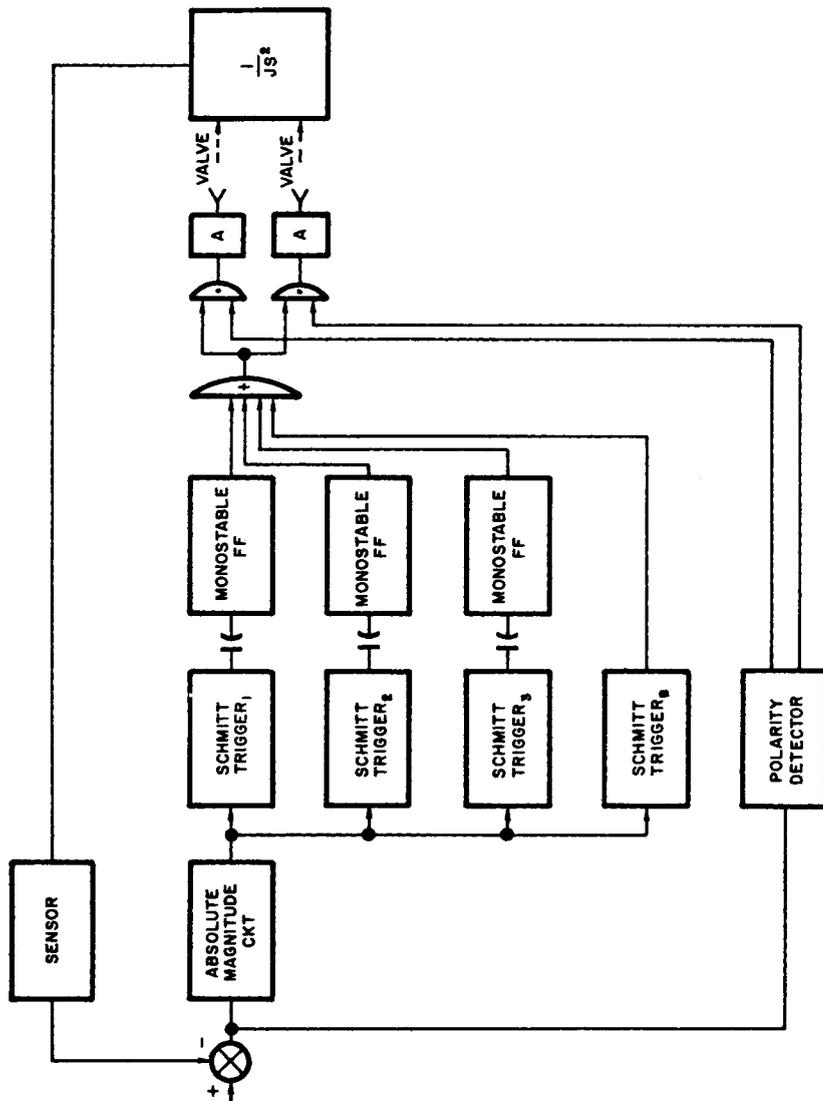


Fig. 9 Pulse-thrust system

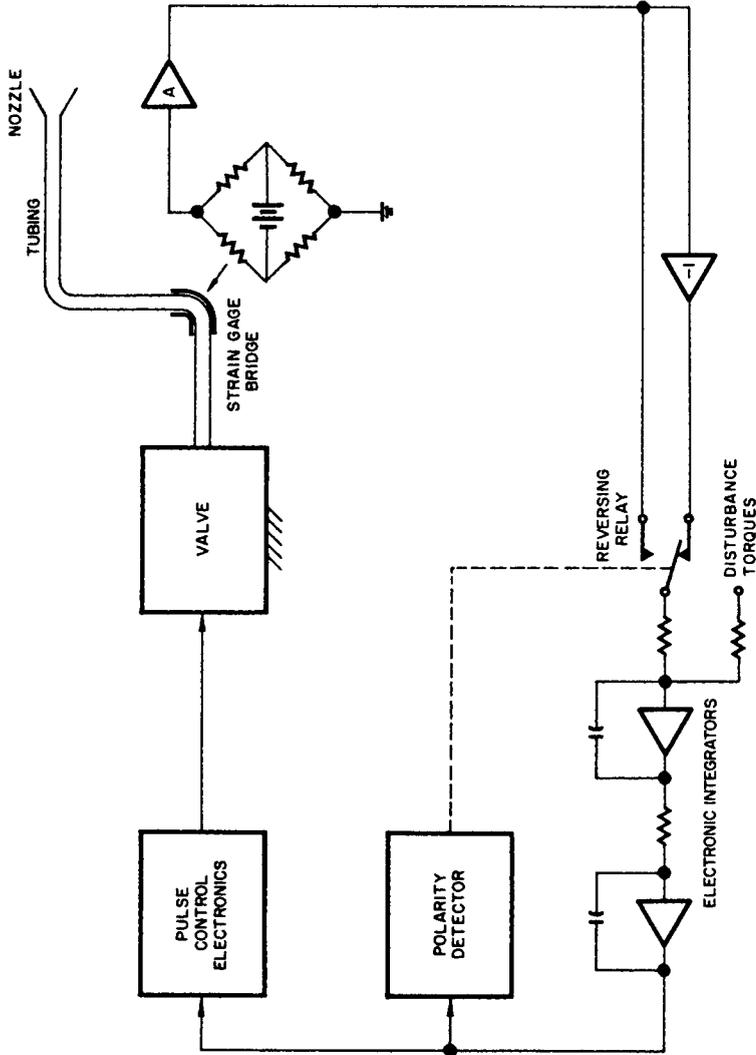


Fig. 10 Simulated systems with valve instrumentation