COMMENT ON "SECOND-ORDER COMPRESSIBLE BOUNDARY LAYER THEORY WITH APPLICATION TO BLUNT BODIES IN HYPERSONIC FLOW" BY MILTON VAN DYKE

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The occasion for this comment is an investigation that was done at Cornell University in the period 1959-1960, under the direction of Nicholas Rott. Viscous stagnation point flow was considered in terms of the same two-fold expansion procedure used by M. Van Dyke. Since attention was limited to the stagnation point flow, the transverse curvature effect was not treated, nor the effect considered of a nonuniform free stream which is identified by Van Dyke as the total enthalpy gradient effect. Numerical solutions were obtained for the remaining five second-order effects, for wall temperatures ranging from adiabatic to 0.1 of free stream stagnation value (Ref. 1). The fluid properties used were very similar to those used by Van Dyke; perfect gas, constant Prandtl number and temperature dependent specific heat and transport properties, corresponding to the properties of nondissociated air at Mach numbers of about 4 to 6.

The five boundary layer correction effects depend on four perturbation parameters, which depend on both Reynolds number and Mach number. The contours of these parameters on a flight velocity vs. altitude diagram have been plotted, as indicated in Fig. 1. Body size will shift the altitude scale, but will leave the relationship between the families of contours unchanged.

The curvature effect depends on δ/R , and the ratio of the boundary layer thickness to the nose radius is δ/Δ , where δ depends on the stagnation fluid properties and the free stream velocity (in agreement with Van Dyke). The displacement effect parameter can not so simply be identified. For hypersonic blunt body flow it will be closely related to the curvature parameter; however, wall temperature also has an important effect on it. The slip and temperature jump parameter is λ/δ , where λ is

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the stagnation mean free path. Finally, the vorticity or entropy gradient effect parameter $V\delta$ is also shown. Here V is the slope of the inviscid stagnation point velocity profile for spheres; the parameter is identical to Kemp's (Ref. 2) parameter which gives the ratio of the inviscid to the average boundary layer vorticity. The strong Mach number dependence of this parameter is due to the fact that for strong shock V varies with $\epsilon^{-3/2}$, where ϵ is the shock density ratio. For reference, the ratio of δ to the inviscid shock standoff distance Δ is also indicated. Viscous hypersonic similitude occurs where the parametric contours become parallel.

The applicability of the expansion procedure is limited to the region in which all the expansion parameters are less than 1. The slip parameter gives, in addition, an approximate indication of the applicability of the continuum equations. The need for a fully viscous shock layer theory is indicated by the parameter δ/Δ.

In the investigation noted by the commentor, heat transfer results of his numerical calculations were compared wiith the available fully viscous shock layer calculations of Probstein and Kemp (Ref. 3), and Hoshisaki (Ref. 4), and also with the experimental results of Tewfik and Geidt (Ref. 5). The theoretical comparisons were inconclusive because, for the available axially symmetric cases the vorticity parameter was too large for the applicability of the expansion procedure. For the two-dimensional case, the available viscous shock layer calculations indicate a gradual increase in heat transfer rate over boundary layer values as the Reynolds number decreased, whereas Van Dyke's and the present computations indicate that both the displacement and curvature corrections are negative for these cases. Comparison with the experiments showed that the theory predicted combined reductions in heat transfer due to curvature and temperature jump ranging from 3 to 13%, whereas the measured reduction (as compared with the stagnation point boundary layer value) ranged from 3 to 33%. One might speculate here that the non-uniform flow conditions at the test section of the wind tunnel might have resulted in a total enthalpy gradient effect described by Van Dyke, which could account for the disagreement between test and theory.

Finally, the commentor should like to mention briefly the most unexpected and interesting result of his investigation; namely, the effect of wall cooling on the temperature jump term. The numerical calculations showed that, for strong cooling, this effect gave very large corrections to heat transfer. However, simultaneously, the rapid shrinkage of the wall mean free path decreased this correction; the combined effect was a weak decrease with wall temperature. Rott showed by an ingenious order of magnitude analysis that the reduction in heat transfer due to this effect depends on $\frac{\lambda_{wall}}{\lambda_{stag.}} \frac{k_{stag.}}{k_{wall}}$, rather than the much stronger dependence on $\frac{\lambda_{wall}}{\lambda_{stag.}}$ that was formerly supposed. This approximation shows excellent agreement with the numerical results. For the fluid properties used by Van Dyke, this gives a $\sqrt{T}$ dependence of this effect on wall temperature (k equals heat conductivity).

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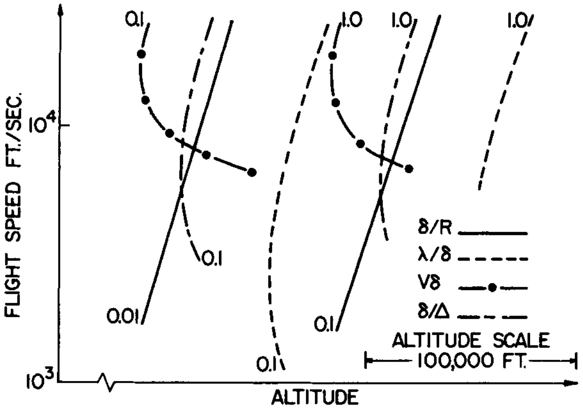

Fig. 1 Perturbation parameters as a function of flight velocity and altitude.