CHAPTER 14

The Measurement of Time

FROM ages immemorial, the means universally used to mark off the passage of time has been the apparent motions of the celestial bodies; and the measurement of time, whether the hours of day and night, or the longer intervals involved in calendarical and chronological reckoning, is still dependent upon the celestial motions. Other means, such as clocks, are only auxiliaries or intermediaries. The problem of the measurement of time is therefore peculiarly one of astronomy.

In astronomy we are not concerned with the elusive question of defining time, but only with the concrete physical problem of *measuring* time. To measure any physical quantity, we need only adopt an arbitrary unit and devise some practicable means of comparison with this unit; for this and, in general, other purposes of the physical sciences, an ultimate definition of the metaphysical nature of the quantity is not essential. We need not know what a quantity *is*, provided we know enough *about* it to be able to deduce anything further about it that may be required for the purpose in hand. A commonplace procedure in the mathematical sciences is the *definition by postulation*, exemplified by the characterization of the undefined elements of geometry by the axioms and postulates from which all the properties and mutual relations of geometric quantities may be deduced. The practical measure of time has been carried on since remote ages, independently of attempted definitions of time or of prevailing philosophical concepts of the nature of time. An essential distinction exists between a definition of time and a definition of a measure of time.

Not only is it unnecessary to define time or to analyze its metaphysical nature in order to measure it, but moreover a system of practical time measurement for the purposes of the physical sciences need not necessarily conform to the criteria that distinguish the ideal uniform scale of measure which is more or less vaguely conceived intuitively. The *essential* characteristic of measurement is the establishment of a unique correspondence between physical magnitudes of a given kind and abstract numbers, by means of a procedure which compares any given magnitude with an adopted unit and in this way always associates a definite number, the measure, with the magnitude. The unit may be any arbitrary magnitude that can be reproduced or identified in some definite manner as occasion demands in effecting the comparison by the prescribed procedure. Ordinarily it is tacitly assumed that only an *invariable* quantity may serve as a unit, i.e., that quantities which on different occasions are identified as of unit magnitude, according to the accepted practice, are actually all equivalent; the scale of measure then generated by successive multiples of the unit is described as *uniform*. However, even though the quantity used as a unit be variable, a process of comparison in which this quantity is conventionally always assigned the *constant measure unity* still associates a measure with every given magnitude. Moreover the concept of uniformity implicitly presupposes an ultimate criterion independent of ordinary measurement. The unit may itself be referred to a different and independent system of measurement, and may or may not be found to have a constant measure in this system, but of course this second system is always subject to the same question.

No means is, in fact, available by which an ultimate test against an *absolute* standard of reference may be made; in practice, the invariability of a unit and the uniformity of the corresponding scale of measure may be established with certainty only relative to another conventional scale, not with reference to an absolute scale, and in general a logically satisfactory system of measurement can be constructed without necessitating the actual use of any scale that is assumed to be an ideal uniform measure in the absolute sense. The measurement of time is an especially good illustration of these principles.

Any feasible procedure by which a given interval of time may always be compared with an adopted standard interval establishes a correspondence between instants of time and abstract numbers, and constitutes a system of time measurement, irrespective of how closely it may represent a measure of the ideal *uniform flow* of time that is intuitively conceived from experience of the physical world. The only requirement that a method of measurement must fulfill is that it be adequate for the needs it is desired to meet; and throughout history, the systems actually used in practice have been determined by current needs and available facilities.

A practical need for some method of indicating and reckoning time has existed at all stages in the development of civilization. Among primitive people in the earliest ages, very simple and rough methods sufficed, but modern scientific systems are only an elaboration and refinement of the same procedures, and remain the same in principle: The fundamental purpose of establishing a numerical time-scale is to order and correlate events and occurrences in a more or less exact way, by relating them in a determinate manner to time as a common basis of reference. In particular, in the physical sciences the primary purpose is to obtain precise descriptions of natural phenomena, expressed in terms of this scale, by *formulating natural laws in mathematical form with the measure of time as the independent variable*. For this purpose, the relations of the adopted measure of time to the intuitively conceived uniform scale of metaphysical time are of secondary importance; the foremost consideration is the practicability and convenience of correlating phenomena with the adopted scale in an effective manner. As intuitive concepts of time develop, the formulation of a system of time measurement is guided by the natural desire to realize in practice, as closely as possible, the ideal uniform scale of absolute time; but only a physically defined scale, logically independent of this conceptual ultimate standard, can actually be effected, and is all that is required.

The physical problem is therefore to (1) define an observable scale of time, and (2) correlate natural phenomena with the measure of time on this scale by devising methods of determining from observation the points on the scale at which given events occur and by formulating natural laws in terms of this measure. A realizable scale of time is most readily defined by adopting some *recurring phenomenon* as a *standard of reference*. A measure of time may be defined by taking as a unit the duration of any recurrent phenomenon; e.g., the apparent diurnal revolution of the Sun, the cycle of lunar phases, the revolution of the Earth around the Sun, or artificially produced phenomena such as the vibrations of a pendulum. An indefinite continuous sequence of successive recurrences in one-to-one correspondence with the integral numbers generates a scale of time. The apparent motions of the celestial bodies, particularly the motions that are related to the recurrence of day and night and to the cycle of the seasons, are a natural basis for the measurement of time in this manner, and they have always spontaneously been used for the purpose.

In primitive ages, celestial phenomena were used merely for roughly marking off the passage of time in a purely empirical manner, as a guide in practical activities. For the purpose of nothing more than a reckoning of time by the lapse of successive intervals, any recurrent phenomenon whatever, whether natural or artificial, could be used, provided effective means for counting its recurrences were available; but the measures of time defined by different phenomena are not all equally well adapted for use as an independent variable in terms of which to describe all other phenomena. For modern scientific purposes, a satisfactory system can be established only on a critical and comprehensive theoretical basis; and it can be put into practice only by means of refined and complex instruments. However, the choice among the scales defined by different phenomena is necessarily based only on their relative ability to meet current needs, and their practicability and convenience. Any actual system is entirely conventional; no means is available for determining with certainty whether it is identical with the ideal concept of uniform absolute time, and for the practical purposes of measurement this question is immaterial. A measure defined by a selected observational procedure must be adopted as a conventional standard, and any other measure may be rated against only this standard.

In fact, one of the *most natural* measures of time—the system, in common use among the ancients, in which the recurrent periods of daylight and darkness are the primary units and are each separately always divided into 12 "*seasonal hours*"—obviously does not correspond to intuitive ideas of a uniform measure; and it gives quite a different scale from "*equinoctial time*," the other most important ancient system, in which the complete period of day and night as a whole was divided always into 24 parts. Relative to the equinoctial scale, the magnitudes of the seasonal hours vary widely with latitude and season, and are different from day to night; and in terms of equinoctial hours, the length of the day varies greatly, although in seasonal measure day and night are each always and everywhere exactly 12 hours long. The progress of the hours, in either system, could be indicated by the positions of the Sun or the stars in their diurnal circuits, although the imperfect means then available prevented much accuracy.

To use the primitive measure of seasonal time in the systematic development of physical science would lead to practically prohibitive complexities; and the seasonal hours dropped out of use for astronomical purposes comparatively early in history, although they remained in popular use until after the introduction of mechanical clocks during the Middle Ages. On the other hand, from the ancient point of view the introduction of "constant" equinoctial hours in place of the more natural seasonal hours likewise introduced the complication of an intricate variability in the measures of the lengths of day and night. The necessity for relating the diurnal motion of the Sun to latitude and season in order to express the varying length of the day from sunrise to sunset in equinoctial measure was perhaps originally one of the principal incentives to investigate the difficult problem of constructing a theory of the apparent annual motion of the Sun among the stars.

The traditional practice of measuring time by the apparent motions of the celestial bodies may now be based on an exact dynamical foundation by adopting as a primary standard the measure of time implicitly defined by the laws of motion. In the form in which these laws are usually stated, they appear to presuppose an independent measure of absolute time; but if the motion of a particle under no forces be adopted as the reference phenomenon that defines the scale of time, then Newton's First Law, the Law of Inertia, becomes in effect the definition of the dynamical standard of time. By hypothesis, equal rectilinear displacements of a particle in motion under no forces then correspond to equal intervals of time. In the terminology of the traditional formulation of the foundations of dynamics in terms of intuitive concepts, the independent variable of the accepted dynamical equations of motion is *uniform time* measured in the invariable unit which by the law of inertia would be determined by successive equal rectilinear displacements of a particle moving under no forces. Since no absolute standard of comparison is attainable, a uniform measure necessarily is uniform only by definition; and for the purposes of the physical sciences, the measure of time defined by the laws of motion may therefore be conventionally considered as uniform.

In the physical sciences, the term *uniform time* must be understood in only this conventional sense. By uniform variation is meant any variation equicrescent with this dynamical measure of time t, i.e., a variation in which equal increments correspond always to equal increments in t, or in which any increment is proportional to the interval of t. Any unit interval in terms of which a measure of time equicrescent with the dynamical measure is expressed is an *invariable* unit of time.

In order to be practicable for actual use, a measure of time must be accessible to determination by observation. The motion of a particle under no forces is an abstraction inaccessible to direct observation. The actual realization of the measure defined by the laws of motion depends upon correlating this measure with an intermediary empirical standard defined by an observable physical motion, by means of a rigorous dynamical theory of this motion. The mathematical theory of any dynamical phenomenon constructed from the laws of motion is ipso facto expressed in terms of the measure of time defined by the motion of a particle under no forces. Abstractly, uniform time is by definition the independent variable of the equations of motion; operationally, uniform time is a measure in terms of which observed dynamical phenomena agree with dynamical theory—in particular, a measure of time in which the observed motions of celestial bodies are in agreement with rigorous gravitational theories of these motions. In practice, the astronomical measures of time are defined and empirically determined by the apparent motions of the Sun, Moon, and stars; the relations of these empirical measures to uniform time are determined from the dynamical theories of the actual motions which the apparent motions reflect.

As to the philosophical problems involved in the fundamental concepts of force and mass, and the difficulties introduced by the unknown absolute motion of the coordinate frames to which observed motions are referred, much the same considerations apply as already discussed in the case of the concept of absolute time. The laws of motion are empirical postulates, inferred from experience of time and motion; their formulation is guided by ordinary intuitive concepts, and involves more or less idealization of actual observations, but essentially they are equivalent to the hypothesis that an appropriate system of concrete procedures of measurement can be found that will determine definite quantities, in terms of which natural phenomena may be represented in accordance with these laws of motion, to within the limits of observation. The practical difficulties in actually realizing these measurements were clearly recognized by Newton, as shown by his discussion of time, space, and motion in the *Principia;* and the possibility of satisfactorily correlating phenomena on this basis must be tested by experiment and observation. Observed phenomena represented in terms of the dynamical measure of time may be compared with their theoretical representations in terms of the independent variable of the laws of motion; the appearance of any discrepancies between theory and observation that cannot reasonably be accounted for by the inevitable errors of observation, or removed by appropriate corrections to adopted numerical parameters, is evidence of inconsistencies among the hypotheses.

Discrepancies of this nature are commonly interpreted as attributable to some defect in the application of the laws of motion to the phenomena, e.g., the neglect of forces that were not recognized to be acting; and if the dynamical theory of the standard reference motion itself is deficient, the relation of the empirical measure of time to the dynamical measure must be corrected. However, it should also be recognized that in order finally to remove all discrepancies which may appear, and obtain a completely consistent representation of natural phenomena, a modification of the laws of motion themselves may conceivably be required. The theory of relativity may be regarded as a more precise formulation of the laws of motion than Newton's formulation, which has the effect of reconciling outstanding differences between theory and observation; and it is not impossible that still further refinements may ultimately be needed.*

The physical motion which has been the basis in the past for the astronomical measurement of time is the rotation of the Earth. The apparent diurnal motions from which sidereal time and mean solar time are derived are principally a reflection of this rotation. Sidereal time, defined by the diurnal motion of the equinox, is obtained in practice from observations of the diurnal motions of stars. Mean solar time, defined in principle by the average rate of the apparent diurnal motion of the Sun, is determined in practice from a conventional relation to the observed sidereal time; the mean solar time obtained from this relation is a numerical measure of the time defined by the rotation of the Earth.

These empirical measures are still the basis of practical timekeeping; but because of small variations which occur in the rate of rotation of the Earth when this rate is measured in terms of the dynamical scale of time, mean solar time is not perfectly uniform. The inequalities due to the

* For the relations of the variables in relativistic planetary theory to the practical astronomical measures of time and length, see G. C. McVittie, Remarks on planetary theory in general relativity. *Astr. Jour.* **63**, 448–452 (1958).

variations in rotation are comparatively slight, but for many scientific and technological purposes they are of practical significance; and over very long intervals their accumulated effects may become of great importance. Accordingly, the measure of time defined by the rotational motion of the Earth has been superseded as the fundamental astronomical standard by a uniform measure which is defined in terms of the orbital motion of the Earth and is called *Ephemeris Time.*

Ephemeris Time is the numerical measure of uniform time that is the independent variable in the gravitational theory of the orbital motion of the Earth; more specifically, it is the argument of Newcomb's *Tables of the Sun.* In principle, it is determined from the apparent annual motion of the Sun which reflects the orbital motion of the Earth. A gravitational ephemeris expresses the position of the Sun as a function of Ephemeris Time, and at any instant the measure of Ephemeris Time is the value of the argument at which the ephemeris position is the same as the actual position at the instant. That is, the measure of time is determined by the inverse relation expressing the time as a function of position, and this relation is the practical means of determining its numerical value. The Ephemeris Time at any instant is obtained from observation by directly comparing observed positions of the Sun, Moon, and planets with gravitational ephemerides calculated from theories expressed in the same numerical measure of uniform time as Newcomb's theory of the Sun; in practice, observations of the Moon are the most effective and expeditious for this purpose.

The variations in the rate of rotation of the Earth cannot be completely determined from theory; consequently, a uniform measure of time cannot be obtained from observations of the apparent diurnal motions, as these motions cannot be expressed in terms of uniform time by means of a dynamical theory. The determination of the variations depends upon comparing the *observed* motions of celestial bodies in terms of *mean solar time* with the *theoretical* gravitational motions in terms of *uniform time.* The practical determination of Ephemeris Time takes the form of determining the correction that must be applied to mean solar time in order to obtain Ephemeris Time.

The determination of sidereal time by observing meridian transits of stars is very literally equivalent to using the rotating Earth as a standard clock. The celestial sphere serves as a dial, and selected stars distributed around the sphere are the graduation marks on the dial. The meridian, moving around the sphere as the plane of the local meridian rotates in space with the rotating Earth, is the clock hand. The celestial clock is read as the meridian passes each star. The numerical reading at any instant is the apparent right ascension of the star which the meridian is passing; this reading is the angular distance of the star eastward from the equinox, or equivalently the hour angle of the equinox which is the measure of sidereal time. The reading of the celestial clock dial determines the correction required to the reading of an artificial clock in order to make the artificial timepiece agree with the measure of time determined by the rotating Earth.

Analogously, for determining uniform time the standard clock is the Sun in its orbital motion, instead of the rotating Earth. In principle, the orbit is the clock dial, graduated in angular measure that represents orbital longitude, and the Sun moving around the orbit is the clock hand. The measure of time at any instant is determined by the orbital position of the Sun, instead of by the position of the meridian plane in its diurnal circuit, and is represented by the apparent position of the Sun among the stars, instead of by position in hour angle. The observed position determines the correction required to the mean solar time of the observation in order to obtain Ephemeris Time.

An essential auxiliary in precise timekeeping is a clock which runs at as nearly a uniform rate as possible. A clock controlled by an oscillating electrical circuit in which the frequency is determined by the vibrations of atoms or molecules is considered to run at a perfectly uniform rate, and to define a standard of uniform time, as long as it remains in a gravitational field of constant intensity. Variations which are large enough to be of practical importance are caused by changes in height above sea level, or in latitude, and by variations in the gravitational attractions of the Sun and other celestial bodies as their distances vary. In principle, these variations may be calculated, and corrections for them applied to an atomic clock. However, it is not yet known with certainty whether the measure of time defined by atomic processes is rigorously uniform relative to the dynamical measure defined by gravitational motions, or whether over sufficiently long intervals the two measures diverge. Moreover, an atomic clock provides only a standard of *frequency*; it determines a unit of time, but not the continuous count of units that is necessary to determine the interval elapsed since any initial epoch in the past. Astronomical time determinations are essential for defining an epoch and referring instants of time to it, as no artificial clock can be indefinitely sustained in continuous operation in the manner of the celestial motions.

It has been plausibly suggested that an atomic resonator, if provided with a suitable counting device, would keep the so-called proper time of the moving observer according to the theory of relativity, and that Ephemeris Time is identical with the so-called coordinate time.

As in all physical measurements, the numerical value obtained by observation for a theoretically defined measure of time is inevitably liable to errors of observation and errors due to imperfections of available methods of determination. For example, sidereal time and the mean solar time derived from it are subject to errors of the adopted star positions. Ephemeris Time is, by definition, rigorously uniform; but as determined in practice from observations of the Moon, it is liable to secular or long-period inequalities due to imperfections in the adopted theory of the motion of the Moon. However, these defects do not invalidate identifying the observed measure with an actual realization of the theoretical definition.