RAREFIED HYPERSONIC FLOW OVER A SPHERE

E. S. Levinsky[1] and H. Yoshihara[2]

Convair Division of General Dynamics, San Diego, California

ABSTRACT

The flow over a sphere at large hypersonic Mach numbers and low Reynolds numbers has been computed by using the Navier-Stokes equations as simplified by Probstein and Kemp. A similarity solution, valid about the stagnation streamline, is used. Both the adiabatic and cold wall cases are treated. Solutions, in particular the shock structure, the inviscid compression zone and the stagnation point boundary layer have been obtained at a Mach number of 10 and Reynolds numbers of 10^4, 10^3 and 10^2 based on the sphere radius, free stream velocity, and adiabatic stagnation values of density and viscosity.

INTRODUCTION

The flow about an object re-entering Earth's atmosphere assumes a myriad of patterns caused by the wide range of density during the descent (Fig. 1). At the higher altitudes the air is sufficiently rarefied so that collisions are highly improbable of air molecules with other air molecules compared with collisions between the air molecules and the body, and molecule-molecule interactions may be neglected. For this "free molecular flow," the trajectories of the molecules and, therefore, the complete flow pattern can be determined, provided that the particle-body surface interactions are known. Here the region of flow influenced by the body extends indefinitely upstream.

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[1]Design Specialist, Fluid Dynamics Research.

[2]Chief of Fluid Dynamics Research.

As the body enters regions of higher density, collisions between air molecules become important. For example, air molecules reflected upstream by the body will now collide with other oncoming molecules instead of traveling unhindered. There will be a tendency now for the molecules to pile up in front of the body. With further increase in ambient density this pile-up will increase, and the upstream extent of this region will contract toward the body. The particle number density in the pile-up region will be much greater than in the undisturbed free stream.

Consider next the more familiar high density extreme of the continuum flow regime encountered by the body. Here one finds the well-known flow pattern of an infinitesimally thin bow shock, a thin viscous boundary layer adjacent to the body and an inviscid region inbetween (Fig. 1e). For sufficiently high free stream densities the flow details within the shock structure need not be considered, and the shock may be bridged, to a good approximation, by the usual shock jump conditions. The remainder of the flow can then be computed in a piecemeal fashion, first by assuming that the flow is inviscid throughout, and then by modifying the flow near the body by a stagnation point boundary layer flow analysis (Ref. 1).

As the free stream density is decreased (for a constant free stream Mach number), the thickness of both the shock wave and boundary layer will grow. In general, the shock wave will increase in thickness more rapidly than the boundary layer, and a flow configuration will be achieved in which the thickness of the shock wave may no longer by ignored (Fig. 1d). The inviscid region separating the shock wave from the boundary layer is now reduced in thickness for this flow pattern which has been termed an "incipient merged layer" by Probstein (Ref. 2).

A further decrease in free stream density causes this inviscid layer to be eliminated entirely, and the shock wave and boundary layer are then said to be fully merged (Fig. 1c). The simplified piecemeal approach described is no longer permissible for the latter two cases, and the flow as a whole must be considered.

For the dilute gases considered, the Boltzmann equation for the one-particle distribution function is, to be sure, adequate to describe the entire spectrum of flows encountered by a reentry body. This equation generally furnishes more information than is frequently required, and except in extremely simplified cases it is impossible to solve.

If the density of the fluid particles, however, is sufficiently high so that averages of the number density and, say, the random kinetic energy of the molecules per unit volume assume steady values, then one may derive a fully determinate system of continuum equations in terms of macroscopic quantities, such as the temperature, pressure and fluid flow velocities. Here the flow is assumed to be macroscopically steady with respect to coordinates fixed with the body, and the averages are taken over an element of space and duration of time each very much less than the corresponding meaningful macroscopic quantities. The larger the gradients of the macroscopic quantities within the flow, the greater will be the complexity of the applicable macroscopic equations.

Attention will be focused in the present paper on the transition of the flow from the high density extreme to a case in which the shock wave and boundary layer have merged. (For a review of previous work on this subject, see Hayes and Probstein in Ref. 3.) In particular, a series of steady flows over a sphere at a fixed hypersonic Mach number and varying free stream densities will be considered which will include the incipient merged layer and fully merged layer cases. The validity of the Navier-Stokes equations will be postulated for this flow sequence. The use of these equations for the high density portion of the shock layer is plausible, but at first glance their use might be questioned within the shock structure, since here the macroscopic quantities such as the temperature vary appreciably over a mean free path. However, there is evidence that the Navier-Stokes equations give excellent agreement with experiments for the structure of normal shocks (Refs. 3 and 4). This correlation is at low Mach numbers, but, nevertheless, the good agreement here lends support for the use of the "Navier-Stokes" equations as a first approximation for the hypersonic case.

To simplify the full "Navier-Stokes" equations a procedure developed by Probstein and Kemp (Ref. 5) will be followed. This procedure is based upon the fact that the shock layer is very thin compared with the body radius for the large hypersonic Mach numbers considered here. Local flow similarity is then enforced, which yields a system of ordinary differential equations which will be valid in the flow domain near the axis of symmetry upstream of the body. The resultant system of equations will thus not describe the entire flow but will yield information on the shock detachment distance and the shock structure, as well as conditions at the stagnation point.

Both the insulated and "cold wall" cases will be treated. The case for which the body is a perfect heat insulator is simplified by assuming the hard sphere model of the gas with a Prandtl number of three-fourths. The energy equation is then decoupled from the remaining equations and may be integrated to yield a particular solution, the so-called Becker's integral, which is valid from the free stream to the body.

A more practical case is one in which the body is a good conductor of heat passing, for example, sufficient heat to maintain the surface at a constant tolerable temperature. The surface temperature will be taken equal to the static free stream temperature for this so called "cold wall" case. The solutions for this case should differ from the insulated case primarily within the temperature boundary layer adjacent to the body, and will effect the shock structure only when the boundary layer and shock are merged.

DISCUSSION

To simplify the problem and avoid real-gas effects and the question of the bulk viscosity, a monatomic gas with a "billiard ball" model for the interaction potential will be considered. The coefficients of viscosity and heat conductivity will then be proportional to the square root of the absolute temperature. In addition, a perfect gas with constant specific heats will be assumed. The value for the Prandtl number Pr = 3/4 was chosen, instead of the consistent value 2/3, to simplify the energy equation.

Basic Equations

The validity of the "Navier-Stokes" approximation for the stress term and the heat fluxes will be assumed. Thus, in spherical coordinates (defined in Fig. 2) the basic equations become

$$(\rho v)_r + \frac{2\rho v}{r} + \frac{1}{r}(\rho u)_\theta + \rho u \frac{\cot\theta}{r} = 0 \tag{1}$$

$$p_r + \rho\left(v v_r + \frac{u}{r} v_\theta - \frac{u^2}{r}\right)$$

$$= 2\left[\mu v_r - \frac{\frac{\mu}{3}}{r}\left(r v_r + 2v + u_\theta + u\cot\theta\right)\right]_r \tag{2}$$

$$+ \frac{1}{r}\left\{\mu\left[r\left(\frac{u}{r}\right)_r + \frac{v_\theta}{r}\right]\right\}_\theta + 2\,\frac{\mu}{r}\left(2v_r - \frac{u_\theta + v}{r}\right)$$

$$+ \frac{\mu}{r}\left\{\cot\theta\left[r\left(\frac{u}{r}\right)_r + \frac{v_\theta}{r}\right] - \frac{2}{r}\,(v + u\cot\theta)\right\} \tag{2}$$

$$\frac{p_\theta}{r} + \rho\left[vu_r + \frac{u}{r}\,u_\theta + \frac{uv}{r}\right]$$

$$= \frac{2}{r}\left[\frac{\mu}{r}\,(u_\theta + v) - \frac{\frac{\mu}{3}}{r}\,(rv_r + 2v + u_\theta + u\cot\theta)\right]_\theta$$

$$+ \left\{\mu\left[r\left(\frac{u}{r}\right)_r + \frac{v_\theta}{r}\right]\right\}_r + 3\,\frac{\mu}{r}\left[r\left(\frac{u}{r}\right)_r + \frac{v_\theta}{r}\right] \tag{3}$$

$$+ 2\,\frac{\mu}{r^2}\cot\theta\,(u_\theta - u\cot\theta)$$

$$\rho\left(vh_r + \frac{u}{r}\,h_\theta\right) = vp_r + \frac{u}{r}\,p_\theta + \frac{1}{rPr}\left[(\mu r h_r)_r + \left(\frac{\mu}{r}\,h_\theta\right)_\theta + \mu h_r + u\,\frac{\cot\theta}{r}\,h_\theta\right]$$

$$+ \mu\left[2v_r^2 + \frac{2}{r^2}(u_\theta + v)^2 + \frac{2}{r^2}(v + u\cot\theta)^2 + \left(r\left(\frac{u}{r}\right)_r + \frac{v_\theta}{r}\right)^2\right]$$

$$- \frac{\frac{2\mu}{3}}{r^2}\left[r^2v_r^2 + 2v^2 + u_\theta^2 + u^2\cot^2\theta + 4rv_rv + 2rv_ru_\theta\right.$$

$$\left. + 2rv_ru\cot\theta + 4vu_\theta + 4vu\cot\theta + 2u_\theta u\cot\theta\right] \tag{4}$$

and

$$p = c_p\left(\frac{\gamma - 1}{\gamma}\right)\rho T \tag{5}$$

where the symbols are defined in the Nomenclature, and the subscripts denote partial differentiation. The boundary conditions are as follows:

1) Far from the body a uniform free stream is assumed with the conditions

$$u = v_\infty \sin\theta$$

$$v = -v_\infty \cos\theta$$

$$p = p_\infty, \quad \rho = \rho_\infty \qquad T = T_\infty$$

2) At the surface of the body the no slip condition is postulated

$$u = v = 0$$

and in addition either the insulated or adiabatic wall condition

$$\frac{\partial T}{\partial r} = 0$$

or the cold wall condition

$$T = T_w \sim T_\infty$$

For the extremely low Reynolds number cases the no slip condition for the adiabatic wall case may be somewhat questionable but should be a good approximation for the cold wall case (Ref. 3).

Simplification of Basic Equations

The boundary value problem as formulated here is, of course, difficult to solve. To simplify the equations the authors shall follow the procedure of Probstein and Kemp. This is based upon on order of magnitude evaluation of the various terms in the differential equations using the fact that for large hypersonic Mach numbers the shock layer is extremely thin and that disturbances near the sonic line do not appreciably influence the flow on the axis. The resultant simplified equations become

$$(\rho v)_r + 2\frac{\rho v}{r_B} + \frac{(\rho u)_\theta}{r_B} + \frac{\rho \frac{u}{\theta}}{r_B} = 0 \tag{6}$$

$$p_r + \rho v v_r + \frac{\rho u v_\theta}{r_B} - \frac{\rho u^2}{r_B} = \left(\frac{4}{3}\mu\, v_r\right)_r - \frac{\left[\frac{2}{3}\ \mu\left(u_\theta + \frac{u}{\theta}\right)\right]_r}{r_B} + \frac{\left(\mu u_r\right)_\theta}{r_B} + \frac{\mu\left(\frac{u}{\theta}\right)_r}{r_B} \tag{7}$$

$$\frac{p_\theta}{r_B} + \rho v u_r + \rho\frac{u}{r_B}(u_\theta + v) = (\mu u_r)_r \tag{8}$$

and

$$\rho v\left(h + \frac{v^2}{2}\right)_r = \left[\frac{4}{3}\mu\left(\frac{\frac{3}{4}}{Pr}h + \frac{v^2}{2}\right)_r\right]_r \tag{9}$$

where r_B is the body radius.

These equations, to be sure, are consistent to the lowest order, but the expansion procedure is not systematic since it does not provide for higher order approximations.

With the Prandtl number of 3/4, the energy Eq. 9 may be integrated to obtain the so-called Becker's integral

$$C_p T + \frac{v^2}{2} = \text{constant}$$

and the flow is said to be isoenergetic.

This integral requires that $T' = 0$ at $v = 0$, so that it may be used only for the adiabatic wall condition. Thus, for this case the number of differential equations may be reduced by one.

For the solution of these equations the following set of similarity postulates is introduced near the axis of symmetry $\theta = 0$

$$u = u_o(r) \sin \theta$$

$$v = v_o(r) \cos \theta$$

$$\rho = \rho_o(r)$$

$$T = T_o(r) \cos^2 \theta + T_2(r) \sin^2 \theta$$

$$h = h_o(r) \cos^2 \theta + h_2(r) \sin^2 \theta$$

$$\mu = \mu_o(r) \cos \theta$$

and

$$p = p_\infty + \left[p_o(r) - p_\infty\right] \cos^2 \theta + \left[p_2(r) - p_\infty\right] \sin^2 \theta$$

where the terms with subscripts 0 and 2 are functions of r only.

Introducing this into the basic Eqs. 5 to 9, and carrying out the nondimensionalization

$$\bar{v} = \frac{v_o}{v_\infty} \qquad \bar{u} = \frac{u_o}{v_\infty} \qquad \bar{p} = \frac{p_o}{\rho_{AW} v^2_\infty} \qquad \bar{p}_2 = \frac{p_2}{\rho_{AW} v^2_\infty}$$

$$\bar{\rho} = \frac{\rho}{\rho_{AW}} \qquad \bar{T} = \frac{T_o}{T_{AW}} \qquad \bar{\mu} = \frac{\mu_o}{\mu_{AW}} \qquad R_e = \frac{v_\infty r_B \rho_{AW}}{\mu_{AW}}$$

$$Pr = \frac{\mu_{AW} C_p}{k_{AW}} \qquad \bar{r} = \frac{r}{r_B}$$

where the subscripts ∞ and AW refer, respectively, to the free stream condition and to the stagnation point of the body with adiabatic wall conditions, one obtains

$$(\overline{\rho v})' + 2\bar{\rho}(\bar{u} + \bar{v}) = 0 \tag{11}$$

$$\bar{p}' + \bar{\rho}\bar{v}\bar{v}' = \frac{4/3}{Re}(\bar{\mu}\bar{v}')' + \frac{2/3}{Re}\bar{\mu}\ \bar{u}' - \frac{4/3}{Re}\bar{\mu}'\ \bar{u} \tag{12}$$

$$\bar{p}_2' - \bar{\rho}\ \bar{u}\ (\bar{u} + \bar{v}) = 0 \tag{13}$$

$$2(\bar{p}_2 - \bar{p}) + \bar{\rho}\ \bar{v}\ \bar{u}' + \bar{\rho}\ \bar{u}\ (\bar{u} + \bar{v}) = \frac{(\bar{\mu}\bar{u}')'}{Re} \tag{14}$$

$$\bar{\rho}\bar{v}\left(A\bar{T}' + \bar{v}\bar{v}'\right) = \frac{4/3}{Re}\left[\bar{\mu}\left(\frac{3/4}{Pr}A\bar{T}' + \bar{v}\bar{v}'\right)\right]' \tag{15}$$

and

$$\bar{p} = \sigma A \bar{\rho} \bar{T} \tag{16}$$

where

$$\sigma = \frac{\gamma - 1}{\gamma} \qquad A = \frac{C_p T_{AW}}{v_\infty^2} \qquad \bar{\mu} = \bar{T}^{\frac{1}{2}}$$

For the adiabatic wall condition, Becker's integral becomes

$$\bar{T} + \frac{\bar{v}^2}{2A} = 1 \tag{17}$$

and the constant A may be evaluated in terms of the free stream Mach number M_∞ as

$$A = \frac{1}{2} + [(\gamma - 1)M_\infty^{\,2}]^{-1}$$

The boundary conditions for the adiabatic wall case are:

1) In the free stream

$$\bar{u} = 1 \quad \bar{v} = -1 \quad \bar{p} = \bar{p}_2 = \bar{p}_\infty \quad \bar{T} = \bar{T}_\infty$$

2) On the body

$$\bar{u} = \bar{v} = 0 \quad \bar{\rho} = \bar{T} = 1 \quad \bar{T}' = 0$$

In addition, the requirement $\bar{v}' = 0$ must be imposed at the body to ensure a finite density gradient. The fulfillment of the conditions $\bar{T} = \bar{T}_\infty$ in the free stream and $\bar{T}' = 0$ at the body is ensured by a suitable choice of the constants of integration in Becker's integral.

For the cold wall case, Becker's integral is no longer valid throughout the flow. The boundary conditions in the free stream remain the same; however, the boundary conditions at the body are now

$$\bar{u} = \bar{v} = 0 \qquad \bar{T} = \bar{T}_\infty$$

Solution of the Problem

The method of integration of a two-point boundary value problem with nonlinear differential equations is to convert the problem into an initial value problem and then to vary parametrically those initial conditions not originally specified until the boundary conditions are fulfilled. An additional difficulty arises, however, because the solutions cannot be integrated all the way from the free stream to the body, or from the body to the free stream, because of instabilities in the integration procedure. The equations are therefore integrated from each end. The unassigned initial conditions in the free stream and at the body are then adjusted such that the two solutions match at a common point between the shock and the boundary layer where the integration procedure is still stable.

1. Free Stream Integration

Because of the singular nature of the free stream, the solution is integrated from this end by perturbing the flow about the free stream conditions, and initiating the integration with the linear solution. By introducing the set of perturbation quantities $\tilde{\rho}$, $\tilde{v}$, $\tilde{u}$, etc., where

$$\bar{\rho} = \frac{\rho_\infty}{\rho_{AW}} (1 + \tilde{\rho}) \quad \bar{v} = -1 + \tilde{v} \quad \bar{u} = 1 + \tilde{u}$$

$$p_2 = \frac{\rho_\infty}{\rho_{AW}} \left(\frac{1}{\gamma M_\infty^2} + \tilde{p}_2 \right) \quad \bar{p} = \frac{\rho_\infty}{\rho_{AW}} \left(\frac{1}{\gamma M_\infty^2} + \tilde{p} \right)$$

$$\bar{T} = \frac{1}{A(\gamma-1)M_\infty^2} + \tilde{T} \qquad \bar{\mu} = \frac{\mu_\infty}{\mu_{AW}} \left(1 + \tilde{\mu} \right)$$

a set of linearized flow equations valid near the free stream is obtained

$$\tilde{v}' - \tilde{\rho}' + 2 (\tilde{v} + \tilde{u}) = 0 \tag{18}$$

$$\tilde{p}' - \tilde{v}' = \frac{4/3}{Re_\infty} \tilde{v}'' \tag{19}$$

$$\tilde{v} - A\tilde{T} = - \frac{4/3}{Re_\infty} (\tilde{v} - A\tilde{T})' \tag{20}$$

$$\tilde{p} = \sigma A \left[\frac{\tilde{\rho}}{A(\gamma-1)M_\infty^2} + \tilde{T} \right] \tag{21}$$

$$\tilde{p}_2' - (\tilde{u} + \tilde{v}) = 0 \tag{22}$$

and

$$2(\tilde{p}_2 - \tilde{p}) - \tilde{u}' + (\tilde{u} + \tilde{v}) = \frac{\tilde{u}''}{Re_\infty} \tag{23}$$

Here Re_∞ is the Reynolds number based on free stream conditions.

This set of equations is readily solved by making the assumptions that the perturbation $\tilde{u} \ll \tilde{v}$, and that the term $\tilde{u}''/Re_\infty$ may be considered small in Eq. 23. Both of these assumptions may be varified a posteriori. The first enables Eqs. 18 to 21 to be separated from the remainder, and permits solutions for $\tilde{v}$, $\tilde{\rho}$,$\tilde{T}$ and $\tilde{p}$ to be found independently of the cross flow quantities $\tilde{u}$ and $\tilde{p}_2$. Thus

$$\tilde{v} = \tilde{v}_i + k_1 A\tilde{T}_i \qquad \tilde{T} = \frac{\tilde{v}}{A} - \tilde{T}_i$$

and

$$\tilde{\rho} = \frac{m+2}{m}\tilde{v}_i + \frac{3/4\, Re_\infty - 2}{\frac{3}{4} Re_\infty} k_1 A\tilde{T}_i \tag{24}$$

where

$$\tilde{v}_i = C_1 e^{m\bar{r}_i} \qquad \tilde{T}_i = C_2 e^{-\frac{3}{4} Re_\infty \bar{r}_i}$$

Here C_1 and C_2 are arbitrary constants of integration, $\bar{r}_i$ is the radius at which the perturbations are evaluated, and

$$m = \frac{3}{8} Re_\infty \left\{\sigma\left[\frac{1}{(\gamma-1)M_\infty^2} + 1\right] - 1\right\}$$

$$- \sqrt{\left[\frac{3}{8} Re_\infty \left\{\sigma\left[\frac{1}{(\gamma-1)M_\infty^2} + 1\right] - 1\right\}\right]^2 + \frac{\frac{3}{2}\sigma Re_\infty}{(\gamma-1)M_\infty^2}} \tag{25}$$

whereas

$$k_1 = \frac{1}{1 + \frac{1}{(\gamma-1)M_\infty^2} - \frac{8/3}{(\gamma-1)M_\infty^2 \, Re_\infty}} \tag{26}$$

For the adiabatic wall case, $\tilde{T}_i = 0$, and the constant C_2 must vanish. For the cold wall case, Becker's integral may still be satisfied in the free stream provided the cold wall effects are restricted to the temperature boundary layer adjacent to the body. Solutions in the free stream for the cold wall case will first be attempted with $\tilde{T}_i = 0$. However, if these solutions do not match the solutions obtained by integrating from the wall, then the value $\tilde{T}_i$ will be varied parametrically. The initial value $\tilde{v}_i$ will be assumed fixed at one per cent of the free stream velocity.

Neglecting the term in $\tilde{u}''/Re_\infty$ permits Eqs. 22 and 23 to be readily integrated giving

$$\tilde{u} = \tilde{u}_i + k_2\tilde{v}_i + k_3 k_1 A\tilde{T}_i$$

and

$$\tilde{p}_2 = \frac{1+k_2}{m}\tilde{v}_i - \frac{1+k_3}{3/4 \, Re_\infty} k_1 A\tilde{T}_i - \tilde{u}_i \tag{27}$$

where

$$\tilde{u}_i = C_3 e^{-\tilde{r}_i}$$

Here C_3 is an arbitrary constant

$$k_2 = - \frac{\frac{\frac{8}{3}}{Re_\infty} m^2 + m - 2}{m^2 - m - 2} \tag{28}$$

and

$$k_3 = - \frac{\frac{3}{4} Re_\infty - 2}{\left(\frac{3}{4} Re_\infty\right)^2 + \frac{3}{4} Re_\infty - 2} \qquad (29)$$

A zero value for the constant C_3 was found to yield solutions which matched those obtained by integrating from the body for all cases considered. Since both k_2 and k_3 are of order $1/Re_\infty$, the assumptions concerning the perturbations $\tilde{u}$ and $\frac{\tilde{u}''}{Re_\infty}$ appear justified.

The linear solutions Eqs. 24 and 27 are used to initiate the integration of the nonlinear system, Eqs. 11 to 17 from the free stream end. This procedure was convergent until the boundary layer was approached.

2. Integration From the Body

The expansion of the solution about the stagnation point offers no essential difficulties, and is similar to the classical incompressible solutions (Ref. 6). The fundamental differences in the present integration result from the variable density and temperature and from the introduction of a variable transverse pressure term $\bar{P}_2$, as given by Eq. 13. In addition, the integration from the stagnation point may be carried out, in principle at least, as far as the free stream, since the nonlinear system is valid throughout the flow domain in front of the body. In practice, however, the solution becomes unstable before the shock is reached, limiting the outward extent of the integration.

3. Review of Parametric Problem

Before discussing the numerical results, the parametric problem required to match the solutions from the free stream and from the body will be reviewed. For the cold wall case, the boundary conditions at the body are four in number, including the requirement that $\bar{v}' = 0$. Since the order of the nonlinear system is eight, four parameters, namely $\bar{u}$, $\bar{T}'$, $\bar{P}_2$ and $\bar{\rho}$, must be adjusted at the wall to match the free stream solution. The parameter $\tilde{T}_1$ in the free stream solution must also be varied from the zero value if the temperature boundary layer is merged with the shock.

The adiabatic wall case is much simpler since only two parameters, $\bar{u}'$ and $\bar{P}_2$, must be adjusted at the body. In addition, $\bar{T}_1$ equals 0 in the free stream solution.

Numerical Results

The system of Eqs. 11 through 17 has been programmed for solution on the IBM 704 digital computer. All solutions were carried out for a monatomic gas with $\gamma = 5/3$; for a free stream Mach number M_∞ equals 10 (which should be representative of the hypersonic conditions encountered during re-entry); and for Reynolds numbers, based on adiabatic stagnation temperature and density, of 10,000, 1000 and 100 to illustrate the transition from relatively dense to rarefied conditions. The results are summarized in Fig. 3 for adiabatic wall conditions, and in Fig. 4 for cold wall conditions. It is seen that for both conditions the shock is essentially a discontinuity at the largest Reynolds number. However, for $Re = 1000$, the shock thickness is of the order of 10% of the detachment distance. For this case, the shock and the boundary layer remain distinct and are separated by an inviscid region. For a Reynolds number of 100, the shock thickness has increased to approximately 50% of the detachment distance, and a distinct inviscid region is no longer present indicating a fully merged layer.

The growth in the upstream influence of the body at the lower densities is shown by the substantial increase in the detachment distance at the lower Reynolds numbers. This increased detachment distance is due to both the thickened shock wave and the increased boundary layer thickness at the lower Reynolds numbers. The shock thickness varies as Re^{-1}, whereas the boundary layer height varies with $Re^{-\frac{1}{2}}$. Cooling the wall reduces the upstream influence of the body by reducing the boundary layer thickness.

The variation in the quantities $\bar{u}$, $\bar{v}$, $\bar{p}$ and $\bar{P}_2$ from the free stream to the body is shown in Figs. 5 to 7 for the adiabatic wall case. Figures 8 to 10 show velocity and temperature profiles for the cold wall cases. The shock temperature profiles for the adiabatic and cold wall cases are identical at the higher Reynolds numbers, but differ for the fully merged case because the influence of the cold body extends to the free stream.

The heat transfer to the stagnation point of the sphere has been computed for the cold wall cases and is presented in Fig. 11. The Stanton number C_H is readily shown to be given as

$$C_H \sqrt{Re_\infty} = \frac{2A}{Pr} \cdot \frac{\bar{T}'(0)}{\sqrt{Re_\infty}} \qquad (30)$$

where the free stream Reynolds number Re_∞ is related to Re by $Re_\infty = \left(\frac{\bar{\rho}_\infty}{\bar{\mu}_\infty}\right) Re$. . A comparison of these results with free molecular flow theory

$$C_H \sqrt{Re_\infty} = \sqrt{Re_\infty}$$

and boundary layer theory (Ref. 1)

$$C_H \sqrt{Re_\infty} = 3.92$$

is included in Fig. 11.

The variation in the stagnation point pressure coefficient with Re is shown in Fig. 12. Within the accuracy of the plot, the variation of the adiabatic and cold wall cases was identical. The decrease in stagnation pressure with decreasing Re is contrary to the experimental trend found by Sherman (Ref. 7) with adiabatic wall conditions, and may result from the neglect of slip effects which could become significant with adiabatic wall conditions (Ref. 8. However, slip effects should be negligible for the cold wall case.

FINAL REMARKS

Solutions have been obtained for the hypersonic flow of a monatomic gas past a sphere under highly rarefied conditions and give the shock structure and stagnation point boundary layer profiles for the incipient merged and fully merged cases. For the incipient merged layer case, the shock wave and boundary layer are distinct, and are separated by an inviscid zone. The flow is isoenergetic in the shock wave for this case, even for a highly cooled wall. With a fully merged layer, the cooling effect of the wall is felt in the shock resulting in nonisoenergetic conditions near the free stream.

On the basis of the result shown, further extensions appear warranted. These include the consideration of diatomic gases with the complicating real-gas effects and the addition of bulk viscosity, as well as the extension to lower Reynolds number (e.g., $Re = 10$) with a more exact slip boundary condition still using the Navier Stokes equations.

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NOMENCLATURE

$( )'$	=	prime, differentiation with respect to $\bar{r}$
$(\bar{\ })$	=	bar, nondimension quantity
$(\tilde{\ })$	=	tilde, perturbation quantity
A	=	constant dependent on free stream Mach number
C_1, C_2, C_3	=	constants of integration, Eqs. 24 and 27
C_H	=	Stanton number
c_p	=	specific heat at constant pressure
h	=	enthalpy
k_1, k_2, k_3	=	constants defined by Eqs. 26, 28 and 29, respectively
k	=	thermal conductivity
M	=	Mach number
m	=	constant defined by Eq. 25
p	=	pressure
Pr	=	Prandtl number
r	=	radius
r_B	=	body (sphere), radius
Re	=	Reynolds number $\frac{r_B\, v_\infty\, \rho_{AW}}{\mu_{AW}}$
T	=	temperature
u	=	velocity component in θ direction
v	=	velocity component in r direction
γ	=	ratio of specific heats
θ	=	angle from axis

μ = viscosity

ρ = density

σ = function of γ, $\left(\sigma = \frac{\gamma-1}{\gamma}\right)$

Subscripts

0,2 = similarity values

AW = adiabatic wall stagnation point values

i = initial perturbation values

∞ = undisturbed free stream values

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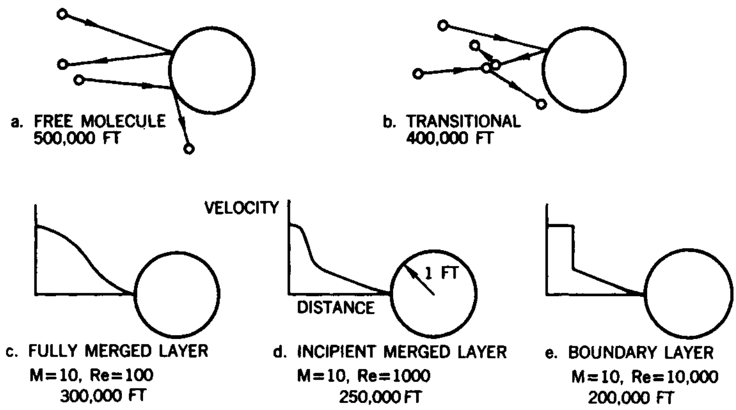

Fig. 1 Flow regimes.

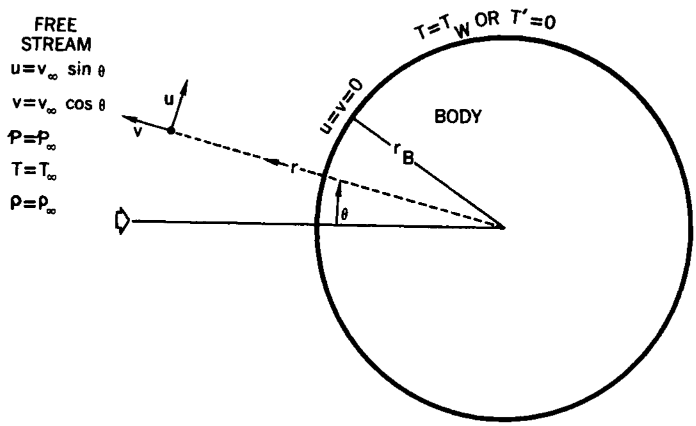

Fig. 2 Coordinate system.

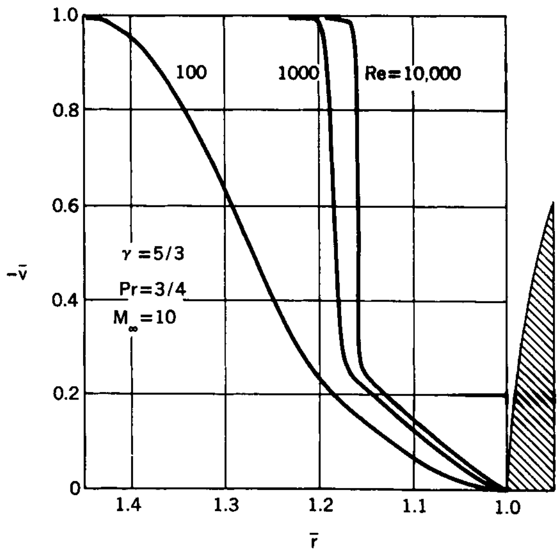

Fig. 3 Effect of Reynolds number on shock thickness and velocity profiles, adiabatic wall.

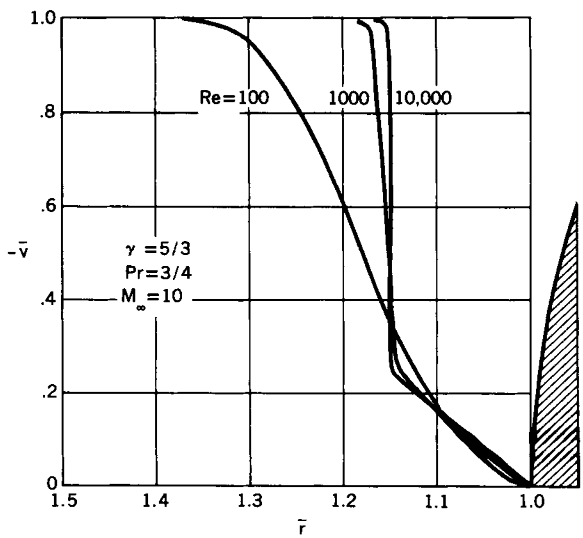

Fig. 4 Effect of Reynolds number on shock thickness and velocity profiles, cold wall.

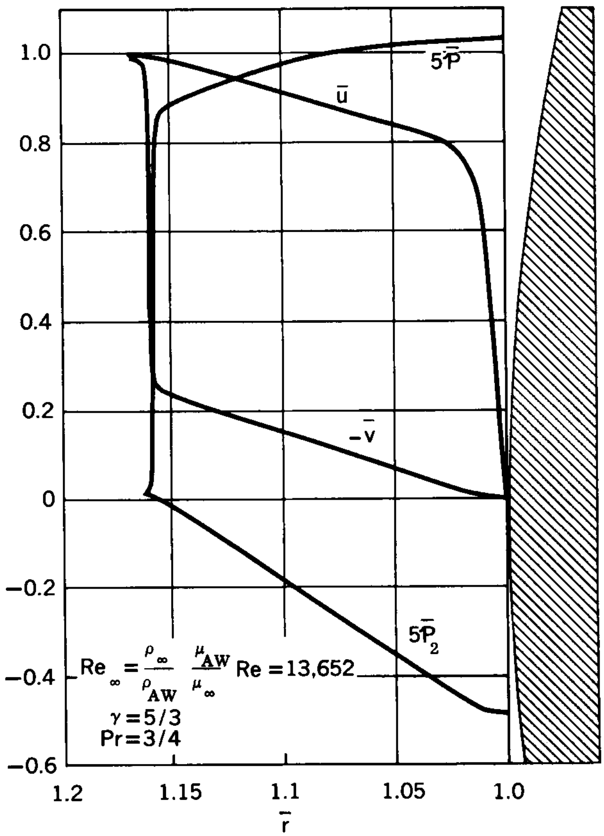

Fig. 5 Adiabatic stagnation line flow characteristics; Re = 10,000, $M_\infty = 10$.

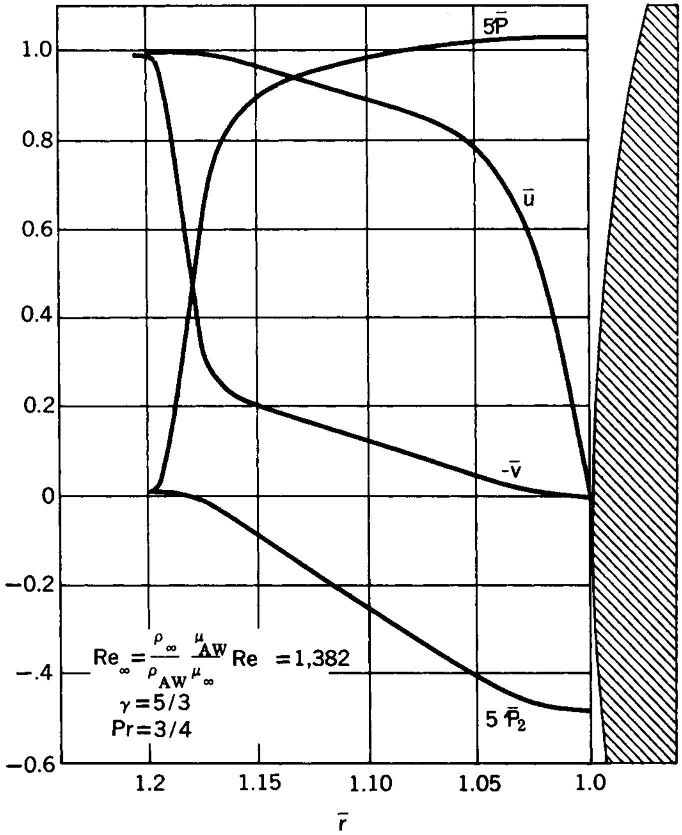

Fig. 6 Adiabatic stagnation line flow characteristics; Re = 1000, $M_\infty = 10$.

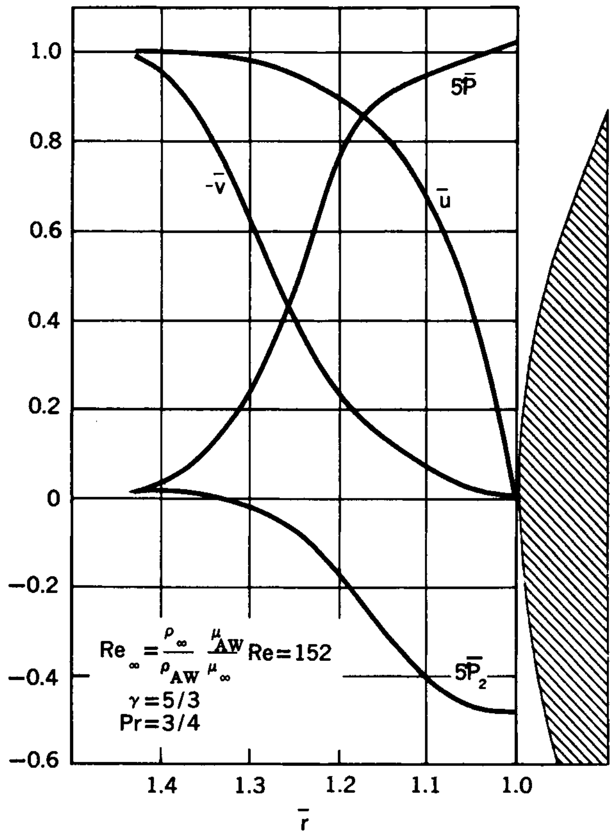

Fig. 7 Adiabatic stagnation line flow characteristics; Re = 100, $M_\infty = 10$.

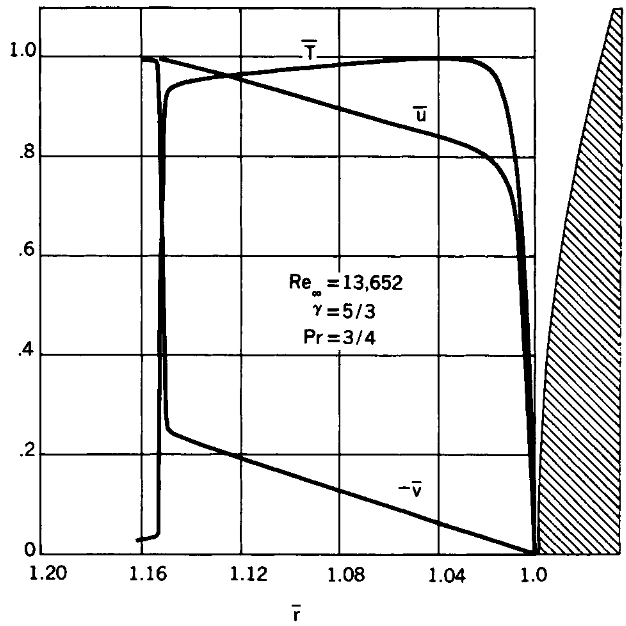

Fig. 8 Cold wall stagnation line flow characteristics; Re = 10,000, $M_\infty = 10$.

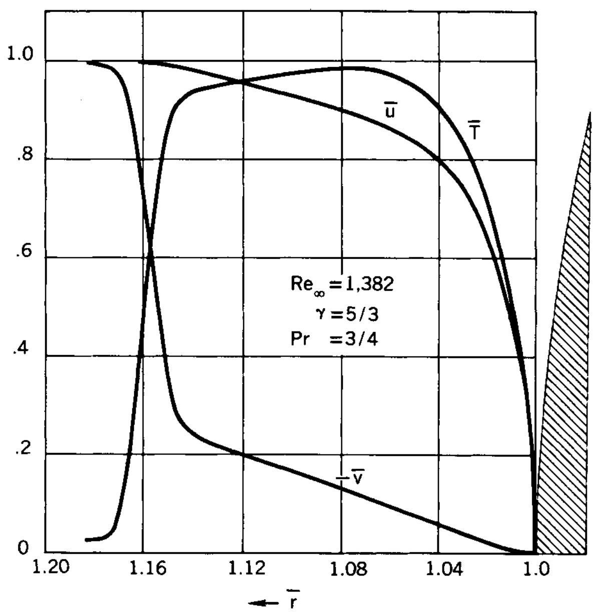

Fig. 9 Cold wall stagnation line flow characteristics; Re = 1000, $M_\infty = 10$.

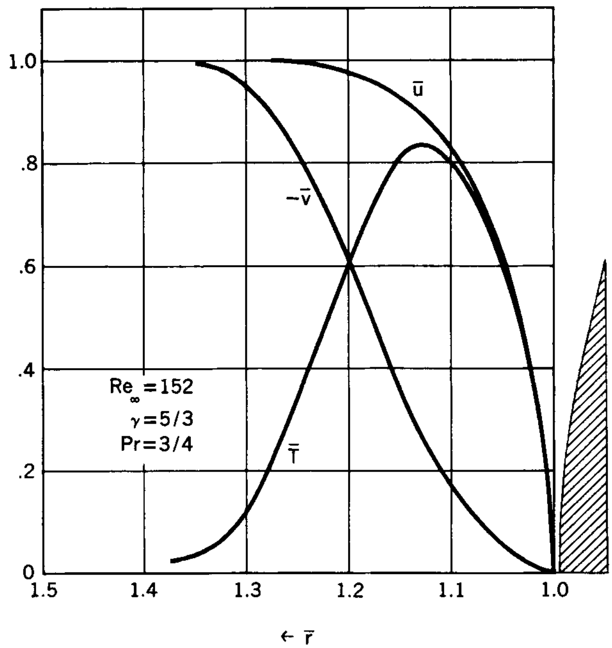

Fig. 10 Cold wall stagnation line flow characteristics; Re = 100,M_∞ =10.

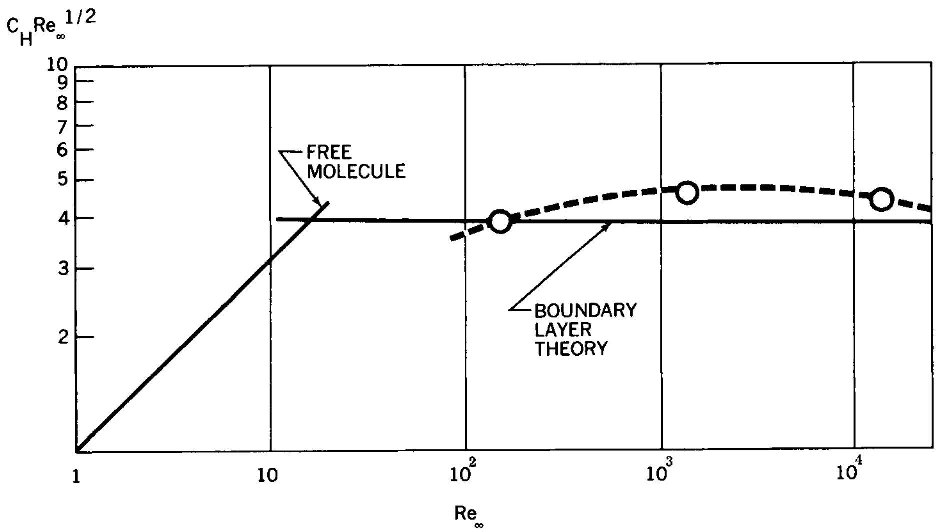

Fig. 11 Heat transfer, cold wall; M_∞=10, Pr = 3/4, γ = 5/3.

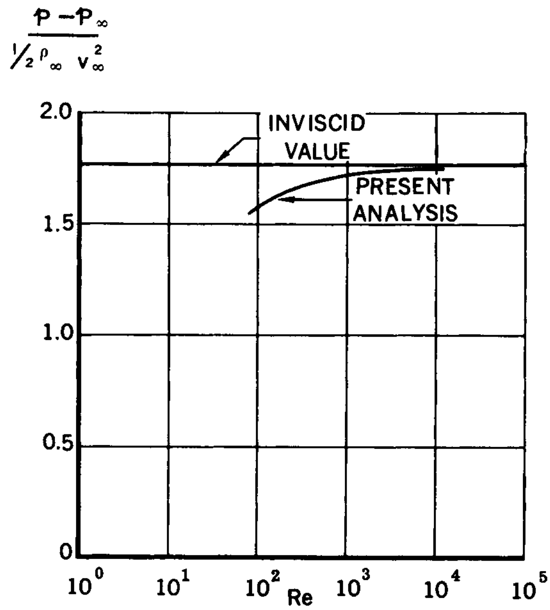

Fig. 12 Stagnation point pressure coefficient; $M_\infty = 10$.