CHAPTER 9

The Variations of the Local Reference System

THE horizon coordinate system and the astronomical system of geographic coordinates, which together form the local reference system for representing the immediate aspect of the celestial sphere at any particular geographic location, both depend in common upon the astronomical vertical and the axis of rotation of the Earth. Consequently, the variations of the vertical and the variations in the position of the axis within the Earth produce effects on these systems which are of direct importance in precise astronomical observations. The precession and nutation of the axis of rotation in space, in which the Earth participates as a whole, have no effect on either the horizon coordinate system or the geographic reference system.

The geographic coordinates depend upon the *direction of the vertical relative to the axis of rotation* and are therefore affected both by the variations of the vertical and by the variations of the position of the axis within the Earth.

The horizon coordinate system is geometrically defined by the astronomical horizon and the celestial meridian. The horizon is determined entirely by the vertical but the meridian depends upon both the vertical and the celestial pole. The horizon system is therefore affected both by the *variations* of the vertical and by the *further displacements* of the vertical in space by the motion of the Earth about its center of mass relative to the axis of rotation. The vertical participates in this motion of the Earth, with no variation relative to the Earth, and the zenith is consequently displaced on the celestial sphere, but the celestial pole is left unchanged among the stars, and therefore the celestial meridian is displaced on the sphere. Moreover, the local astronomical meridian line on the surface of the Earth is likewise displaced, since this line is the intersection of the plane of the celestial meridian with the surface of the Earth.

The Local Horizon Coordinate System

At a fixed position O on the surface of the Earth, the local meridian plane, which determines both the celestial meridian and the local geographic meridian line, is the plane through the astronomical vertical and the line from O toward the celestial pole. The great circle in which the celestial sphere is intersected by this plane is the celestial meridian, passing through the zenith and the pole. The intersection of the plane with the surface of the Earth is the local astronomical meridian line, or local north-south line, which coincides locally with the irregular astronomical meridian of longitude through the point O.

The local meridian plane does not in general pass through the axis of rotation of the Earth, because of the irregularities in the direction of local gravity, but it is always parallel to the instantaneous axis, and rotates with the Earth around this axis; this rotation of the plane in space is reflected in the apparent diurnal motions of the stars relative to the celestial meridian. However, because of the variations of the vertical, and the motion of the Earth about its center of mass relative to the axis of rotation, the local meridian plane is not fixed within the Earth. The intersection with the surface of the Earth, and the rotating intersection with the celestial sphere, are the primary immediate reference lines on the Earth and on the celestial sphere in actual astronomical observations; but the local meridian line on the Earth is not perfectly stationary on the surface, and the diurnal rotation of the celestial meridian on the sphere relative to the stars is not entirely due to the angular motion of the Earth around its instantaneous axis.

Variations of the Meridian

The motion of the local meridian plane that determines the variations of the celestial meridian and of the geographic meridian line is a rotation in space around the line from O to the celestial pole (Fig. 35). This line is left fixed in space, both by the variations of the vertical and by the motion of the Earth about its center of mass relative to the axis of rotation. The meridian plane necessarily moves with the vertical, but remains parallel to the axis of rotation, and continues to pass through the fixed line to the celestial pole; the plane consequently rotates in space around this line.

The rotation produced by the lunisolar displacement of the vertical is due to the east-west component of the variation in the direction of the vertical *relative to the Earth* caused, not by any motion of the Earth, but by the action of the lunisolar tide-generating forces. The vertical also moves *with the Earth* in the motion of the Earth relative to the axis of rotation. This motion directs the vertical toward a different point on the celestial sphere, with no change in its direction relative to the surface of the Earth; but the axis of rotation remains directed toward the same point on the celestial sphere, and since the direction from O to the celestial pole is therefore left unchanged *in space*, it is necessarily altered *relative to the surface of the*

Earth. The east-west component of this motion of the vertical therefore produces a further rotation of the local meridian plane in space around the line to the celestial pole.

The rotation of the local meridian plane around the direction to the celestial pole produces a displacement of the celestial meridian in hour angle on the celestial sphere, which is added to the diurnal motion around the instantaneous axis of rotation. Furthermore, the local meridian line on the surface of the Earth is rotated slightly around the vertical, over the moving

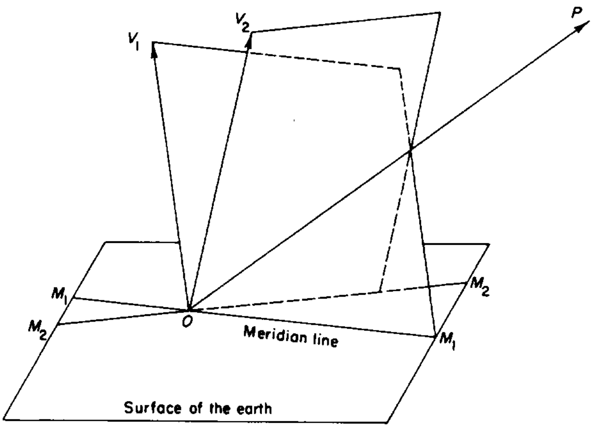

FIG. 35. Variation of the meridian plane.

surface; and this variation in the direction of the meridian line through O produces variations in the astronomical azimuths of lines that are fixed on the surface. These variations of the celestial meridian and the geographic meridian line are due to the *east-west* components of the variations in the direction of the vertical in space produced by the lunisolar tidal forces and by the motion of the Earth about its center of mass that causes the variation in the position of the axis of rotation within the Earth and the motion of the geographic poles over the surface. The *north-south* components alter the angle between the astronomical vertical and the axis of rotation, and therefore produce variations of the astronomical latitude of the point O. Both components displace the astronomical zenith among the stars, but the positions of the celestial poles and equator on the celestial sphere are not affected.

Variations in the Local Aspect of the Celestial Sphere

The local hour angle of a celestial body referred to the instantaneous meridian depends upon the variations of the meridian due to the lunisolar variation of the vertical and to the motion of the geographic poles over the surface of the Earth.

The variations of the local hour angle that are caused by the polar motion may be represented in terms of the relation of the plane of the astronomical meridian to the plane of the meridian of figure determined by the dynamical theory of the motion of the Earth about its center of mass. Because of the varying departure of the geographical pole from the pole of figure, and also

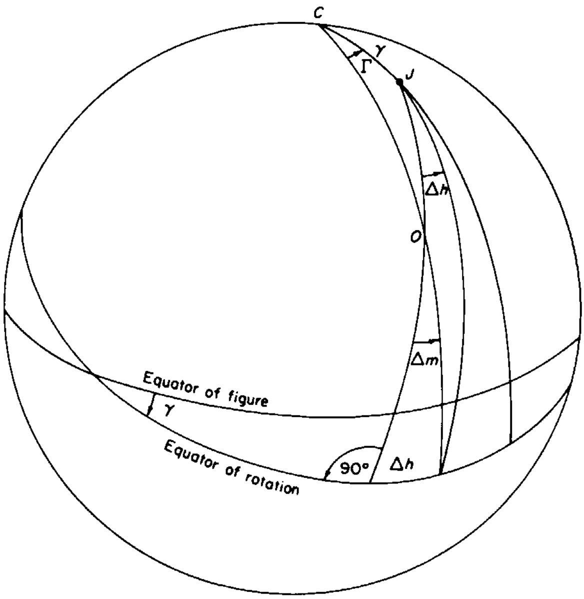

FIG. 36. Variation of the celestial meridian: C, pole of figure; J, pole of rotation; O, zenith; $OC = 90° - \Phi_0$; $OJ = 90° - \Phi$.

because of the constant deflection of the vertical, the local astronomical meridian through a point O on the surface of the Earth forms an angle Δm at O with the meridian of figure through this point (Fig. 36); Δm is the angle at the astronomical zenith between the celestial meridian and the intersection of the sphere with the plane of the meridian of figure, reckoned *eastward* from the instantaneous celestial meridian.

At any particular point fixed on the surface of the Earth at latitude of figure Φ_0, the instantaneous position of the local astronomical meridian relative to the meridian of figure through this point is represented by the angle Δm at which the two meridians intersect. This angle depends upon the varying position of the instantaneous pole of rotation J on the surface of the Earth relative to the pole of figure C; the astronomical azimuth Δm of the celestial meridian, westward from the southern arc of the meridian of figure, due to the displacement of the pole of rotation from the pole of figure, is given by

$$\sin \Delta m = \frac{\sin \gamma \sin \Gamma}{\cos \Phi}, \tag{111}$$

in which γ is the angular distance of the pole of rotation from the pole of figure, Γ is the angle at C eastward from the meridian of figure to CJ, and Φ is the instantaneous astronomical latitude.

Consequently, the point where the astronomical meridian intersects the equator of rotation is westward from the point where the meridian of figure intersects the equator, by an angular distance Δh given by

$$\tan \Delta h = \sin \Phi \tan \Delta m;$$

and the angle Δh between the instantaneous meridian and the hour circle through the point where the equator is intersected by the meridian of figure is

$$\Delta h = \gamma \sin \Gamma \tan \Phi + \gamma^3 \sin^3\Gamma \tan \Phi \frac{1 + 2\cos^2\Phi}{6\cos^2\Phi} + \cdots, \tag{112}$$

in which Γ is reckoned from the meridian of figure through O.

The angle Δh is the amount by which the celestial meridian is displaced westward in hour angle from the meridian of figure, due to the displacement Δm in azimuth, while the celestial pole J remains fixed among the stars. The departure of J from C decreases local west hour angles by the amount Δh.

The further variation of the local hour angle due to the lunisolar variation of the vertical is produced by the prime vertical component of the displacement of the zenith by the horizontal tide-generating force. Were the Earth rigid, the *eastward* displacement of the zenith at latitude Φ would be, from (110),

$$\begin{aligned} \Delta z &= -\frac{3}{2}\frac{M}{M_\oplus} \sin^3\pi \sin 2z \sin A \\ &= +\frac{3}{2}\frac{M}{M_\oplus} \sin^3\pi\{\cos \Phi \cos^2\delta \sin 2H + \sin \Phi \sin 2\delta \sin H\}, \end{aligned} \tag{113}$$

where the azimuth A is reckoned from north through east, H is the west hour angle of the attracting mass M, and δ is its declination. With sufficient accuracy, this eastward displacement may be regarded as an arc of the parallel of declination through the zenith, and therefore the eastward displacement of the celestial meridian in hour angle is $\Delta z/\cos \Phi$; it increases the hour angle of a star and causes the time of meridian transit to be earlier by this amount. From the action of the Moon, the eastward displacement of the zenith would be $-0''.0173 \sin 2z \sin A$, and from the action of the Sun $-0''.0082 \sin 2z \sin A$. However, this theoretical displacement of the zenith is modified by the departures of the Earth from an invariable rigid body.

Variations of the altitude and the azimuth of a celestial body are likewise produced by the variations of the zenith and the meridian. The altitude of the celestial pole remains continually equal to the astronomical latitude; and since the celestial pole is left fixed among the stars, the variations of its altitude cause slight variations in the geometric relations of the diurnal motions to the astronomical horizon.

The variations of the horizon reference system appreciably affect precise astronomical measurements, since observations are necessarily referred to the astronomical vertical; but they have no perceptible effects on the immediately visible appearance of the celestial sphere. The lunisolar variation in the positions of the astronomical zenith and horizon on the sphere is entirely a geometric effect, which produces no visible change. The variation due to the polar motion is produced by an actual displacement of the observer in space by the motion of the Earth about its center of mass, which displaces the visible horizon among the stars, and shifts the cardinal directions to different points of the visible horizon. The stars culminate above different points of the visible horizon, and different stars culminate in the zenith. However, the effects are not cumulative, and these changes in the visible aspects of the celestial sphere are too slight to be noticeable to ordinary observation.

Comparison of absolute and differential observations of stars shows that the variations of zenith distance at transit from the variations of the local reference system are due entirely to a variation of the zenith, and in no part to a simultaneous variation of the zenith and the celestial pole.

Variations of Geographical Coordinates

The variations which have been well established by observation in the astronomical latitudes and longitudes of fixed points on the Earth are produced virtually entirely by the motion of the geographic poles and the lunisolar variations of the vertical, although many of their characteristics are due to effects of departures of the Earth from rigidity. The principal part of the variations is caused by the polar motion.

From time to time, other variations of uncertain origin have been reported, but have not been satisfactorily confirmed. However, appreciable effects may possibly sometimes be caused by differential displacements of the lithosphere by seismic and tectonic actions, or by large-scale variations in physiographic conditions on the surface, such as thermal deformations of the lithosphere. Geological displacements of mass may occasionally exert direct effects in addition to their indirect effects on the polar motion. Observations can give only the resultant geographic coordinates determined by the concurrent actions of the polar motion, the variations of the vertical, and geophysical processes. The observations are inevitably subject to errors of observation, effects of extraneous conditions on the instruments, and errors in the adopted apparent places of the stars that are observed for the purpose. The elimination of errors, detection of spurious effects, and determination of the component variations and the individual causes of each require an appropriate analysis of long continuous series of highly accurate observations.

The variations produced by the polar motion may be expressed in terms of the rectangular coordinates x, y of the geographic pole J relative to the pole of figure C. With the angle Γ reckoned eastward from the meridian of figure through Greenwich (Fig. 37), these coordinates are $x = +\gamma \cos \Gamma$ toward Greenwich, and $y = -\gamma \sin \Gamma$ along the meridian 90°W. Denoting the latitude of figure of a point O on the surface of the Earth by Φ_m, and its longitude of figure reckoned *westward* from Greenwich by λ_m, the angle between the meridian of figure through O and the meridian CJ is $\Gamma + \lambda_m$.

To obtain the variation of latitude due to the displacement γ of J from C, we have from the triangle formed on the celestial sphere by C, J, and the zenith at O,

$$\sin \Phi = \sin \Phi_m \cos \gamma + \cos \Phi_m \sin \gamma \cos(\Gamma + \lambda_m),$$

where Φ is the instantaneous observed astronomical latitude of O. With sufficient accuracy, therefore,

$$\sin \Phi - \sin \Phi_m = \gamma \cos \Phi_m \cos(\Gamma + \lambda_m),$$

or

$$(\Phi - \Phi_m) \cos \tfrac{1}{2}(\Phi + \Phi_m) = \gamma \cos \Phi_m \cos(\Gamma + \lambda_m),$$

which may be written

$$\begin{aligned} \Phi - \Phi_m &= \gamma \cos(\Gamma + \lambda) \\ &= x \cos \lambda + y \sin \lambda, \end{aligned} \qquad (114)$$

in which for λ on the right, either the instantaneous astronomical longitude or the mean value λ_m may be used.

From this relation and the expressions (105) for x and y that have been obtained from dynamical theory, the variations of Φ may be determined that are due to the free Eulerian and the forced annual motions of the pole, and to the diurnal motion caused directly by the gravitational attractions of the Sun and the Moon on the Earth. In addition, a further lunisolar diurnal variation may be expected, due indirectly to the lunisolar tide-generating forces because of the effects of oceanic and earth tides on the figure and moments of inertia of the Earth. In practice, because of the great accuracy with which astronomical latitudes may be directly observed, the

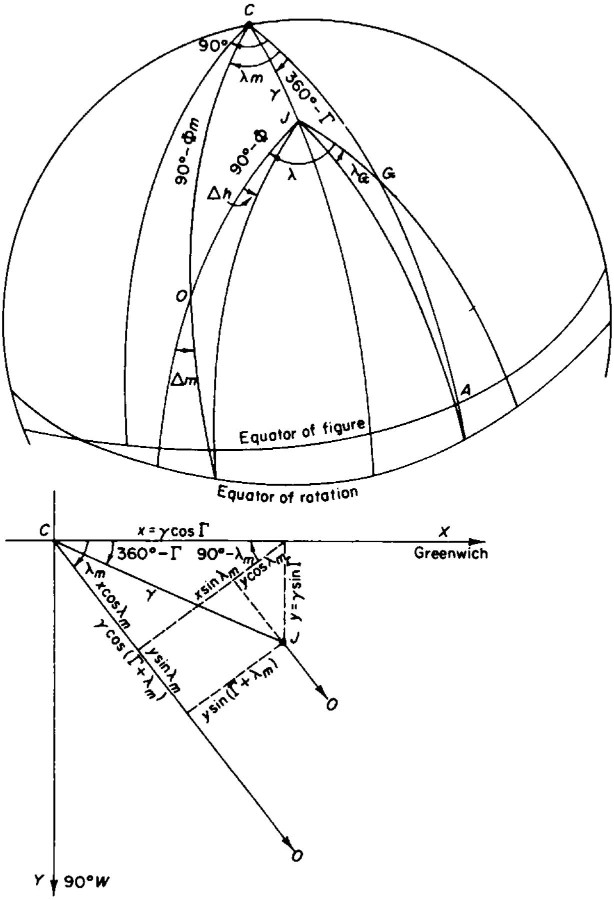

FIG. 37. Variations of latitude and longitude.

observation of these variations of Φ is the principal means of determining the actual motion of the geographic poles; only the direct lunisolar diurnal variation can be accurately calculated from theory, or reliably predicted in advance.

De Sitter* called attention to the lunisolar terms in the variation of latitude, pointing out that the term in twice the mean longitude of the Moon was not beyond the possibility of being detected in the observations of the International Latitude Service; and that the other terms, together with the mean annual parallax of the stars, could explain about half of the Kimura term in

* W. de Sitter, Über die von der Anziehung von Sonne und Mond herrührenden Breitenvariationen. *A. N.* **166**, 331 (1904).

the observed variations of latitude. The lunar term has since actually been found in latitude observations.*

The variation of Φ due to the lunisolar variation of the vertical is represented by the north-south component of the displacement of the zenith by the tide-generating forces. Were the Earth rigid, the *northward* displacement, which *increases* the latitude, caused by the direct action of the tide-generating forces in compounding with gravity, would be, by (110),

$$\Delta z_n = \frac{3}{2}\frac{M}{M_\oplus}\sin^3\pi\{\tfrac{1}{2}\sin 2\Phi\cos^2\delta\cos 2h - \cos 2\Phi\sin 2\delta\cos h + \tfrac{1}{2}\sin 2\Phi(1 - 3\sin^2\delta)\}. \quad (115)$$

From the action of the Moon, this increase in Φ would be $0''.0173 \sin 2z \cos A$, and from the action of the Sun, $0''.0082 \sin 2z \cos A$, where the azimuth A is reckoned from south through west. On the actual Earth, the magnitude is modified by the factor due to the redistribution of mass in the tidal deformation of the Earth. In addition, a further variation of the vertical is produced by the direct gravitational attraction of the oceanic tide. The direct attraction of the ocean tide, and the deformation of the surface of the Earth by tidal loading, vary greatly from place to place, and they extend inland to surprising distances.

The variation of longitude due to the variation in the direction of the local astronomical meridian caused by the polar motion is equal in magnitude to the variation Δh of local hour angles on the celestial sphere. The meridian through the fixed point O is rotated around O by the motion of the pole J over the surface of the Earth, and oscillates on either side of a mean position; and meanwhile, it likewise oscillates in direction around the moving pole. The angle Δh measures the variation from the mean direction at J, and Δm represents the variation from the mean direction at O. The displacement Δh of the moving meridian from its mean position *increases* the *west* longitude of O by the amount

$$\gamma \sin(\Gamma + \lambda_m) \tan\Phi = \tan\Phi\,(x \sin\lambda_m - y\cos\lambda_m), \quad (116)$$

where Γ is reckoned from the meridian of figure through Greenwich; this is the amount by which local west hour angles are decreased. The variation of longitude due to the lunisolar variation of the vertical is similar.

The astronomical meridian through Greenwich is likewise affected by these variations. Accordingly, because of the polar motion, the instantaneous Greenwich meridian, referred to its mean position, is at west longitude

$$\lambda_G = \gamma \sin\Gamma \tan\Phi_G.$$

* H. R. Morgan, The short period terms in the nutation as given by observation. *Astr. Jour.* **57**, 232 (1952).

The instantaneous west longitude of O from the instantaneous Greenwich meridian is therefore

$$\lambda = \lambda_m + \gamma\{\tan\Phi \sin(\Gamma + \lambda_m) - \tan\Phi_G \sin\Gamma\}. \tag{117}$$

The variation of the astronomical azimuth of a fixed line on the surface of the Earth is represented by the angle Δm. The polar motion decreases the azimuth A, reckoned from south through west, by the amount

$$\begin{aligned}\Delta A &= \frac{\gamma \sin(\Gamma + \lambda)}{\cos\Phi} \\ &= (x \sin\lambda - y\cos\lambda)\sec\Phi.\end{aligned} \tag{118}$$

For example, a fixed meridian instrument has a variable azimuth error because of the polar motion.