Electron Magnetic Moment and Atomic Magnetism*

VERNON W. HUGHES
Yale University, New Haven, Connecticut

I. Introduction

The original stimulus for recent precise measurements of atomic electronic magnetic moments or $g_J$ values was the interest in the spin magnetic moment of the electron. The discovery of the Lamb shift\(^1\) and the disagreement of the measured value of the hyperfine structure splitting in hydrogen\(^2\) with the value computed from the Fermi formula led to the suggestion that the spin magnetic moment of the electron is larger than 1 Bohr magneton.\(^3\) Comparisons of experimentally determined $g_J$ values in different atoms in states involving both orbital and spin electronic magnetic moments proved the existence of an anomalous magnetic moment of the electron about 0.1% larger than the Bohr magneton.\(^4\)

The anomalous spin magnetic moment of the electron arises from virtual radiative processes involving photons and electron-positron pairs. On the basis of modern quantum electrodynamics its value has been computed to order $\alpha^2$ ($\alpha = $ fine structure constant), and hence to an accuracy of about 1 part in a million.\(^5\) The most accurate experimental determination is obtained from a comparison of the $g_J$ value of hydrogen in its ground state with the orbital $g$ value, $g_l$, of the free electron, and has an uncertainty of about 10 parts in a million.\(^6\)

Atomic $g_J$ values of many multielectron atoms have been measured to an accuracy of about 1 part in a million and the theory of atomic magnetism has been extended to order $\alpha^2$ to include the relativistic as well as the virtual radiative contributions to atomic magnetism.\(^7\) Particularly for the case of helium a strong confirmation of the theory of

* The preparation of this article was aided in part by the Office of Naval Research, the Air Force Office of Scientific Research and Development, and the National Science Foundation.
the relativistic contributions has been achieved. Substantial discrepancies exist between the experimental and theoretical $g_J$ values for many other multielectron atoms. The most likely explanation of these discrepancies appears to be the inadequacy of the atomic wavefunctions available for the computations, and hence experimentally determined $g_J$ values can be used as a test for atomic wavefunctions. From another viewpoint, if adequate atomic Schroedinger wavefunctions were available for multielectron atoms, the highly precise experimental $g_J$ values could be used to obtain the value of the spin magnetic moment of the electron to a higher precision than it is presently known from experiments on hydrogen and the free electron.

THEORY OF ATOMIC MAGNETISM

II. Electron spin magnetic moment

The Dirac theory of the electron predicts that the spin magnetic moment of the electron is 1 Bohr magneton

$$\left( \mu_0 = \frac{e\hbar}{2mc} \right)$$

and hence that the spin $g$-value of the electron is 2.

$$\left[ g_s = \frac{\text{spin magnetic moment (in units of } \mu_0)}{\text{spin angular momentum (in units of } \hbar)} \right]$$

Corrections to the Dirac theory value of $g_s = 2$ arise due to virtual radiative processes. The order $\alpha$ and $\alpha^2$ corrections have been computed by the modern covariant, renormalized theory of quantum electrodynamics and the Feynman diagrams which contribute are shown in Fig. 1. The theoretical value is:

$$g_s = 2 \left[ 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} \right] \quad (1)$$

$$= 2 \left[ 1 + 0.0011614 - 0.0000018 \right] \quad (1a)$$

$$= 2 \left[ 1.0011596 \right] \quad (1b)$$
III. Atomic $g_j$-values

The theory of atomic magnetism for a multielectron atom is based on a generalized Dirac-Breit equation with the addition of a term to represent the interaction of the anomalous spin magnetic moment of each electron with the external magnetic field. The equation reads:

$$\left\{ \sum_{k=1}^{n} [\beta_k mc^2 + \alpha_k \cdot (cp_k + eA_k)] + V + \sum_{l>k=1}^{n} B_{kl} + \Omega H_0 \right\} \Psi = E\Psi \quad (2)$$

$E$ is the stationary state energy, including the rest energy of the $n$ electrons; $V$ is the sum of the electrostatic interactions:

$$V = -\sum_{k=1}^{n} \frac{Ze^2}{r_k} + \sum_{l>k=1}^{n} \frac{e^2}{r_{kl}} \quad (2a)$$
in which \(-\varepsilon\) is the electron charge, \(Z\varepsilon\) is the nuclear charge; \(\mathbf{p}_k\) is the momentum of the \(k\)th electron; \(\mathbf{A}_k\) is the vector potential of the external field at the \(k\)th electron and for a field \(\mathbf{H}_0\) constant in space and time,

\[
\mathbf{A}_k = \frac{1}{2} \mathbf{H}_0 \times \mathbf{r}_k;
\]

(2b)

\(\alpha_k, \beta_k\) are the Dirac matrices; \(B_{k\ell}\) is the Breit interaction between the \(k\)th and \(\ell\)th electrons,

\[
B_{k\ell} = \frac{-\varepsilon^2}{2r_{k\ell}} \left[ \alpha_k \cdot \alpha_\ell + \frac{(\alpha_k \cdot r_{k\ell})(\alpha_\ell \cdot r_{k\ell})}{r_{k\ell}^2} \right]
\]

(2c)

in which \(r_{k\ell} = r_k - r_\ell\);

\[
\Omega_{H_0} = \mu_0 \left( \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} \right) \sum_{k=1}^{n} \sigma_k' \cdot \mathbf{H}_0
\]

(2d)

is the interaction of the anomalous part of the electron spin magnetic moments with the external field \(\mathbf{H}_0\), in which \(\sigma_k'\) is the \(4 \times 4\) Pauli spin matrix; the wave function \(\Psi\) depends on \(n\) space coordinates \(\mathbf{r}_k\) and on \(n\) 4-component spinor variables.

The magnetic interaction term linear in \(H_0\) is computed with first-order perturbation theory by evaluating the diagonal matrix element of:

\[
\mathcal{H}_{H_0} = \sum_{k=1}^{n} e \alpha_k \cdot \mathbf{A}_k + \Omega_{H_0}
\]

(3)

for the zeroth order wavefunction \(\Psi_0\) obtained from Eq. (2) with the omission of the Hamiltonian terms (3) which depend on the magnetic field. Another equivalent procedure is to derive a Schroedinger-Pauli equation from Eq. (2) and then to evaluate by first-order perturbation theory the energy term which depends linearly on \(H_0\). Higher-order terms in \(H_0\) correspond to diamagnetic corrections and are usually negligible. The result for the magnetic interaction energy can be expressed:

\[
E_{H_0} = \mu_0 g_J H_0 M_J
\]

(4)

in which \(M_J\) is the magnetic quantum number for the total electronic angular momentum. The quantity \(g_J\) can be expressed as a power series in \(\alpha\) and the calculations are correct to the order of \(\alpha^2\).
ELECTRON MAGNETIC MOMENT

The lowest order term for $g_J$ is the value obtained from the Schroedinger-Pauli theory. If the atom obeys Russell-Saunders coupling so that the total orbital angular momentum $L$ and the total spin angular momentum $S$ are separately constants of the motion, then

$$g_J = g_L \frac{J(J+1)+L(L+1)-S(S+1)}{2J(J+1)} + g_S \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$$

(5)

in which $g_L = 1$ and $g_S = 2$. If the atom obeys the $jj$ coupling scheme then the atomic magnetic moment results from the coupling of all the electronic magnetic moments characterized by their $g_J$ values. For the simple case of two electrons with $jj$ coupling

$$g_J = g_{j_1} \frac{J(J+1)+j_1(j_1+1)-j_2(j_2+1)}{2J(J+1)} + g_{j_2} \frac{J(J+1)+j_2(j_2+1)-j_1(j_1+1)}{2J(J+1)}$$

(6)

in which $j_1$ and $j_2$ are the angular momentum quantum numbers for the two electrons.

For many of the atoms studied the Russell-Saunders coupling is a good approximation. However, slight deviations from this coupling scheme cause important perturbations in the theoretical $g_J$ values from the viewpoint of high precision measurements. If several states with the same configuration and the same total angular momentum quantum number $J$ are present, the spin-orbit interactions lead to an admixture of these states and hence to an alteration in the $g_J$ value obtained for a pure $LS$ state. An example is the oxygen atom\(^9\) for which the ground state electronic configuration is $1s^22s^22p^4$, and two of the $LS$ states, $^3P_2$ and $^1D_2$, are admixed by spin-orbit interactions. The amplitude $c_k$ of the admixture of the foreign $LS$ state is given by first-order perturbation theory as the matrix element of the spin-orbit interaction between the two states divided by the energy difference between the two states, and the change in $g_J$ is $c_k^2 \Delta g$ in which $\Delta g$ is the difference between $g_J$ for the initial and final states. For the case of oxygen the admixture $c_k$ of $^1D_2$ state with $^3P_2$ state is 0.0065, and the change in $g_J$ for the $^3P_2$ state is $-21 \times 10^{-6}$.

More generally, a combination of electrostatic interactions between electrons which leads to configuration mixing and of spin-orbit interactions can cause the admixture of states of different $L$ and $S$ and hence can alter the $g_J$ value.\(^{24}\) The amplitude $c_k$ of the admixture of the foreign $LS$ state is given by second-order perturbation theory as:

$$c_k = \frac{V_{0l}\xi_{lk}}{E_{0l}E_{0k}}$$

(7)
$V_{0i}$ is the matrix element of the electrostatic interaction between the zeroth-order state 0 and the state $i$ having a different configuration but the same $L$ and $S$ values. $\xi_{ik}$ is the matrix element of the spin-orbit interaction between the state $i$ and the state $k$ which have the same configuration but different $L$, $S$ values. $E_{0i}$ and $E_{0k}$ are the energy differences between states 0 and $i$ and 0 and $k$, respectively. The corresponding change in $g$ value is

$$\Delta g_J = \frac{V_{0i}^2 \xi_{ik}^2 \Delta g}{E_{0i}^2 E_{0k}^2}$$

(8)

in which $\Delta g = g_J(k) - g_J(0)$. As an example, consider Na in its ground state with the zeroth-order configuration $1s^22s^22p^63s^2$. State $i$ can be a configuration $1s^22s^22p^63p^2$, and state $k$ can then be a $2P_\pm$ state. In Na the fractional change in $g_J$ is estimated to be about 1 part in a million; the changes in $g_J$ are larger for the heavier alkalis, and for Cs it is about 7 parts in 10$^5$. The effect of virtual radiative corrections is taken into account by using instead of $g_s = 2$, the value given in Eq. (1) from the quantum electrodynamical calculation for the scattering of a free electron by an external magnetic field:

$$g_s = 2 \left[ 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} \right] = 2(1.0011596)$$

Relativistic contributions to the magnetic interaction energy of order $\alpha^2 \mu_0 H_0$ arise from the Dirac-Breit equation (2). These contributions can be expressed as the following expectation values:

$$\Delta E_R = -\frac{\mu_0}{2mc^2} \left< U \sum_{k=1}^n \left( L_k + \sigma_k \right) \mathbf{p}_k \right>$$

(9)

$$\Delta E_S = \frac{\mu_0}{2mc^2} \left< U \sum_{k=1}^n \sigma_k \cdot \nabla_k V \times \mathbf{A}_k \right>$$

(10)

$$\Delta E_B = \left< U \sum_{k=1}^n \left( -\frac{e^2}{2mc^2} \frac{\mathbf{p}_l}{r_{kl}} \cdot \mathbf{A}_k \left( 1 + \frac{r_{kl}}{mc} \right) \cdot \mathbf{p}_k + \mathbf{A}_l \left( 1 + \frac{r_{kl}}{mc} \right) \cdot \mathbf{p}_k \right) + \frac{\mu_0 e^2}{mc^2 r_{kl}^3} \left( \sigma_k \cdot \mathbf{r}_{kl} \times \mathbf{A}_l + \sigma_l \cdot \mathbf{r}_{lk} \times \mathbf{A}_k \right) \right>$$

(11)
$p_k$, $L_k$, $\sigma_k^2$ are the linear momentum, orbital angular momentum, and spin angular momentum operators for the $k$th electron; $V$ is the electrostatic potential energy and $A_k$ is the vector potential at the position $r_k$ of the $k$th electron; $r_{kl} = r_k - r_l$; $U$ is the normalized wavefunction which satisfies the Schroedinger-Pauli approximation equation:

$$\left( V + \sum_{k=1}^{n} \left( \frac{p_k^2}{2m} - \frac{p_k^4}{8m^3c^2} - \frac{i\hbar}{4mc} \sigma_k \cdot \nabla_k V R_k \right) \right) - \frac{-mc^2}{4} \sum_{i>k}^{n} \left( R_k R_i M_{kl} + M_{ki} R_k R_i + R_k M_{kl} R_i + R_i M_{kl} R_k \right) U = W U \tag{12}$$

$$R_k = \frac{\sigma_k \cdot p_k}{mc}; \quad M_{kl} = \frac{e^2}{2mc^2 r_{kl}} \left( \sigma_k \cdot \sigma_l + \frac{\sigma_k \cdot r_{kl} \sigma_l \cdot r_{kl}}{r_{kl}^2} \right); \quad W = E - nmc^2$$

The contributions $\Delta E_R$, $\Delta E_S$, and $\Delta E_B$ are of the order $\alpha^2 \mu_0 H_0$ and correspond to familiar contributions to fine structure of order $\alpha^2 r_y$. Such a correspondence is to be expected in view of the occurrence of $A_k$ in the Dirac-Breit equation (2) only in the combination $p_k + (e/c) A_k$. Since this combination occurs also in the classical Hamiltonian and equations of motion for electrons, the effect of a given external vector potential can be regarded as inducing the diamagnetic momentum $(e/c) A_k$ in electron $k$ to give it the total momentum $p_k + (e/c) A_k$. A general diamagnetic classical interpretation for magnetic interaction energies is thus possible. For example, the usual nonrelativistic contribution of order $\mu_0 H_0$ has the well-known classical interpretation of diamagnetically-induced kinetic energy due to the product of mechanical momentum $p$ and the diamagnetically induced momentum $eA/c$ ($p$ may be taken as the original momentum for $A = 0$ if $eA/c \ll p$). The contribution $\Delta E_R$ of Eq. (9) to $\alpha^2 \mu_0 H_0$ is due to the "relativistic increase of mass" in which one factor of the fourth power momentum term is diamagnetically induced momentum. The contribution $\Delta E_S$ of Eq. (10) has the classical spin-orbit energy explanation with $eA/c$ replacing $p$. The contribution $\Delta E_B$ of Eq. (11) corresponds to the spin $k$-orbit $l$ and to the orbit-orbit interaction energies with $eA/c$ replacing $p$. From this point of view it is clear that spin-spin contributions to fine structure have no analogous $\alpha^2 \mu_0 H_0$ contributions. Accurate evaluation of $\Delta E_R$, $\Delta E_S$, and $\Delta E_B$ requires a knowledge of the
Schroedinger wavefunction both as to its dependence on angular and radial variables.

The nucleus of the atom is treated as a point charge Ze of infinite mass in the Dirac-Breit equation. The effects of the nuclear mass, size, and moments can be treated by perturbation theory. The effect of the finite mass of the nucleus\(^\text{10}\) is obtained from the Schroedinger-Pauli equation by including terms for the total momentum of the nucleus and of each electron of the form \(p - (q/c)A\) in which \(p\) is the linear kinetic momentum of the particle and \(A\) is the vector potential \((A = (H_0 \times r/2))\) for a uniform applied field \(H_0\), \(r\) = position coordinate with respect to a fixed point, \(q\) = charge of particle. The coordinates of the center of mass of the atom can be separated out, and the following Hamiltonian magnetic interaction term is obtained due to the nuclear motion:

\[
\mathcal{H} = \frac{e}{2mc} \left( -\frac{m}{M} \right) \left( \sum_{k=1}^{n} (s_k \times p_k) \cdot H_0 + \sum_{j \neq k} (s_j \times p_k + s_k \times p_j) \cdot H_0 \right)
\]  

\(p_k\) = linear momentum of \(k\)th electron; \(s_k = r_k - r_N\) in which \(r_N\) = radius vector for the nucleus; \(M\) = mass of nucleus. The first term in Eq. (13) is a simple reduced mass correction to the interaction of the orbital magnetic moment of the electrons with the external magnetic field. The second term involves a cross product between position and momentum coordinates of two different electrons and corresponds to the "specific isotope shift" or the Hughes-Eckart effect in atomic optical spectra. These effects of the nuclear mass are of order \((m/M)\mu_0H_0\).

The finite size of the nucleus and the nuclear magnetic dipole and electric quadrupole moments alter the Schroedinger atomic wavefunction obtained for a point charge nucleus, and admixture of states with different \(g_J\) values can occur.\(^\text{11}\) Furthermore, the change of the nuclear electrostatic potential due to the finite nuclear size or the nuclear electric quadrupole moment will alter the relativistic contributions to the magnetic interaction. These effects are much smaller than the present experimental errors. For a mu-mesonic atom, however, the radius of an atom with large \(Z\) is of the order of the nuclear radius, and the size of the nucleus will influence appreciably the value of the atomic magnetic moment. Precise measurements of the magnetic moments of mu-mesonic atoms, which are now possible, may thus yield useful information about nuclear structure.\(^\text{25}\)
Considerations based on the uncertainty principle emphasize the difficulties of experiments to measure the spin magnetic moment of the free electron by classical type experiments, e.g. a magnetometer or a Stern-Gerlach experiment.\textsuperscript{12,13} Of the many attempts to measure the electron spin magnetic moment by quantum-mechanical type experiments, two experiments have been successful thus far, but their accuracy is considerably less than that obtained in measurements of atomic magnetic moments.

The spin magnetic moment of the free electron was first measured in an experiment which utilized Mott scattering for the production and analysis of the electron polarization and compared the electron spin precession frequency to the electron orbital cyclotron frequency.\textsuperscript{13} A schematic diagram of the apparatus is shown in Fig. 2. The incoming electron beam of \((420 \pm 0.8)\)kev energy is obtained from a Cockroft-Walton generator. Mott scattering at \(90^\circ\) by a gold foil produces a partial polarization for the scattered beam transverse to the direction of the scattering. In the longitudinal magnetic field produced by a solenoid the electron spin precesses at the rate \(\nu_s = (g_s\mu_0 H_0)/\hbar\). At
the position of the analyzing foil the orientation of the electron spin is determined from a study of the asymmetry in azimuthal angle of the electron scattering. Approximately 5 revolutions of the electron spin are obtained between the first and second scatterers in the field of 400 gauss. The magnetic field is measured by the cyclotron orbital frequency \( \nu_I = (2g_I\mu_0H_0/\hbar) \) with use being made of the focusing property of the orbital motion at integral numbers of revolutions. Relativistic corrections to \( \nu_s \) and \( \nu_I \) cancel in the ratio \( \nu_s/\nu_I \) to a sufficient accuracy. The result of the experiment is \( g_s = 2.00 \pm 0.01 \), where the error is due primarily to inaccuracy in the determination of the spin orientation by the analyzer.

A spin resonance experiment on electrons in a plasma has recently been achieved using optical methods for the production and detection of the electron polarization. A schematic diagram of the experimental arrangement is shown in Fig. 3. Sodium atoms at a partial pressure of about \( 10^{-7} \) mm Hg and argon at a pressure of 70 mm Hg are contained in a bulb and illuminated with circularly polarized sodium resonance radiation. A net polarization of the sodium atoms is produced thereby. Periodically a discharge is excited to produce an electron density of some \( 3 \times 10^8 \) electrons/cm\(^3\), and the electrons become polarized by exchange collisions with the polarized sodium atoms. In a magnetic field \( H_0 \) an electron spin resonance transition can be induced by an rf magnetic field of frequency \( \nu_s = \mu_0g_sH_0/\hbar \). The resulting change in net polarization then causes a change in polarization of the sodium atoms through the mechanism of electron exchange collisions. Since the transmission of polarized sodium resonance radiation through the sodium atoms depends on the amount of polarization of the sodium atoms, the electron spin resonance can be detected.
by the resulting change in light transmission. The width of the electron spin resonance is largely determined by the electron spin polarization relaxation associated with electron-argon collisions. A Zeeman transition $\Delta M_F = \pm 1$ in the $F = 2$ hyperfine structure level of the ground state of sodium can be induced in the magnetic field $H_0$ also, and detected by the change in light transmission. The result of the experiment is the ratio of the spin $g$ value of the electron $g_e$ to the value $g_J(Na,^3S_1)$,

$$\frac{g_e}{g_J(Na,^3S_1)} = 0.999974 \pm 0.00003$$

in which the error is due to electron spin relaxation processes and inhomogeneity of the magnetic field.

V. Hydrogen

The most precise value for the electron spin magnetic moment is obtained from a comparison of a Zeeman transition frequency in the ground state of H and the cyclotron orbital frequency of a free electron in the same magnetic field. This comparison of $g_J(H,^3S_1)$ to $g_I$ has been obtained by two experiments. One is a measurement of $g_J(H,^3S_1)/g_p$, in which $g_p$, the $g$-value for a proton, is obtained from a proton resonance absorption experiment; the second is a measurement of $g_I/g_p$.

The hyperfine energy levels of the ground state of hydrogen in a magnetic field are shown in Fig. 4, and the energy levels are given by the Breit-Rabi formula:

$$W_P-J,M = \mu_0 g_p H_0 M \pm \frac{\Delta W}{2} \left[ 1 + 2Mx + x^2 \right]$$

(14)

where $F$ is the quantum number for total angular momentum and $M$ is the associated magnetic quantum number; $\Delta W$ is the energy difference at zero magnetic field between the $F = 1$ and $F = 0$ hyperfine levels; $x = \mu_0 (g_J - g_p) H_0 / \Delta W$.

One measurement of $g_J(H,^3S_1)/g_p$ was done by the atomic beam magnetic resonance method. The transition $(F, M) = (1,0) \leftrightarrow (1, -1)$ was observed in a magnetic field near 1430 gauss at a frequency of about 3655 Mc/sec. With extensive shimming of the magnetic field the natural line width of 60 kc/sec corresponding to a 4 cm rf transition region was obtained. A proton resonance absorption measurement was made in the same magnetic field. Since the hfs energy difference is known from atomic beam and microwave absorption experiments to an accuracy of about 0.1 ppm, the above experiment determines $g_J(H,^3S_1)/g_p$. Due to diamagnetic shielding the magnetic field at the
protons differs slightly from the applied magnetic field and hence in the measured ratio \( g_J(H,^2S_1)/g_p \), \( g_p \) should be interpreted as the \( g \)-value of protons in the sample used, which was a spherical sample of mineral oil. An accuracy of \( \pm 1 \) ppm was obtained for \( g_J(H,^2S_1)/g_p \).

A measurement of \( g_J(H,^2S_1)/g_p \) has also been made by a microwave magnetic resonance absorption method using hydrogen gas. The experimental arrangement is shown in Fig. 5. Hydrogen atoms formed

Fig. 4. Energy levels of the ground state of hydrogen in a magnetic field.

Fig. 5. Cutaway view of apparatus used for microwave magnetic resonance absorption experiments.
in a discharge tube diffuse through a high $Q$ microwave cavity placed between the pole faces of an electromagnet. Fixed frequency microwave power is fed into the cavity whose TE$_{011}$ mode is such as to have a component of $H_\parallel$ perpendicular to the static magnetic field $H_0$ of the electromagnet. If the static magnetic field is at the resonant value for a transition $\Delta M = \pm 1$ in hydrogen, microwave power transmitted to the bolometer will be reduced. At a frequency of about 9000 Mc/sec, the resonant magnetic fields for the two transitions with $\Delta m_J = \pm 1$, $\Delta m_I = 0$ were observed with a line width of about 200 kc/sec, which was due to magnetic field inhomogeneity and collision broadening. The proton resonance absorption frequency for a mineral oil sample was measured in the same static magnetic field so the ratio $g_J(H,^2S_1)/g_p$ is determined. A statistical accuracy of $\pm 0.3$ ppm was obtained.

The orbital cyclotron frequency of the free electron has been compared with the proton resonance absorption frequency in two independent experiments. In the first$^{17}$ the electrons were obtained from a hot filament and the electron cyclotron resonance was observed by its effect on the electron trajectories. In the second$^{18}$ the electrons were obtained by the photoelectric effect and the resonance was observed by a microwave absorption method. The sources of error in these measurements arise from inhomogeneity and nonconstancy of the magnetic field, diamagnetic effects, space charge effects, and the relativistic dependence of mass on velocity. The accuracy for each of these measurements is about $\pm 10$ ppm.

Combination of the average experimental values of $g_J(H,^2S_1)/g_p$ and $g_I/g_p$ yields the result:

$$\frac{g_J(H,^2S_1)}{g_I} = 2[1.001139 \pm 0.000010]$$

Use of the theoretical relativistic contributions $\Delta E_R$ and $\Delta E_S$ of Eqs. (9) and (10) gives the relation:

$$g_J(H,^2S_1) = g_s(1 - \alpha^2/3) = g_s(1 - 17.7 \times 10^{-6})$$

Hence the experimentally determined value of $g_s/g_I$ is

$$g_s/g_I = 2[1.001156 \pm 0.000010]$$

This experimental value is in agreement with the theoretical value $g_s/g_I = 2[1.0011596]$, but it is not sufficiently accurate to test the fourth-order $\alpha^2$ radiative correction to the spin magnetic moment of the electron.

To ascertain experimentally that the $g_J$ value of hydrogen does not depend significantly on nuclear properties, the electron $g$ value in the
ground state of deuterium $g_J(D,^2S_1)$ has been measured. An early measurement by the atomic beam magnetic resonance method compared directly $g_J(D,^2S_1)$ with $g_J(H,^2S_1)$ and found them to be equal to within the experimental error of ± 10 ppm.¹⁹ A recent measurement by the microwave magnetic resonance absorption method of $g_J(D,^2S_1)/g_p$ again yielded $g_J(D,^2S_1)/g_J(H,^2S_1) = 1$ to within the experimental accuracy of 1 ppm.²⁰

VI. Other Atoms

Accurate measurements have been made of $g_J$ values of many atoms by the methods of radiofrequency and microwave spectroscopy. Although some of the earlier measurements on multielectron atoms were used for the determination of $g_s/g_1$ for the electron, at present $g_J$ values for multielectron atoms are used primarily to test the theory of relativistic contributions to atomic magnetism or to provide a criterion for the excellence of an atomic wavefunction.

The $g_J$ value of the two electron system in the 1s2s,^3S_1 metastable state of helium has been studied particularly carefully both experimentally and theoretically. In an atomic beam magnetic resonance experiment the transitions $\Delta M = \pm 1, \pm 2$ in helium and the transitions $(F,M) = (1,0)\leftrightarrow(1,-1)$ in hydrogen were observed in the same magnetic field, and thus the ratio $g_J(He,^3S_1)/g_J(H,^2S_1)$ was determined. The first such experiment²¹ used a transition region with a single oscillating field and determined the ratio to an accuracy of ± 16 ppm. The method of separated oscillating fields was used in a more recent experiment⁸ and allowed the achievement of narrow natural line widths despite the presence of magnetic field inhomogeneities in the C-region. A schematic diagram of the apparatus used for the latter experiment is given in Fig. 6. A block diagram of the radiofrequency system is shown in Fig. 7. The "hairpin" arrangement for the separated oscillating fields is shown in Fig. 8. Line shapes for hydrogen and helium transitions obtained by the method of separated oscillating fields are shown in Figs. 9 and 10. For hydrogen, only transitions between the two levels $(F,M) = (1,0)$ and $(1,-1)$ are involved. For helium all three Zeeman levels $M = 0, \pm 1$ are involved. Theoretical line shapes for the three-level case near the central resonance are shown in Fig. 11. The experimental value obtained is

$$
\frac{g_J(He,^3S_1)}{g_J(H,^2S_1)} = 1 - (23.3 \pm 0.8) \times 10^{-6}
$$
Fig. 6. Schematic diagram of atomic beam magnetic resonance apparatus used for measurement of \( g_J(\text{He}, ^3S_1)/g_J(\text{H}, ^3S_1) \).

Fig. 7. Block diagram of the radiofrequency system used for measurement of \( g_J(\text{He}, ^3S_1)/g_J(\text{H}, ^3S_1) \).
Fig. 8. Construction of separated oscillating fields system. The parts labelled "b" are loops made of Cu; they are fed from a single rf source and carry the rf currents which produce the separated oscillating fields. The parts labelled "a" are auxiliary Cu loops used for producing two independent single oscillating fields, which allow measurement of the magnetic field in the neighborhood of both "b" loops. The brass plates "c" serve as rf shields for the loops and also as the outer conductor for the coaxial line feeding the loops. The beam passes parallel to the groove "d". At the right of the figure an end view along the propagation direction of the beam is shown.

Fig. 9. An observed $\Delta M_s = \pm 1, \pm 2$ Zeeman line of the $^3S_1$ state of helium taken with separated oscillating fields spaced 4.2 cm apart.
Fig. 10. Central, "near resonance" portions of Zeeman lines for hydrogen and helium taken with separated oscillating fields spaced 4.2 cm apart. For hydrogen the transition observed was \((F, M) = (1, 0) \leftrightarrow (1, -1)\) in the ground \(^1S_0\) state; the amplitude of the radiofrequency field was adjusted to give maximum intensity at resonance. For helium the transition observed was \(\Delta M_J = \pm 1, \pm 2\) in the metastable \(^3S_1\) state (i.e. any transition in which \(M_J\) changes for any of the three \(M_J\) states); the amplitude of the radiofrequency field was adjusted to be about 60\% of the value which gives maximum intensity at resonance.
Fig. 11. Theoretical transition probabilities near resonance for Zeeman transitions in the \(^{3}S_1\) state of helium by the method of separated oscillating fields, averaged over the velocity distribution in the beam. \(P_1\) is the probability of a transition from the initial magnetic substate \(M_J = +1\) to either of the magnetic substates \(M_J = 0\) or \(M_J = -1\); \(P_T\) is the probability of a transition from any of the magnetic substates \(M_J = 0, +1,\) or \(-1\) to a different magnetic substate. 

\(a\) = most probable velocity of atoms in the source; \(L\) = distance between oscillating fields; \(l\) = length of each oscillating field; \(b\) is the matrix element for the transition \(\Delta M_J = \pm 1\) divided by \(\hbar\) and it is proportional to the amplitude of the oscillating field (the two oscillating fields have the same amplitude and phase); \(\delta \nu\) is the frequency width at half height between the peak and first minimum; \(\omega_0 = \mu_H J_H / \hbar\); \(\omega\) = applied angular frequency. The cases \((\omega / a) = 3.6\) and \((\omega / a) = 2.8\) correspond to conditions for maximum transition probability at exact resonance for \(P_1\) and \(P_T\), respectively.

The theoretical value for \(g_J(\text{He}, ^{3}S_1)\) is given by:

\[
g_J(\text{He}, ^{3}S_1) = 2 \left[ 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} - \frac{1}{3} \frac{\langle T \rangle}{mc^2} - \frac{1}{6} \frac{\langle e^2/r_{12} \rangle}{mc^2} \right] \\
= 2 \left[ 1 + 1161.4 \times 10^{-6} - 1.8 \times 10^{-6} - 38.7 \times 10^{-6} - 2.3 \times 10^{-6} \right] \\
= 2 \left[ 1.0011186 \right]
\]

The two terms involving \(\alpha\) constitute the virtual radiative correction to the spin magnetic moment of the electron. The term \(-\frac{1}{3} \langle T \rangle/mc^2\)
in which $\langle T \rangle$ is the expectation value for the kinetic energy of the two electrons, arises from the relativistic contributions $\Delta E_R$ and $\Delta E_S$ of Eqs. (9) and (10). The term $\frac{-\hbar}{4} \langle \psi^2 | r_{12} | \psi \rangle/mc^2$ arises from the relativistic contribution $\Delta E_B$ of Eq. (11). The theoretical value for $g_J(H,^3S_1)$ is given by:

$$g_J(H,^3S_1) = 2 \left[ 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} - \frac{\alpha^2}{3} \right]$$

Hence,

$$\frac{g_J(He,^3S_1)}{g_J(H,^3S_1) \text{ theor}} = 1 - (23.3 \pm 1.0) \times 10^{-6}$$

in which the uncertainty in the theoretical value is estimated from the neglected radiative effects of order $\alpha^3 \mu_a H_0$. The excellent agreement between the experimental and theoretical values provides strong confirmation of the theory of relativistic contributions to atomic magnetism as based on the Dirac-Breit equation up to order $\alpha^2 \mu_a H_0$. Furthermore, the “additivity” of the anomalous magnetic moments of the two electrons as treated in Eq. (2) is confirmed. The comparison of the $g_J$ values for He and H does not provide any test of the anomalous magnetic moment of the electron, because this contribution cancels in the ratio $g_J(He,^3S_1)/g_J(H,^3S_1)$. Theoretically, it remains to justify the use of Eq. (2) for helium by a more fundamental field theoretical approach involving the use of the Bethe-Salpeter equation for two electrons in the presence of the nuclear Coulomb field.

Another carefully studied atom is oxygen in its ground configuration $2p^4$ for both fine structure states $^3P_2$ and $^3P_1$. The experimental values were obtained by the microwave magnetic resonance absorption method. Figure 12 shows a resonance line for the transition $M_J = 0\leftrightarrow M_J = 1$ in the $^3P_2$ state at a frequency of 9100 Mc/sec. The oxygen resonance frequencies are compared with the proton resonance frequency. Use of the measured value of $g_J(H,^3S_1)/g_p$ and the theoretical value for $g_J(H,^3S_1)$ determines the results:

$$g_J(O,^3P_1)_{\text{expt.}} = 1.500986 \pm 0.000002$$
$$g_J(O,^3P_2)_{\text{expt.}} = 1.500921 \pm 0.000002$$

The various contributions to the theoretical oxygen $g_J$ values are shown in Table 1, and the resulting theoretical values are:

$$g_J(O,^3P_1)_{\text{theor.}} = 1.500995$$
$$g_J(O,^3P_2)_{\text{theor.}} = 1.500931$$
Fig. 12. An observed $M_J = 1 \rightarrow M_J = 0$ line in the microwave magnetic resonance spectrum of the $^3P_1$ ground state of atomic oxygen. The apparatus records what is essentially the derivative of the absorption line. The observed points have been fitted with a theoretical curve which corresponds to a Lorentz absorption line having a half width of 5 parts in 10$^6$.

**Table 1**

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$\Delta g_1$</th>
<th>$\Delta g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure from LS coupling</td>
<td>0</td>
<td>-21</td>
</tr>
<tr>
<td>Virtual radiative effect</td>
<td>+1160</td>
<td>+1160</td>
</tr>
<tr>
<td>$\Delta E_B + \Delta E_S + \Delta E_B$</td>
<td>-167</td>
<td>-210</td>
</tr>
<tr>
<td>Motion of nucleus</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>Total</td>
<td>+995</td>
<td>+931</td>
</tr>
</tbody>
</table>

$\Delta g_1(\Delta g_2)$ are the contributions to the $g_J$ value of the $^3P_1(\ ^3P_2)$ state of O. Hence $g_J(^3P_1) = 1.500995$ and $g_J(^3P_2) = 1.500931$.

Both experimental values are lower than the corresponding theoretical values by 7 parts in 10$^6$. The experimental and theoretical values for the differences $g_J(^3P_2) - g_J(^3P_1)$ agree within the experimental error. The most likely cause for the discrepancies between the experimental
and theoretical $g_J$ values is inaccuracy in the atomic wavefunction used for the computation of the relativistic contributions $\Delta E_R$ and $\Delta E_S$. It should be noted, however, that computations of $\Delta E_R$ with several different wavefunctions yielded values considerably closer than could account for the discrepancies.

The $g_J$ values for the alkali atoms have been measured and the theoretical $g_J$ values for these single valence electron atoms have been computed. Substantial discrepancies exist between the experimental and theoretical $g_J$ values for Cs and Rb. The breakdown of Russell-Saunders coupling due to a combination of electrostatic and spin-orbit interactions produces appreciable positive contributions to the theoretical $g_J$ values, which is difficult to calculate accurately and which may be capable of accounting for the discrepancies.\textsuperscript{24}

Table 2 (page 87 et seq.) summarizes the experimental and theoretical information on atomic $g_J$ values.

In conclusion, the excellent agreement between the experimental and theoretical values of $g_J(\text{He}^8\text{S}_1)/g_J(\text{H}^2\text{S}_1)$ provides strong support for the theory of atomic magnetism based on the generalized Dirac-Breit equation with the addition of a magnetic interaction term for the anomalous electron spin magnetic moments. Several atomic $g_J$ values involving both orbital and spin contributions are known sufficiently well experimentally so that $g_s/g_t$ for the electron could be

\textsuperscript{*}W. Perl, Phys. Rev. 91, 852 (1953). Relativistic contributions to the $g_J$ values of alkali atoms are calculated in this reference. The general theory of these contributions to the $g_J$ value of any atom and its specialization to alkali atoms in Eq.(33) are correct. However, a calculational error was made in evaluating $\Delta E_R$ and $\Delta E_S$. The error first appears in Eq. (42), which should read

\begin{equation}
\left\langle \frac{z_{1t}^2}{r_{1t}^3} \right\rangle = \frac{1}{3} \left\langle \frac{1}{r_1} \right\rangle
\end{equation}

Correct to terms of relative order $r_{1t}^s/r_{1t}^3$ . Evaluation of $\Delta E_{RBB}$ can be made directly from Eq. (33) using an effective central potential for $V$ defined by the effective quantum numbers in Perl's paper. The result is that in Table 1 the quantities $(\Delta g/2)_+$ are identically zero and $\Delta g/2$ is given by:

\begin{equation}
\frac{\Delta g}{2} = \left( \frac{\Delta g}{2} \right)_M + \left( \frac{\Delta g}{2} \right)_L
\end{equation}

The values for $(\Delta g/2)_M$ and $(\Delta g/2)_L$ are correctly given in Table 1. The theoretical $g_J$ values for alkali atoms given in Table 2 of the present paper incorporate this correction to Perl's paper. The error in Eq. (42) of Perl's paper also invalidates the numerical results given for $^2P$ states of Ga and In. Relativistic contributions are not included in our table for the theoretical $g_J$ values of Ga and In.
obtained to an accuracy of 1 part in a million, if adequate atomic wavefunctions were available. Such an accuracy would represent an improvement by a factor of 10 over the accuracy with which \( g_J \) is known from experiments on hydrogen and the free electron. Alternatively, the theoretical quantum electrodynamic radiative corrections to the electron spin magnetic moment up to order \( \alpha^2 \) can be regarded as correct, and then accurate experimental atomic \( g_J \) values can be used as a test for atomic wavefunctions.

REFERENCES

### Table 2

<table>
<thead>
<tr>
<th>Atom</th>
<th>State&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Quantity measured&lt;sup&gt;i&lt;/sup&gt;</th>
<th>Experimental value&lt;sup&gt;e&lt;/sup&gt;</th>
<th>Theoretical value&lt;sup&gt;f&lt;/sup&gt;</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>free</td>
<td>$\frac{g_s}{g_i}$</td>
<td>$g_s = 2.00 \pm 0.10$</td>
<td>$g_e = 2(1 + \frac{a}{2\pi} - 0.328 \frac{a_s}{\pi})$</td>
<td>54L1, 50K1, 57S1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{g_s}{g_{J(Na)}}$</td>
<td>0.999974(30)</td>
<td>2.00226(40)</td>
<td>58D1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{g_s}{g_p}$</td>
<td>$\frac{2g_s}{g_p} = 657.469(9)$&lt;sup&gt;(f)&lt;/sup&gt;</td>
<td>2.002277(10)</td>
<td>51G1, 56F1</td>
</tr>
<tr>
<td>H&lt;sup&gt;1&lt;/sup&gt;</td>
<td>$^2S_1$</td>
<td>$\frac{g_J(H)}{g_p}$</td>
<td>$\frac{g_J}{g_p} = 658.2176(0.8)$&lt;sup&gt;(f)&lt;/sup&gt;</td>
<td>2.0022838</td>
<td>52K1, 28B1, 54B1, 40M1</td>
</tr>
<tr>
<td></td>
<td>$^2S_1$</td>
<td>$\frac{g_J(H)}{g_p}$</td>
<td>1.000016(12)</td>
<td>2.002309(16)</td>
<td>55W1, 28B1, 40M1</td>
</tr>
<tr>
<td>D&lt;sup&gt;3&lt;/sup&gt;</td>
<td>$^3S_1$</td>
<td>$\frac{g_J(D)}{g_p}$</td>
<td>0.9999997(2.3)</td>
<td>2.002276(10)</td>
<td>49N1, 57G1</td>
</tr>
<tr>
<td></td>
<td>$^3S_1$</td>
<td>$\frac{g_J(D)}{g_J(H)}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>He&lt;sup&gt;4&lt;/sup&gt;</td>
<td>$1s2^2S_1$</td>
<td>$\frac{g_J(He)}{g_J(H)}$</td>
<td>0.9999767(0.8)</td>
<td>2.002230(10)</td>
<td>53H1, 53P1, 58D2</td>
</tr>
<tr>
<td>Atom</td>
<td>State(b)</td>
<td>Quantity measured(c)</td>
<td>Experimental value(e)</td>
<td>Theoretical value(g)</td>
<td>Reference</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>----------------------</td>
<td>-----------------------</td>
<td>----------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Li⁴⁺</td>
<td>⁴S¹</td>
<td>( \frac{g_J(Li)}{g_J(Na)} )</td>
<td>1.000016(25)</td>
<td>2.002309(27)</td>
<td>2.002297</td>
</tr>
<tr>
<td>N¹⁴</td>
<td>⁴S¹</td>
<td>( \frac{g_J(N)}{g_J} )</td>
<td>0.9999184(1.7)</td>
<td>2.002114(10)</td>
<td>2.002</td>
</tr>
<tr>
<td>O¹⁶</td>
<td>⁴P¹</td>
<td>( \frac{g_J(O)}{g_J} )</td>
<td>0.7496368(0.9)</td>
<td>1.500981(10)</td>
<td>1.500995</td>
</tr>
<tr>
<td></td>
<td>⁴P¹</td>
<td>( \frac{g_J(F)}{g_J} )</td>
<td>0.666170(2)</td>
<td>1.333857(11)</td>
<td>1.3341</td>
</tr>
<tr>
<td>Ne²⁸</td>
<td>2p³3s³P₂</td>
<td>( \frac{g_J(Ne)}{g_J(He)} )</td>
<td>0.7495883(3)</td>
<td>1.500883(11)</td>
<td>1.50088</td>
</tr>
<tr>
<td>Na²⁵</td>
<td>⁴S¹</td>
<td>( \frac{g_J(Na)}{g_J(K)} )</td>
<td>1.000016(25)</td>
<td>2.002309(27)</td>
<td>2.002293</td>
</tr>
<tr>
<td>P¹⁴</td>
<td>⁴S¹</td>
<td>( \frac{g_J(P)}{g_J(N)} )</td>
<td>0.99981(200)</td>
<td>2.0019(200)</td>
<td>2.002</td>
</tr>
<tr>
<td>A¹⁰</td>
<td>3p³4s³P₂</td>
<td>( \frac{g_J(A)}{g_J(He)} )</td>
<td>0.7496260(5)</td>
<td>1.500959(12)</td>
<td>1.5011</td>
</tr>
<tr>
<td>Atom</td>
<td>State(b)</td>
<td>Quantity measured(i)</td>
<td>Experimental value(e)</td>
<td>Theoretical value(g)</td>
<td>Reference Exper'l. Theor'l.</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>----------------------</td>
<td>-----------------------</td>
<td>----------------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>K&lt;sup&gt;39&lt;/sup&gt;</td>
<td>2S&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(K)}{g_J(H)} )</td>
<td>1.000016(4)</td>
<td>2.002309(11)</td>
<td>2.0020</td>
</tr>
<tr>
<td>Cr&lt;sup&gt;52&lt;/sup&gt;</td>
<td>2S&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Cr)}{g_J(K)} )</td>
<td>0.999536(78)</td>
<td>2.001348(79)</td>
<td>2.002</td>
</tr>
<tr>
<td>Cu&lt;sup&gt;52,66&lt;/sup&gt;</td>
<td>2S&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Cu)}{g_J(Cs)} )</td>
<td>1.00015(500)</td>
<td>2.0026(500)</td>
<td>2.002</td>
</tr>
<tr>
<td>Ga&lt;sup&gt;49,71&lt;/sup&gt;</td>
<td>2P&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Ga,2P_1)}{g_J(Na)} )</td>
<td>0.332534(66)</td>
<td>0.665825(66)</td>
<td>0.66589</td>
</tr>
<tr>
<td></td>
<td>2P&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Ga,2P_1)}{g_J(Ga,2P_1)} )</td>
<td>0.666212(87)</td>
<td>1.333941(88)</td>
<td>1.33411</td>
</tr>
<tr>
<td>Br</td>
<td>2P&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Br,2P_1)}{g_J(Cl,2P_1)} )</td>
<td>( \frac{0.99977(400)}{} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rb&lt;sup&gt;85,87&lt;/sup&gt;</td>
<td>2S&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Rb)}{g_J(Na)} )</td>
<td>1.000066(27)</td>
<td>2.002409(29)</td>
<td>2.00234</td>
</tr>
<tr>
<td>Ag&lt;sup&gt;107,109&lt;/sup&gt;</td>
<td>2S&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(Ag)}{g_J(Cs)} )</td>
<td>1.00002(105)</td>
<td>2.00232(105)</td>
<td>2.002</td>
</tr>
<tr>
<td>In&lt;sup&gt;115&lt;/sup&gt;</td>
<td>2P&lt;sub&gt;1&lt;/sub&gt;</td>
<td>( \frac{g_J(In,2P_1)}{g_J(Na)} )</td>
<td>0.332531(102)</td>
<td>0.665819(104)</td>
<td>0.6659</td>
</tr>
<tr>
<td>Atom</td>
<td>State(^{(b)})</td>
<td>Quantity measured(^{(d)})</td>
<td>Experimental value(^{(c)})</td>
<td>Theoretical value(^{(g)})</td>
<td>Reference Exper'l. Theor'l.</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>(^{3})P(_{1})</td>
<td>(g_J)((\text{In},^{3})P(<em>{1})) (g_J)((\text{In},^{3})P(</em>{1}))</td>
<td>(0.666392(120))</td>
<td>(1.334301(12))</td>
<td>(1.3341)</td>
<td>57 F1</td>
</tr>
<tr>
<td>Xe(^{(a)})</td>
<td>(^{5})P(<em>{5}) (^{6})S(</em>{4}) (^{5})P(_{1})</td>
<td>(g_J)((\text{Xe},^{5})P(<em>{1})) (g_J)((\text{He},^{5})S(</em>{1}))</td>
<td>(0.749593(60))</td>
<td>(1.50089(60))</td>
<td>(2.002577(29))</td>
</tr>
<tr>
<td>Cs(^{133})</td>
<td>(^{5})S(_{1})</td>
<td>(g_J)((\text{Cs})) (g_J)((\text{Na}))</td>
<td>(1.000150(27))</td>
<td>(2.002577(29))</td>
<td>(2.00244)</td>
</tr>
<tr>
<td>Pr(^{141})</td>
<td>(^{4})I(_{9})</td>
<td>(g_J)((\text{Pr})) (g_J)((\text{Cs}))</td>
<td>(0.365 \pm 0.002)</td>
<td>(0.731 \pm 0.004)</td>
<td>(0.727)</td>
</tr>
<tr>
<td>Au(^{197})</td>
<td>(^{5})S(_{1})</td>
<td>(g_J)((\text{Au})) (g_J)((\text{Cs}))</td>
<td>(1.00096(57))</td>
<td>(2.00420(58))</td>
<td>(2.002)</td>
</tr>
<tr>
<td>Hg(^{(b)})</td>
<td>(^{6})S(<em>{3}) (^{8})P(</em>{1})</td>
<td>(g_J)((\text{Hg})) (g_J)((\text{p}))</td>
<td>(0.74106(270))</td>
<td>(1.4838(270))</td>
<td>(1.50)</td>
</tr>
</tbody>
</table>

\(^{(a)}\) Experiment was done on xenon with its natural isotopic abundance.

\(^{(b)}\) The atomic state is the ground state electronic configuration unless a different electronic configuration is indicated.

\(^{(c)}\) The quantity \(g_J/g_J\)\((H)\) is obtained by combining the appropriate measured quantities.

\(^{(d)}\) The experimental value of \(g_J\) for an atom is obtained by combining the directly measured quantity involving the \(g_J\) of the atom with other measured quantities given in the Table in order to obtain \(g_J/g_J\)\((H)\). (The unit is \(g_J = 1\).) For all cases, the measured value of \(g_J/g_J\)\((H)\) is required, and for several atoms with the best known \(g_J\) values the inaccuracy of about 9 ppm for this quantity provides the principal part of the quoted errors.

\(^{(e)}\) Errors are indicated by the numbers in parentheses in parts per million (ppm).

\(^{(f)}\) This number is the unweighted average of the two experimental values in the references.

\(^{(g)}\) The latest value \(\alpha^{-1} = 137.0390 \pm 0.0012\) has been used in calculating theoretical \(g_J\) values. See reference 57 S1.

\(^{(h)}\) Result applies to the naturally occurring odd A isotopes of mercury.

\(^{(i)}\) The quantity \(g_J\) is the nuclear \(g\)-value of the proton in a spherical sample of mineral oils.
REFERENCES TO TABLE 2

48 K1 P. Kusch and H. M. Foley, Electron $g_\text{J}$ from atomic $g_\text{J}$ values experiment, *Phys. Rev.* **74**, 250 (1948).
53 W1 G. Wessel and H. Lew, $g_\text{J}(\text{Au})$, $g_\text{J}(\text{Ag})$ experiment, *Phys. Rev.* **92**, 641 (1953).