DENSITY BEHAVIOR ALONG THE STAGNATION LINE OF A BLUNT BODY IN HYPERTHERMAL FLOW

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ABSTRACT

The present paper is concerned mainly with the density behavior along the stagnation line of a highly cooled flat disk, as a simple example of a blunt nosed body, in a nearly free molecular flow. The flow is assumed "hyperthermal." The present analysis was worked out following the iterative procedure of Willis without any assumption regarding the importance of any particular class of molecular collisions.

An approximate expression for the density profile along the stagnation line has been obtained. Numerical calculations are presented for speed ratios S = 3.0 and 5.0 with $\lambda_\sigma/R = 5.0$ and 10.0, respectively, where R is the disk radius and λ_σ the mean free path at the surface of the emitted molecules with respect to the incident molecules for free molecule conditions.

The results show that, as was found by Probstein in an analysis of rarefied flow past a sphere based on a first-collision-type theory, there is no indication of the appearance of a shock-like structure in the density behavior along the stagnation line, and that the density profile is not too strongly dependent on S for λ_σ/R fixed. The density near the surface has a nonanalytic part which behaves as $\lambda \log \chi$, where χ is the distance measured from the surface along the stagnation line made dimensionless with respect to R.

INTRODUCTION

A vehicle re-entering the atmosphere at hypersonic speeds will encounter various types of flow regimes, from the free molecule

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flow as one limit to the usual continuum flow as another limit. As is well known, there is no shock-wave-like behavior in the free molecule flow regime, whereas there certainly exists a shock-wave-like structure in the continuum region. Naturally a question arises of when and how a shock wave will begin to appear in the transition region when a vehicle is re-entering from the very rarefied upper atmosphere.

Probstein (Ref. 1) was the first to investigate the whole picture of these flow regimes. According to his results, a shock-wave-like structure begins to appear in the regime between near free molecule flow and the beginning of the continuum regime. The width of this regime was estimated to be less than one decade in free stream density change. On the other hand, however, he conjectured from an examination of the density behavior along the stagnation line of a sphere, that in the first collision or near free molecule regime there is no indication of the appearance of any shock-wave-like structure. His analysis assumed an exponential decay law with distance for the collisions of the incident molecules with the emitted molecules. This is analogous to a first-collision-type assumption in which only a certain class of collisions between incident and emitted molecules is taken into account.

As noted previously, the shock-wave-like structure from these results will be expected to appear somewhere in a narrow region between the limit of the first-collision regime and the lower density limit of the continuum regime. In this paper the author shall examine the density behavior along the stagnation line of a blunt body, with particular reference to the problem of shock wave formation, for the case where collisions other than the first are considered. The role of various classes of collisions was discussed, for example, by Welander in Ref. 2.

The analysis of rarefied flow becomes extremely difficult as soon as collisions between molecules enter into the problem, because of the complicated nonlinear collision term which appears in the Boltzmann equation. One of the methods by which collisions can be taken into account is the integral iteration method which was developed by Willis (Refs. 3 and 4) for the case where the flow is close to free molecule. The method leads to a simple mathematical structure when it is combined with the single relaxation time model of the Boltzmann equation, which was introduced by Krook et al. (Ref. 5).

As a simple example of a blunt body, one may choose a flat disk oriented normal to a uniform stream with a mean macroscopic velocity at infinity which is very large in comparison with the mean random speed of the molecules emitted from the surface. For simplicity the present considerations are restricted to the case of a highly cooled body and, in particular, the case shall be considered where the surface temperature is taken equal to the free stream temperature. In addition the molecules are assumed to be hard spheres and to be emitted diffusely from the surface. Such a flow is termed "hyperthermal" by Schramberg (Ref. 6). It is clear that not all of the restrictions mentioned are essential and that their removal would only increase the technical complexity of the calculation. It is hoped that the present analysis based on these simplified assumptions will provide a clearer picture whether there is the beginning of a definable shock wave in the near free molecule hyperthermal flow past a blunt body.

MATHEMATICAL FORMULATION AND BOUNDARY CONDITIONS

The simplifying assumptions, on which the present analysis is based, are summarized as follows:

1) The flow at infinity has a mean macroscopic velocity U very large in comparison with the mean random speed $\overline{C}_e$ of the molecules emitted from the surface.

2) The surface condition is taken to be such that all of the molecules striking the surface are re-emitted diffusely, so that the thermal energy of these molecules is completely accommodated to the surface temperature.

3) All of the molecules are assumed to be hard spheres with a finite collision cross section $\pi\sigma^2$.

4) The body is assumed to be highly cooled, so that the body temperature is assumed equal to the free stream temperature.

In a steady flow in the absence of external forces the Boltzmann equation may be written symbolically as

$$C \cdot \nabla f = G(f) - fL(f) \tag{2.1}$$

where f is the distribution function, C the molecular velocity, and G and L denote gain and loss operaters of f, respectively. According to the iteration scheme presented by Willis (Refs. 3 and 4), the first iteration can be obtained by solving Eq. 2.1 with $G(f_o)$, $L(f_o)$ in place of $G(f)$, $L(f)$, respectively, where f_o, an equilibrium distribution, is assumed to be a free molecule distribution of the form

$$f_o = n_o \left(\frac{B_o}{\pi}\right)^{3/2} e^{-Bo(C - V_o)^2} \tag{2.2}$$

with n_o, β_o and V_o to be determined from the free molecule solution.

A point off the body is now considered. Among the specified molecular velocities at the point one may distinguish two categories: If the backward ray with the direction of the specified velocity through the point intersects the body surface, the velocity belongs to one category, and if it does not, the corresponding velocity belongs to the second category. Therefore the velocity space at the point in question is conveniently divided into the subspaces Ω_2 and Ω_1 corresponding to the two molecular velocity categories mentioned, respectively. If the velocity space is observed with the point in question as the origin, the subspace Ω_2 is identified with the forward cone subtended by the body whose apex is at the point (see Fig. 1).

In order to obtain f_o one must derive the free molecule solution. The velocity of molecules emitted from the surface belongs to the Ω_2 space, whereas that of molecules incident on the surface belongs to the Ω_1 space. The distribution of incident and emitted molecules may be denoted by f_i and f_e, respectively. Then

$$f_i = n_\infty \left(\frac{\beta_\infty}{\pi}\right)^{3/2} e^{-\beta_\infty (C - iU)^2} \tag{2.3a}$$

$$f_e = n_e \left(\frac{\beta_\infty}{\pi}\right)^{3/2} e^{-\beta_e C^2} \tag{2.3b}$$

where i is a unit vector parallel to the direction of the mean macroscopic velocity at infinity, and β_∞ and β_e are related to the temperatures at infinity T_∞ and on the surface T_e, respectively

$$\beta_\infty = \frac{1}{2RT_\infty} \tag{2.4a}$$

$$\beta_e = \frac{1}{2RT_e} \tag{2.4b}$$

with R the gas constant.

With the assumption of a hyperthermal flow the condition of no mass flux through the surface element gives for the stagnation point

$$n_e \approx 2\sqrt{\pi}\ \ U\sqrt{\beta e}\ \ n_\infty = 2\sqrt{\pi}\ Se\ n_\infty \tag{2.5}$$

where S_e is the speed ratio defined by

$$S_e = U\sqrt{\beta_e}$$

One can thus determine n_o , V_o and β_o involved in f_o from the following relations

$$n_o = \int_{\Omega_1} f_i\, d\, C + \int_{\Omega_2} f_e\, d\, C \tag{2.6a}$$

$$n_o V_o = \int_{\Omega_1} C\, f_i\, d\, C + \int_{\Omega_2} C\, f_e\, d\, C \tag{2.6b}$$

$$\frac{3}{2}\,\frac{n_o}{\beta_o} = \int_{\Omega_1} (C - V_o)^2\, f_i\, d\, C + \int_{\Omega_2} (C - V_o)^2\, f_e\, d\, C \tag{2.6c}$$

In view of the very complicated form of $G(f_o)$ and $L(f_o)$, for computational purposes it is convenient to use the single relaxation time model suggested by Krook et al. in Ref. 5. In this case

$$G(f_o) = An_o f_o \tag{2.7a}$$

$$L(f_o) = An_o \tag{2.7b}$$

where A is a constant and where An_o has the dimension of collision frequency.

The distribution functions f^+ and f^-, respectively, are now introduced related to Ω_1 and Ω_2. That is

$$f = f^+ \ \text{for}\ \ C \epsilon\, \Omega_1 \tag{2.8a}$$

$$f = f^- \ \text{for}\ \ C \epsilon\, \Omega_2 \tag{2.8b}$$

Similarly

$$f_o = f_o^+ \ \text{for}\ \ C \epsilon\, \Omega_1 \qquad f_o = f_o^- \ \text{for}\ \ C \epsilon\, \Omega_2 \tag{2.9}$$

With G and L defined by Eqs. 2.7 the following equations for f^+ and f^- are obtained from Eq. 2.1

$$C \cdot \nabla f^+ = An_o f_o^+ - An_o f^+ \ \text{for}\ \ C \epsilon\, \Omega_1 \tag{2.10a}$$

$$C \cdot \nabla f^- = An_o f_o^- \ \text{for}\ \ C \epsilon\, \Omega_2 \tag{2.10b}$$

Consideration is given the boundary conditions on f^+ and f^- specified, respectively, at infinity and on the surface. The distribution f becomes f^+ at infinity because there the Ω_2 space vanishes. If the flow is assumed free molecular at infinity, then

$$f^+ \rightarrow f^- \tag{2.11}$$

at infinity. On the other hand it follows from the present basic definition that f^- on the surface is itself just the distribution of emitted molecules. As a result of the assumption of completely diffuse reemission, f^- on the surface is identical with a Maxwellian f_b which includes the only unknown n_b, that is

$$f^- = f_b = n_b\left(\frac{\beta_e}{\pi}\right)^{3/2} e^{-\beta_e C^2} \tag{2.12}$$

on the surface. Since the condition of no mass flux through surface must be satisfied, at the stagnation point

$$\int_{\Omega_1} C_\chi f^+ \, dC = \left| \int_{\Omega_2} C_\chi f^- \, dC \right| \tag{2.13}$$

where C_χ is the velocity component parallel to the inward normal to the surface (see Fig. 1). The right-hand side can readily be evaluated with f^- of Eq. 2.12 as

$$\left| \int C_\chi f^- \, dC \right| = \frac{1}{2\sqrt{\beta_e \pi}} n_b \tag{2.14}$$

Therefore, at the stagnation point

$$n_b = 2\sqrt{\beta_e \pi} \int_{\Omega_1} C_\chi f^+ \, dC \tag{2.15}$$

If the mass flux of incident molecules at the stagnation point is known, the only unknown n_b, and therefore the Maxwellian of the emitted molecules at the surface, can be determined from the forementioned obtained relation.

As was mentioned previously, An_o has the dimension of collision frequency. If the flow is assumed free molecular at infinity, it follows from Eq. 2.6a that n_o tends to n_∞ so that at infinity An_o becomes An_∞. It is reasonable therefore to choose

$$A = \frac{\overline{C_\infty}}{\lambda_\infty n_\infty} \tag{2.16}$$

so that An_∞ is identified with the collision frequency at infinity.

THE DETERMINATION OF THE NUMBER DENSITY OF EMITTED MOLECULES AT THE STAGNATION POINT

In the present section the investigator shall obtain the number density of emitted molecules, n_b at the stagnation point, which is the only unknown involved in the Maxwellian distribution of emitted molecules on the surface. One may consider that a velocity C belongs to the Ω_1 space at the stagnation point and that a radial vector r is measured from the point along the backward ray with the specified velocity and is made dimensionless with respect to the radius of a flat disk R. Then Eq. 2.10 is written along this ray in a scalar form as

$$C_r \frac{\partial f^+}{\partial r} = -ARn_o f_o^+ + Arn_o f^+ \qquad (3.1)$$

where C_r and r are the magnitude of C and r, respectively. With the condition 2.11 at infinity it is possible to readily obtain $f^+_{r=o}$ for velocities with a specified direction, at the stagnation point as

$$f^+_{r=o} = f_i + \int_o^\infty \frac{ARn_o}{C_r} (f_o^+ - f_i) \exp. [-\int_o^{r'} \frac{ARn_o}{C_r} dr''] dr' \qquad (3.2)$$

The number flux of incident molecules $\dot{n}_i$ per unit time and per unit surface area at the stagnation point is given by

$$\dot{n}_i = \int_{\Omega_1} C_\chi f^+_{r=o} \; dC$$

With $f^+_{r=o}$ of Eq. 3.2 this becomes in polar coordinates

$$\dot{n}_i = (\dot{n}_i)_F + 2\pi AR \int_o^\infty dr' \int_o^\infty dC_r \int_o^{\pi/2} d\theta \cdot C_r^2 \sin\theta \cos\theta \; x \qquad (3.3)$$
$$n_o (f_o^+ - f_i) \exp. [-\int_o^{r'} \frac{ARn_o}{C_r} dr'']$$

where $(\dot{n}_i)_F$ denotes the free molecule flow value, that is

$$(\dot{n}_i)_F = \int_{\Omega_1} C_\chi f_i \, dC \approx n_\infty U \qquad (3.4)$$

The integral in Eq. 3.3 is now considered. For C_r and θ fixed the integrand is predominant in the region of r small not only due to the exponential term, but also due to the term $n_o (f^+ - f_i)$ which rapidly tends to zero with increasing r. For r small, the quantities n_o, V_o, β_o and therefore the distribution f_o are weakly dependent upon θ, because the domains of integration Ω_1 and Ω_2 in Eqs. 2.6 do not change appreciably with θ when r is small. With this situation in mind one may write the behavior of the variables n_o, V_o, β_o for r small in a way similar to that pointed out by Narasimha (Ref. 7) regarding the calculation of the nearly free molecule mass flow through an orifice, as

$$\left.\begin{aligned} n_o(r,\theta) &\approx n_o(r,o) \\ V_o(r,\theta) &\approx V_o(r,o) = [U_o(r,o), o, o] \\ \beta_o(r,\theta) &\approx \beta_o(r,o) \end{aligned}\right\} \quad (3.5)$$

Since the free stream is assumed as uniform free molecule, the behavior of these variables for r large is quite the same as for r small. Therefore, the approximation given by Eqs. 3.5 may be expected to be applied to the entire region under consideration in the present calculation. The analysis based upon such an approximation belongs to the category of quasi one-dimensional analyses, some of which were performed for a rarefied mass flow problem through an orifice by Narasimha (Refs. 7 and 8) and by Probstein (Ref. 9). Willis (Ref. 10) thereafter performed a complete analysis for the orifice problem and showed that, in comparison with his results, the quasi one-dimensional analyses involve some uncertainty in their results. However, it should be stressed that in contrast to the present case, in the orifice problem, the free molecule flow variables n_o, V_o and β_o behave as shown in Eqs. 3.5 for r small, while these behave as in a radial flow for r large, so that no quasi one-dimensional approximations may be expected to be valid throughout the region of interest without further considerations.

The values of n_o, U_o and β_o are to be found along the stagnation line by the use of Eqs. 2.6. The integrals are considered which include f_i, in Eqs. 2.6. It follows from the expression for f_i that the integrands dominate in the neighborhood of the point $C_\chi = U$ which is very large in comparison with the mean random speed at infinity $\overline{C}_\infty$. Therefore the predominant part of the integrands lies far from the origin in the Ω_1 space and thus one can approximately replace in Eqs. 2.6 the integral domain Ω_1 by the entire domain Ω. With this approximation for n_o, U_o and β_o along the stagnation line

$$n_o \approx \int_\Omega f_i \, d\,C + \int_{\Omega_2} f_e \, d\,C$$

$$n_oU_o \approx \int_\Omega C_\chi f_i \, d\,C + \int_\Omega C_\chi f_e \, d\,C$$

$$\frac{3}{2}\frac{n_o}{\beta_o} \approx \int_\Omega (C - i\,U_o)^2 f_i \, d\,C + \int_{\Omega_2} (C - i\,U_o)^2 f_e \, d\,C$$

Indeed, it can easily be shown that the replacement of Ω_1 by Ω causes only minor errors of the order of e^{-S^2}/S, e^{-S^2} and Se^{-S^2} in the values of n_o/n_∞, $\sqrt{\beta_o}/U_o$ and β_o/β_∞, respectively. For the case when $T_\infty = T_e$, as is assumed throughout the present paper, one may obtain from Eqs. 2.6 after some calculation

$$n_o \approx n_\infty + (n_o)_e = n_\infty[\sqrt{\pi}\,S\,(1 - \frac{r}{\sqrt{1 + r^2}}) + 1] \tag{3.6a}$$

$$n_oU_o \approx n_\infty\,U\,\frac{r^2}{1 + r^2} \tag{3.6b}$$

$$\beta_o \approx [\,\frac{1}{\beta_\infty} + \frac{2}{3}(\frac{n_\infty}{n_o}\,U^2 - U_o^2\,)\,^{-1} \tag{3.6c}$$

With these values of n_o, U_o and β_o, the free molecule distribution f_o of Eq. 2.2 becomes along the stagnation line in polar coordinates

$$f_o = n_o\,(\frac{\beta_o}{\pi})^{3/2}\,e^{-\beta_o(C_r^2 - 2C_rU_o\cos\theta + U_o^2\,)} \tag{3.7}$$

Applying n_o and f_o previously obtained to Eq. 3.4 and integrating with respect to θ brings

$$\begin{aligned} \dot{n}_i = (\dot{n}_i)_F + \frac{\pi}{2}AR\int_0^\infty dr'\int_0^\infty dC_r \cdot n_o\,[n_o\,(\frac{\beta_o}{\pi})^{3/2}\Bigg\{\frac{e^{-\beta_o(C_r^2 + U_o^2}}{\beta_o^2U_o^2} \\ + (\frac{2C_r}{\beta_oU_o} - \frac{1}{\beta_o^2U_o^2})\,e^{-\beta_o(C_r-U_o)^2} - n_\infty\,(\frac{\beta_\infty}{\pi})^{3/2}\Bigg\{\frac{e^{-\beta_\infty(C_r^2 + U^2)}}{\beta_\infty^2U^2} \\ + (\frac{2C_r}{\beta_\infty U} - \frac{1}{\beta_\infty^2U^2})\,e^{-\beta_\infty(C_r-U)^2}\Bigg\}] \cdot \exp.[\,-\int_0^{r'}\frac{ARn_o}{C_r}\,dr''] \end{aligned} \tag{3.8}$$

Consider the integral part in Eq. 3.8, which represents the deviation of $\dot{n}_i$ from the free molecule value. The quantity AR is inversely proportional to λ_∞/R, the Knudsen number referred to the mean free path at infinity, which is assumed sufficiently large so that the flow is nearly free molecular. With C_r not too small or r not too large, the exponential term exp. $[-\int_0^{r'} ARn_o/C_r\, dr'']$ in the integral of Eq. 3.8 can be approximately replaced by 1. On the other hand, for C_r small the integrand can readily be shown to be proportional to C_r^2 exp. $[-\int_0^{r'} ARn_o/C_r\, dr'']$. This means that the approximation of replacing the exponential term by 1 leads to only a negligible error in the integral which is at most of the order of C_r^3. Similarly, for r large it follows that the term in the square bracket in Eq. 3.8 is proportional to $1/r^2$, so that the approximation of replacing the exponential term by 1 again results in only a negligible error in the integral which is at most of the order of $1/r$.

With the foregoing approximation one may obtain after integration with respect to C_r for the integral of Eq. 3.8.

$$\dot{n}_i = (\dot{n}_i)_F + AR\int_0^\infty dr'\, n_o\, (n_o F - n_\infty F_\infty) \tag{3.9}$$

where for abbreviation

$$F = \frac{1}{4\beta_o U_o^2}\left[\left(1 + 2\sqrt{\frac{\beta_o}{\pi}}\, U_o\right) e^{-\beta_o U_o^2} + (2\beta_o U_o^2 - 1)\left(1 + \operatorname{erf}\sqrt{\beta_o}\, U_o\right)\right] \tag{3.10a}$$

$$F_\infty = \frac{1}{4\beta_\infty U^2}\left[\left(1 + 2\sqrt{\frac{\beta_\infty}{\pi}}\, U\right) e^{-\beta_\infty U^2} + (2\beta_\infty U^2 - 1)\left(1 + \operatorname{erf}\sqrt{\beta_\infty}\, U\right)\right] \tag{3.10b}$$

With n_o given by Eqs. 3.6a and 3.9 is rewritten as

$$\dot{n}_i = (\dot{n}_i)_F + AR\int_0^\infty dr' \left[(n_o)_e^2\, F + n_\infty\, (n_o)_e\, (2F - F_\infty) + (F - F_\infty)\right] \tag{3.11}$$

As can be shown from Eq. 3.10b, F_∞ is almost equal to 1 for a hyperthermal flow where S >> 1 and, as is seen from Eq. 3.6a, n_o decreases rapidly with increasing r. Therefore Eq. 3.11 becomes approximately

$$\dot{n}_i = (\dot{n}_i)_F + AR\int_0^\infty dr'[(n_o)_e^2 F + n_\infty (n_o)_e (2F-1) + n_\infty^2 (F-F_\infty)] \qquad (3.12)$$

Introduction is now made, at the stagnation point, of a mean free path λ_σ of emitted molecules with respect to incident molecules. With the hyperthermal assumption

$$\lambda_\sigma = \overline{C_e}/(n_\infty \pi\sigma^2 U) \qquad (3.13)$$

where $\overline{C}_e$ is the mean random speed of the emitted molecules at the surface. With A of Eq. 2.16 and with $\dot{n}_i$ of Eq. 3.4

$$\frac{ARn_\infty n_e}{(\dot{n}_i)_F} = 4 \quad 2 \quad (\frac{\overline{C_e}}{U}) \frac{R}{\theta\sigma}$$

Equation 3.13 may therefore be rewritten as

$$\frac{\dot{n}_i}{(\dot{n}_i)_F} = 1 + \frac{R}{\lambda\sigma} [4\sqrt{2} \int_0^\infty (1 - \frac{r}{\sqrt{1+r^2}})^2 F \, dr + \frac{1}{\pi S^2} \int_0^\infty$$

$$(F - F_\infty) dr + 2\sqrt{2} \frac{\overline{Ce}}{U} \int_0^\infty (1 - \frac{r}{\sqrt{1+r^2}}) \quad (2F-1) \, dr] \qquad (3.14)$$

Examination is now made of the order of magnitudes of the three terms in the square bracket in Eq. 3.14. It follows from Eq. 3.10a that the value of F increases monotonically with increasing r, taking the limit values of 1/4 and 1, respectively, as $r \to o$ and $r \to \infty$. Therefore the value of F is of the order of 1 over the integral domain. With respect to the first integral in the square bracket of Eq. 3.14, the contribution of the integral over the domain from any large r, say r_o, to infinity is of the order of $1/r_o^3$. The corresponding contribution of the last integral is of the order of $\overline{C}_e/Ur_o$ ($\approx 1/r_o S$). . With respect to the middle integral, one may find the order of magnitude of the integrand

$$F - F_\infty = O\,(S/r^2)$$

and on taking the limit for r large with the relation obtained from Eqs. 3.6

$$\frac{1}{\beta_o U_o^2} \approx \frac{1}{S^2} + \frac{\sqrt{\pi}}{3} \frac{S}{r^2}$$

This obtains

$$\int_0^{\infty} (F - F_{\infty})\, dr = \int_0^{S^2} (F - F_{\infty})\, dr + O(1/S)$$

In view of this relation the second term in the square bracket of Eq. 3.14 may be not negligible in comparison with the other terms unless the value of S is extremely large, whereas it may be small from the fact that this term represents collisions between the incident molecules. These arguments will be checked later from the numerical calculation.

The unknown n_b, or the Maxwellian of emitted molecules at the stagnation point, is related to the number flux of incident molecules n_i by the relation from Eqs. 2.13 and 2.14

$$\frac{n_b}{n_e} = \frac{\dot{n}_i}{(\dot{n}_i)_F} \tag{3.15}$$

where here $(\dot{n}_i)_F$ is given from Eq. 3.4. Therefore the unknown n_b can be evaluated from Eqs. 3.14 and 3.15 in terms of $R/\lambda\sigma$, $\bar{S}$ and $\bar{C}_e/U$.

Numerical calculations have been carried out for the cases of S = 3.0 and 5.0, respectively. The function F for these cases is plotted against r in Fig. 2. It can be seen from this figure that F is only weakly dependent upon the value of S. The ratio of the number flux of incident molecules to that in free molecule flow or correspondingly the ratio of the number density of emitted molecules to that in free molecule flow has been evaluated as follows

For S = 3.0

$$\left.\begin{aligned} \frac{n_b}{n_e} = \frac{\dot{n}_i}{(\dot{n}_i)_F} &= 1 + \frac{R}{\lambda\sigma}\left[0.72 - 0.08 - \frac{\bar{C}_e}{U}\, 0.45\right] \\ &= 1 + \frac{R}{\lambda\sigma}\left[0.64 - \frac{\bar{C}_e}{U}\, 0.45\right] \end{aligned}\right\} \tag{3.16a}$$

For S = 5.0

$$\left.\begin{aligned} \frac{n_b}{n_e} = \frac{\dot{n}_i}{(\dot{n}_i)_F} &= 1 + \frac{R}{\lambda\sigma}\left[0.71 - 0.03 - \frac{C_e}{U}\, 0.47\right] \\ &= 1 + \frac{R}{\lambda\sigma}\left[0.68 - \frac{\bar{C}_e}{U}\, 0.47\right] \end{aligned}\right\} \tag{3.16b}$$

where the three terms in the square brackets have been evaluated from the corresponding terms in Eq. 3.4 and where the basic assumptions $\bar{C}_e/U = 2/\sqrt{\pi S}$. These results show that n_b/n_e or $\dot{n}_i/(\dot{n}_i)_F$ is only weakly dependent upon the value of S, as is the function F.

DENSITY BEHAVIOR ALONG THE STAGNATION LINE OF A FLAT DISK

In this section the question under consideration is whether there is a shock-wave-like structure in a nearly free molecule flow. In order to investigate this point the author shall examine the density behavior along the stagnation line ahead of a flat disk. Based on a first-collision-type analysis, Probstein in Ref. 1 has considered the same problem for a sphere. The number density of incident molecules along the stagnation line was found by applying the exponential decay law to the free molecule result. Within the approximations of his analysis the results show no indication of the appearance of shock-like behavior in the density profile. In the present analysis, however, considerations are not restricted to any particular class of collisions between molecules. Therefore the results presented here should lead to a more definitive conclusion regarding this question.

Consideration is given a distribution f^- appropriate to the Ω_2 space at a point near the surface on the stagnation line. An inward directed ray parallel to a specified velocity C in the Ω_2 space through the point in question intersects the flat disk. Here the radial distance r is measured from the intersection point along the ray and is made dimensionless with respect to the body radius R. For the specified velocity C whose magnitude is C_r, Eq. 2.10b is rewritten in a scalar form along the specified ray direction

$$C_r \frac{\partial f'}{\partial \hat{r}} = ARn_o f_o' - ARn_o f' \qquad (4.1)$$

With the boundary condition 2.12 one readily obtains the solution

$$f' = f_b + \int_0^r \frac{ARn_o}{C_r} (f_o' - f_b) \exp. [- \int_r^1 \frac{ARn_o}{C_r} dr''] dr' \qquad (4.2)$$

If by θ the angle is denoted between the ray and the stagnation line and by χ the dimensionless distance measured from the stagnation point along the axis (see Fig. 3)

$$\chi = r \cos\theta \qquad C_\chi = C_r \cos\theta$$

Then Eq. 4.2 becomes

$$f^- = f_b + \int_o^r \frac{ARn_o}{C_r} (f_o^- - f_b) \exp.[-\int_r^r 1 \frac{ARn_o}{C_r} dr''] dr' \qquad (4.3)$$

The number density of molecules n^- with velocities related to the Ω_2 space can be determined from performing the integration of Eq. 4.3 with respect to the velocity over the Ω_2 space. That is

$$n^- = \int_{\Omega_2} f_b dC + \int_{\Omega_\lambda} dC \int_o^\chi \frac{ARn_o}{C_\chi} (f_o^- - f_b) \exp.[-\int_{\chi'}^{\chi} 1 \frac{ARn_o}{C_\chi} \qquad (4.4)$$

$$d\chi''] d\chi'$$

In view of the geometry of the Ω_2 space, it now is convenient to introduce cylindrical coordinates (C_χ, C_n, ϕ) about the C_χ axis, where C_n is the magnitude of the velocity projected onto the plane normal to the C_χ axis. By the use of cylindrical coordinates, Eq. 4.4 is written after performing a simple integration with respect to ϕ, as

$$n^- = \int_o^\chi d\chi' \int_o^\infty dC_\chi \int_o^{C_\chi \tan\theta_1} dC_n \cdot 2\pi C_n f_b + \int_o^\chi d\chi^1 \int_o^\infty dC_\chi$$

$$\int_o^{C_\chi \tan\theta_1} dC_n \cdot \frac{2\pi C_n}{C_\chi} ARn_o (f_o^- - f_b) \exp.[-\int_{\chi'}^{\chi} \frac{ARn_o}{C_\chi} d\chi''] \qquad (4.5)$$

where θ_1 is the semi-apex angle of the cone subtended by the flat disk, that is

$$\theta_1 = \tan^{-1} 1/\chi = \tan^{-1} Cn/C\chi$$

Since, in general, the quantities n_o, V_o and β_o depend on r and θ, the quantity n_o and velocity distribution f_o^- appearing in Eq. 4.5 depend not only on χ but also on C_χ/C_n. Therefore it seems very difficult to evaluate exactly the second integral on the right-hand side of Eq. 4.5. The behavior is considered of the quantities n_o, V_o and β_o as $r \to o$ or $\chi \to 0$ for any fixed θ less than θ_1, obtaining

$$n_o(\chi,\theta) \approx n_o(0,\theta)$$

$$V_o(\chi,\theta) \approx V_o(0,\theta)$$

$$\beta_o(\chi,\theta) \approx \beta_o(0,\theta)$$

Since, as can be easily shown from Eqs. 2.6, at the surface the quantities n_o, V_o and β_o are constants independent of θ, one may reasonably make the assumption

$$\left.\begin{aligned} n_o(\chi,\theta) &\approx n_o(\chi,0) \\ V_o(\chi,\theta) &\approx V_o(\chi,0) = [u_o(\chi,0),\, 0,\, 0] \\ \beta_o(\chi,\theta) &\approx \beta_o(\chi,0) \end{aligned}\right\} \quad (4.6)$$

The given relations are quite similar to these assumed by Probstein (Ref. 9) in his analysis of the mass flow through an orifice in rarefied flow. In the present calculation, however, the quasi one-dimensional assumption may be expected to well approximate the free molecule flow feature throughout the concerning region, which is identified with the cone subtended by a flat disk, because this region is assumed to be confined closely to the surface.

Equation 4.5 shall now be examined. Since f_b has no dependence on the spacial location, the first integral of Eq. 4.5 can be performed easily. That is

$$\int_0^{\chi} d\chi' \int_0^{\infty} dC_\chi \int_0^{C_x\tan\theta,} dC_n \cdot 2\pi C_n f_b = \frac{N_b}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right) \quad (4.7)$$

Applying the foregoing assumption given by Eqs 4.6 to the second integral on the right-hand side of Eq. 4.5, f_o^- included reduces to

$$f_o^- = n_o\left(\frac{\beta_o}{\pi}\right)^{3/2} \exp.\left[-\beta_o\{(C_x + u_o)^2 + C_n^2\}\right] \quad (4.8)$$

where without any confusion one can denote $n_o(\chi,0)$, $u_o(\chi,0)$, $\beta_o(\chi,0)$ simply by n_o, u_o, β_o, respectively.

Now one may obtain from Eq. 4.5 with Eqs. 4.7 and 4.8 after integration with respect to C_n

$$n^- = \frac{n_b}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right) + AR\int_o^{\chi} d\chi \int_o^{\infty} \frac{dC_\chi}{C_\chi} \cdot n_o \left[n_o\left(\frac{\beta_o}{\pi}\right)^{3/2} \times \right.$$

$$\left\{\exp.\left(-\beta_o(C_x+u_o)^2 - \frac{ARn_e g}{2C_\chi}\right) - \exp.\left(-\beta_o(C_\chi+u_o)^2 - \beta_o C_\chi^2\tan^2\theta_1 - \frac{ARn_e g}{2C_\chi}\right)\right\} -$$

$$n_b\left(\frac{\beta_e}{\pi}\right)^{3/2}\left\{\exp.\left(-\beta_e C_\chi^2 - \frac{ARn_e g}{C_\chi}\right) - \exp.\left(-\frac{\beta_e C_\chi^2}{\cos^2\theta_1} - \frac{ARn_e g}{2C_\chi}\right)\right\}\Bigg] \tag{4.9}$$

where for abbreviation

$$\frac{n_e}{2} g(\chi,\chi') = \int_{\chi'}^{\chi} n_o d\chi'' \tag{4.10}$$

In the neighborhood of the stagnation point it is possible to set $\sqrt{\beta_o}\ u_o$ equal to zero because, as seen from Eqs. 3.6, it is of the order of $\chi^2/\sqrt{S}$ for χ small. Therefore

$$n^- = \frac{n_b}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right) + AR\int_o^{\chi} d\chi' n_o \left[n_o\left(\frac{\beta_o}{\pi}\right)^{3/2} (I_1 - I_2)\right.$$

$$\left. - n_b\left(\frac{\beta_e}{\pi}\right)^{3/2} (I_3 - I_4)\right] \tag{4.11}$$

where for abbreviation

$$I_1 = \int_o^\infty \frac{dC_x}{C_x} \exp.\left[-\beta_o C_x^2 - \frac{ARn_e g}{2C_x}\right] \tag{4.12a}$$

$$I_2 = \int_o^\infty \frac{dC_x}{C_x} \exp.\left[-\frac{\beta_o C_x^2}{\cos^2\theta_1} - \frac{ARn_e g}{2C_x}\right] \tag{4.12b}$$

$$I_3 = \int_o^\infty \frac{dC_x}{C_x} \exp.\left[-\beta_e C_x^2 - \frac{ARn_e g}{2C_x}\right] \tag{4.12c}$$

$$\int_o^\infty \frac{dC_x}{C_x} \exp.\left[-\frac{\beta_e C_x^2}{\cos^2\theta_1} - \frac{ARn_e g}{2C_x}\right] \tag{4.12d}$$

Since, in the present case, χ is small and S is large, the approximate expressions A-1 and A-3 for the I_n's which are derived in the Appendix can be applied to a good approximation when $R/\lambda\sigma$ is sufficiently small. Thus

$$I_1 - I_2 \approx -\log\theta_1 = -\log\frac{\chi}{\sqrt{1+\chi^2}}$$

$$I_3 - I_4 \approx -\log\theta_1 = -\log\frac{\chi}{\sqrt{1+\chi^2}}$$

With these relations and with $ARn_e\sqrt{\beta_e\pi} = 8\sqrt{2}\ R/\lambda_\sigma$ Eq. 4.11 reduces to

$$n^- \approx \frac{n_b}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right) - \frac{8\sqrt{2}}{\pi}\, n_e\, \frac{R}{\lambda_\sigma} \log \frac{\chi}{\sqrt{1+\chi^2}}\ x$$

$$\int_o^\chi \left[\left(\frac{n_o}{n_e}\right)^2 \left(\frac{\beta_o}{\beta_e}\right)^{1/2} - \frac{n_o}{n_e} \right] d\chi^1 \qquad (4.13)$$

where within the present first-order approximation n_b involved in the integral has been replaced by n_e. Here the variables n_o and β_o are given by Eqs. 3.6a and 3.6c, respectively, and n_b is given by Eq. 3.15 with $\dot{n}_i$ evaluated from Eq. 3.14. Therefore the number density along the stagnation line of molecules with velocities appropriate to the Ω_2 space can be evaluated from Eq. 4.13.

For S sufficiently large, n_o and β_o given by Eqs. 3.6a and 3.6c become approximately

$$n_o \approx \frac{n_e}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right), \quad \frac{\beta_o}{\beta_e} \approx \frac{3\sqrt{\pi}}{2S}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right.$$

With these values of n_o and β_o, the approximate expression for n^- for S sufficiently large are obtained

$$n^- \approx \frac{n_b}{2}\left(1 - \frac{\chi}{\sqrt{1+\chi^2}}\right) + \frac{4\sqrt{2}}{\pi} n_e \frac{R}{\lambda_\sigma} \left[\left(\chi + 1 - \sqrt{1+\chi^2}\right) - \right.$$

$$\left(\frac{3\sqrt{\pi}}{2S}\right)^{1/2} \left\{ 3 - \frac{3+\chi/\sqrt{1+\chi^2}}{(1+\chi/\sqrt{1+\chi^2})^{1/2}} \right\} \Big] \log \frac{\chi}{\sqrt{1+\chi^2}} \qquad (4.14)$$

As can be seen from Eq. 3.6a, in a hyperthermal free molecule flow, along the stagnation line in the neighborhood of the surface the number density n_e of emitted molecules is very large in comparison with the number density of incident molecules. Therefore, along the stagnation line in the neighborhood of the surface the number density n^- of the molecules with velocities appropriate to the Ω_2 space may be very large in comparison with the number density of the molecules with velocities appropriate to the Ω_1 space. One can say, therefore, that in the neighborhood of the surface the total number density may be identified with the number density n^-, which has been investigated in the present section.

Numerical calculations for n^- have been carried out using Eq. 4.13 for values of S = 3.0 and 5.0. In Fig. 4 are shown the values of the ratio of n^- to $n_b/2$ which is the number density of emitted molecules at the stagnation point. Since, as mentioned before, near the surface along the stagnation line n^- is almost equal to the total number density, the ratio of n^- to $n_b/2$ may also be identified with the ratio of the density ρ to the stagnation density ρ_{st}: In Fig. 4 are also shown the limit values for $S \to \infty$ of the ratio $n^-/n_b/2)$ $(\approx \rho/\rho_{st.})$, which have been evaluated from Eq. 4.14, with the corresponding value in free molecule flow. It can be seen in Fig. 4 that the density behavior is not too strongly dependent on S for λ_σ/R fixed, whereas it changes appreciably depending on λ_σ/R.

Here it is worthwhile noting that, as can be seen from Eq. 4.13 or 4.14, the first-order correction term of the order of R/λ_σ involved in n_b results in a higher-order term of order $(R/\lambda_\sigma)^2$ in the ratio of $n^-/n_b/2)$ and, therefore, the ratio $n^-/(n_b/2)$ is not affected by the first-order term in n_b which has been calculated in the previous section, at least in so far as higher-order terms of order $(R/\lambda_\sigma)^2$ are concerned. Therefore it is possible to say that, in spite of the crude estimate of the n_b which was presented in the previous section, the estimate for the ratio $n^-/(n_b/2)$ that has been obtained will be well approximated to the first order in R/λ_σ, so far as the neighborhood of the stagnation point is concerned.

As is seen from Eq. 4.13, the number density of emitted molecules n^- is a monotonically decreasing function of χ with increasing χ. Indeed this can be seen from the results given in Fig. 4. The author concluded that within the framework of the present analysis along the stagnation line there is no indication of the appearance of shock-like structure in the density behavior for a flat disk oriented normal to a hyperthermal flow. This is in agreement with the results for a sphere found by Probstein (Ref. 1) on the basis of a first-collision-type theory. Furthermore Fig. 4 shows a trend for the region of higher density to be confined more closely to the surface for Knudsen number smaller.

The expression in Eq. 4.15 for n^- has a nonanalytic term in χ of the form $\chi \log \chi$. It is worthwhile noting that the velocity profile near the surface in a plane Couette flow has a similar singularity as was shown by Willis (Refs. 3 and 12). Such a singularity seems to occur due to the nonanalyticity of the velocity distribution which contains a term of the form exp. $[-C^2 - a/C_\chi]$ where a is a parameter related to the collision frequency. Detailed discussions on this point were presented by Lees in Ref. 11 and by Willis in Ref. 12. Owing to the singularity mentioned in the foregoing, the density gradient is logarithmically infinite at the stagnation point. The numerical results, however, indicate that the density has only a weakly singular behavior except in the region very close to the stagnation point (see Fig. 4).

APPENDIX: APPROXIMATE EXPRESSIONS FOR THE INTEGRALS I_n

The integrals of Eqs. 4.12 are now examined. These integrals have a similar form so that the integral I_4 is first considered. The exponential term included takes a maximum at $C_x = C_{x4}$ given by

$$C_{x_4} = \left(\frac{ARn_e g \cos^2\theta_1}{4\beta_e}\right)^{1/3}$$

For convenience, the integral I_4 is rewritten as

$$I_4 = \int_0^{C_{x_4}} \frac{dC_x}{C_x} \exp.\left(-\frac{ARn_e g}{2C_x}\right)\left[\exp.\left(-\frac{\beta_e C_x^2}{\cos^2\theta_1}\right)\right]$$

$$\int_{C_{x_4}}^{\infty} \frac{dC_x}{C_x} \exp.\left(-\frac{\beta_e C_x^2}{\cos^2\theta_1}\right)\left[\exp.\left(-\frac{ARn_e g}{2C_x}\right)\right]$$

If the following quantities

$$\frac{ARn_e g}{2C_{x4}} = \left(\frac{ARn_e\ \beta_e\ g}{2\cos\theta_1}\right)^{2/3}, \quad \frac{\beta_2 C_{x4}^2}{\cos^2\theta_1} = \left(\frac{ARn_e\ \beta_e\ g}{4\cos\theta_1}\right)^{2/3}$$

are sufficiently small in comparison with one, the exponential terms in the square bracket can be expanded in terms of their arguments. One then obtains by retaining only the leading terms the approximate expression for I_4

$$I_4 \approx \int_0^{C_{x4}} \frac{dC_x}{C_x} \exp.\left(-\frac{ARn_e g}{2C_x}\right) + \int_{C_{x_4}}^{\infty} \frac{dC_x}{C_x} \exp.\left(-\frac{\beta_e C_x^2}{\cos^2\theta_1}\right)$$

$$\int_{ARn_e g/2C_{x4}}^{\infty} \frac{e^{-t}}{t}\, dt + \frac{1}{2}\int_{\beta_e C_{x_4}/\cos^2\theta_1}^{\infty} \frac{e^{-t}}{t}\, dt$$

By the use of the formula

$$\int_{\chi}^{\infty} \frac{e^{-t}}{t}\, dt \approx \log\frac{1}{\gamma\chi} \quad \text{for } \chi \text{ small}$$

where $\log\gamma$ is the Euler's constant, there is

$$I_4 \approx -\log \frac{ARn_e g\ \beta_e}{2\cos\theta_q} - \frac{3}{2}\log\gamma \qquad \text{A-1}$$

Proceeding to higher-order quardratures the terms neglected in Eq. A-1 can be shown to be of the order of $ARn_e g\sqrt{\beta_e/2}\cos\theta_1$. The expression obtained in the foregoing by this elementary procedure has been checked and found to agree exactly to this order with the series expression for the integral of the same form as I_4, which was obtained by Abromowitz in Ref. 13.

The order of magnitude of the quantity $ARn_e g\sqrt{\beta_e/2}\cos\theta_1$ is here examined. With n_e given by Eq. 2.6a, taking into account that $n_e \gg n_\infty$ in the neighborhood of the stagnation point, the function g defined by Eq. 4.10 is given by

$$g \approx \left(1 - \frac{\chi'}{\chi}\right)\cos\theta_1$$

where χ' is smaller than or equal to χ. With this value of g and with A given by Eq. 2.16

$$\frac{ARn_e g\sqrt{\beta_e}}{2\cos\theta_1} \approx \frac{4\sqrt{2}}{\pi}\left(1 - \frac{\chi'}{\chi}\right)\frac{R}{\lambda_\sigma} \qquad \text{A-2}$$

Indeed, as was assumed previously, the arguments $ARn_e g/2C_{x_4}$, $\beta_e C_{x_4}/\cos^2\theta_1$ are small for the case of R/λ_σ small. It follows from Eq. A-2 that the contribution of the terms neglected in Eq. A-1 is of the order of R/λ_σ which is assumed small in the present paper.

In quite the same way as for I_4, the following approximate expression for I_1, I_2 and I_3 are obtained

$$\left.\begin{aligned}
I_1 &\approx -\log \frac{ARn_e g\sqrt{\beta_o}}{2} - \frac{3}{2}\log\gamma \\
I_2 &\approx -\log \frac{ARn_e g\sqrt{\beta_o}}{2\cos\theta_1} - \frac{3}{2}\log\gamma \\
I_3 &\approx -\log \frac{ARn_e g\sqrt{\beta_e}}{2} - \frac{3}{2}\log\gamma
\end{aligned}\right\} \qquad \text{A-3}$$

Owing to the same reason as for I_4 these expressions are also valid within a good approximation, so long as the following quantities

$$ARn_e g \sqrt{\beta_o}\ ,\quad ARn_e g \sqrt{\beta_o}/\cos\theta_1\ ,\quad ARn_e \sqrt{\beta_e}\ g$$

are small in comparison with 1. These quantities are expressed in terms of R/λ_σ as

$$\frac{ARn_e g\ \beta_o}{2} \approx \frac{4\sqrt{2}}{\sqrt{\pi}}\left[\frac{3\sqrt{\pi}}{2\ S}(1-\chi)\right]^{3/2}(\chi-\chi')\ \frac{R}{\lambda\sigma}$$

$$\frac{ARn_e g\ \beta_o}{2\cos\theta_1} \approx \frac{4\sqrt{2}}{\sqrt{\pi}}\left(\frac{3\sqrt{\pi}}{2\ S}\right)^{3/2}\left(1-\frac{\chi'}{\chi}\right)\frac{R}{\lambda\sigma}$$

$$\frac{ARn_e g\ \beta_e}{2} \approx \frac{4\sqrt{2}}{\sqrt{\pi}}(\chi-\chi')\ \frac{R}{\lambda\sigma}$$

For χ small and S large, all of these arguments are always small in comparison with the maximum value of $ARn_e g\sqrt{\beta_e}/2\cos$ ($\approx 4\sqrt{2}\ R/\sqrt{\pi}\ \lambda\sigma$). Therefore one can say that the expressions for I_n's obtained in the foregoing are valid to a high degree of approximation for the case when $R/\lambda\sigma$ is sufficiently small.

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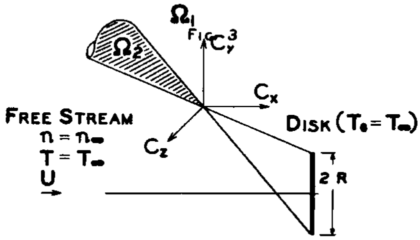

Fig. 1   Nearly free molecular flow past a flat disk.

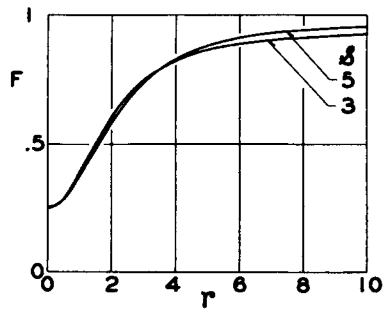

Fig. 2   Values of the function F for S = 3.0 and 5.0.

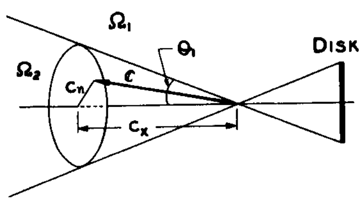

Fig. 3   Velocity space appropriate to a point on the stagnation line.

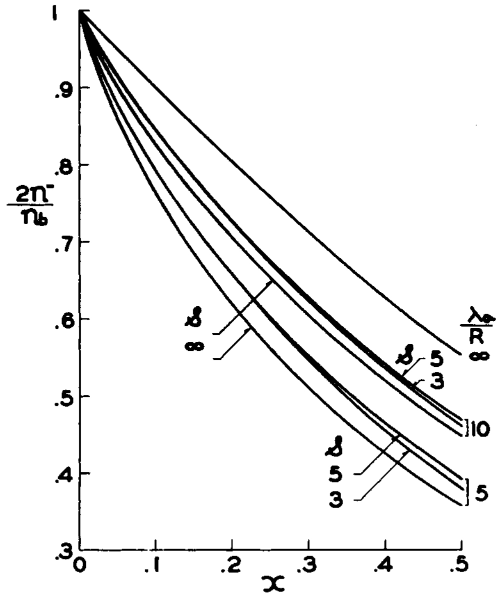

Fig. 4 Ratio along the stagnation line of number density to the stagnation value or the corresponding ratio of density to the stagnation value.